Neurocomputing 125 (2014) 142–147

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Second-order consensus of multi-agent systems with nonlinear dynamics via impulsive control

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ARTICLE INFO

Article history:
Received 21 April 2012
Received in revised form
21 August 2012
Accepted 1 October 2012
Available online 28 February 2013

Keywords:
Second-order consensus
Multi-agent systems
Impulsive control
Virtual leader

ABSTRACT

In many real-world multi-agent systems, the intrinsic dynamics of velocity for each agent is usually nonlinear dynamic rather than static. Moreover, it is often difficult to obtain the continuous velocity information of multi-agent systems. To overcome the above essential difficulties, this paper aims at investigating the second-order consensus problem of multi-agents systems with nonlinear dynamics by using impulsive control signal protocol. In detail, by using the impulsive signals from agents and virtual leaders, several impulsive control protocols are designed for reaching the second-order consensus of multi-agent systems with fixed or switching topologies. The theoretical analysis is also given to guarantee the second-order consensus based on algebraic graph theory and stability theory of impulsive differential equations. Finally, two typical examples are used to validate the above developed theoretical results.

1. Introduction

Over the last decade, the consensus problem for multi-agent systems has attracted increasing attention due to its extensive applications in real-world distributed computation, rendezvous tasks, flocking, swarming, biological systems, sensor networks, and so on [2,11,18–20].

In many cooperative multi-agent systems, a group of agents usually share local information with their neighbors so as to reach agreement on some certain states asymptotically or in finite time. Many existing literatures focus on the first-order consensus [1,4,6,15,26,31]. Recently, the second-order consensus has received increasing attention due to various real-world applications [5,7,8,14,16], where all the agents are governed by second-order dynamics, such as the position and velocity states. Note that there are few results reported on second-order consensus with time-varying velocities. Moreover, in many real-world multi-agent systems, the agents usually have a time-varying intrinsic velocities rather than constant, even after a velocity consensus has been reached. During the process of consensus, each agent will automatically adjust its own dynamics based on the information of its neighbors or some leaders. However, it is very difficult for the agents to obtain the continuous velocity information. Therefore, it is very important to further investigate second-order consensus of multi-agent systems with time-varying intrinsic velocities by using some suitable techniques.

The impulsive phenomena are ubiquitous in nature. It is well known that the impulsive control is very effective, robust, and lost-cost. Over the last few decades, it has been widely applied into the consensus and synchronization problems of various complex networks [3,13,20,23–25,28,30,31]. In [12], several fundamental criteria were introduced for the impulsive synchronization of complex dynamical systems. It indicates that the impulsive synchronization depends on the eigenvalues and eigenvectors of corresponding coupling matrices. Following this line, some basic synchronous or consensus criteria were established for various complex systems via impulsive control approaches, such as the multi-agent systems with switching topology [22]. In [29,32,33], it is shown that the consensus algorithms based on the first-order impulsive control have the much faster convergence speeds than the standard consensus algorithms [21,25]. However, the conditions for impulsive synchronization or consensus are rather conservative based on algebraic graph theory and stochastic matrix theory [9,10]. This is because the impulsive intervals are often very narrow and the impulsive gains usually have very strict constraints. In [27], Lu and his colleagues investigated the synchronization of impulsive dynamical networks using the concept of “average impulsive interval”.

This paper aims to further investigate the second-order consensus problem of multi-agents systems with nonlinear dynamics by using impulsive control signal protocol. Several fundamental consensus criteria are obtained based on algebraic graph theory and stability theory of impulsive differential equations by designing the suitable impulsive control protocols.

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http://dx.doi.org/10.1016/j.neucom.2012.10.027

This paper is organized as follows. Some basic preliminaries are introduced in Section 2. Section 3 further investigates the second-order consensus of multi-agent systems with fixed or switching topologies. Numerical examples are given to show the effectiveness of theoretical results in Section 4. Finally, the concluding remarks are also given in Section 5.

2. Preliminaries

In this section, an undirected graph will be used to characterize the multi-agent systems with N agents. To begin with, some basic concepts and lemmas on algebraic graph theory [17] are briefly introduced.

The following notations are used throughout the paper. The superscripts "T" means the transpose of a matrix, $\otimes$ denotes the Kronecker product of matrices, $\|\cdot\|$ indicates the Euclidean norm, I_N (O_N) is the N-dimensional identity (zero) matrix, $1_N(0_N)$ denotes the N-dimensional column vector whose elements are all ones (zeros), and $\mathbb{N}_+$ denotes the set of positive integers.

Let $\mathcal{G}=(\mathcal{V},\mathcal{E},A)$ be a weighted undirected graph with order N, $\mathcal{V}=\{v_1,v_2,\dots,v_N\}$ be the set of all nodes, and $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ be the set of all edges. $A=[a_{ij}]_{N\times N}$ is the adjacency matrix with nonnegative elements a_{ij}, where $a_{ii}=0$ for $i=1,2,\dots,N$. The undirected edge is denoted by $e_{ij}=(v_i,v_j)$, which means that node i and node j of the graph can exchange information with each other. The neighboring set of node i is defined by $N_i=\{v_j\in\mathcal{V}:(v_j,v_i)\in\mathcal{E}\}$, where the edge e_{ij} exists if and only if $a_{ij}=a_{ji}>0$. A path of length r from i to j is defined by a sequence of edges $(v_i,v_{i_1}),(v_{i_1},v_{i_2}),\dots,(v_{i_{r-1}},v_j)$. An undirected graph $\mathcal{G}$ is connected if and only if there exists an undirected path between any two vertices in $\mathcal{G}$.

The Laplacian matrix $L=(l_{ij})_{N\times N}$ of graph $\mathcal{G}$ is defined by

$$l_{ij}=\begin{cases}-a_{ij}, & i\neq j,\\ -\sum\limits_{j=1,j\neq i}^{N} l_{ij}, & i=j.\end{cases}$$

It means that all the row sums of L are zero, that is, $1_N=[1,1,\dots,1]^{\mathrm{T}}\in R^N$ is a right eigenvector of L associated with the eigenvalue $\lambda=0$.

Lemma 1 (Horn and Johnson [9]). *The Laplacian matrix L of an undirected graph $\mathcal{G}$ is semi-positive definite. It has a simple zero eigenvalue and all the other eigenvalues are positive if and only if the graph $\mathcal{G}$ is connected, that is, all the eigenvalues of L satisfy $0=\lambda_1(L)<\lambda_2(L)\leq\cdots\lambda_N(L)$.*

Lemma 2 (Horn and Johnson [9]). *The second smallest eigenvalue $\lambda_2(L)$ of the Laplacian matrix L of an undirected graph $\mathcal{G}$ satisfies*

$$\lambda_2(L)=\min_{x^{\mathrm{T}}1_N=0,x\neq 0_N}\frac{x^{\mathrm{T}}Lx}{x^{\mathrm{T}}x}. \tag{1}$$

Lemma 3 (Horn and Johnson [10]). *For matrices A, B, C, and D with appropriate dimensions, the Kronecker product $\otimes$ has the following properties*:

$$\begin{aligned}&(1)\quad(\alpha A)\otimes B=A\otimes(\alpha B),\\&(2)\quad(A+B)\otimes C=A\otimes C+B\otimes C,\\&(3)\quad(A\otimes B)(C\otimes D)=(AC)\otimes(BD),\\&(4)\quad(A\otimes B)^{\mathrm{T}}=A^{\mathrm{T}}\otimes B^{\mathrm{T}}.\end{aligned}\tag{2}$$

3. Main results

The main results are given in this section.

The general second-order consensus protocol is described by

$$\begin{aligned}&\dot{x}_i(t)=v_i(t),\\&\dot{v}_i(t)=u_i(t),\\&u_i(t)=\alpha\sum_{j\in N_i}^{N}a_{ij}(x_j(t)-x_i(t))+\beta\sum_{j\in N_i}^{N}a_{ij}(v_j(t)-v_i(t)),\end{aligned}\tag{3}$$

where $x_i\in R^n$ and $v_i\in R^n$ are the position and velocity of the ith agent, respectively. α and β are the coupling strengths. $A=[a_{ij}]_{N\times N}$ is the symmetric adjacency matrix characterizing the topology structure of undirected network.

Definition 1. The multi-agent system (3) is said to achieve second-order consensus, if for any initial conditions

$$\lim_{t\to\infty}\|x_j(t)-x_i(t)\|=0,\quad \lim_{t\to\infty}\|v_j(t)-v_i(t)\|=0,\quad \forall i,\ j=1,2\dots,N. \tag{4}$$

As we know now, the second-order consensus can be reached if the coupling strengths and spectra of Laplacian matrix satisfy some suitable conditions. However, for the real-world multi-agent systems, the intrinsic dynamics of velocity for each agent is often nonlinear. Furthermore, it is much more difficult to obtain the continuous velocity information compared with the position information. To cope with the above essential difficulty, an impulsive control technique is introduced, where each agent can update its position and velocity states at impulsive instants. Therefore, the multi-agent system with impulsive control signals is described by

$$\begin{cases}\dot{x}_i(t)=v_i(t),\\ \dot{v}_i(t)=f(v_i,t)+\alpha\sum\limits_{j\in N_i}a_{ij}(x_j(t)-x_i(t)),\ t\neq t_k,\\ x_j(t_k^+)-x_i(t_k^+)=B_k(x_j(t_k^-)-x_i(t_k^-)),\\ v_j(t_k^+)-v_i(t_k^+)=B_k(v_j(t_k^-)-v_i(t_k^-)),\\ \forall i,\ j=1,2\dots,N,\end{cases}\tag{5}$$

where $f(v_i,t)$ is a nonlinear continuously differentiable vector-valued function and B_k is the impulsive gain at time t_k. Assume that $x_i(t)$, $v_i(t)$ are left continuous at t_k. That is, $x_i(t_k)=x_i(t_k^-)$ and $v_i(t_k)=v_i(t_k^-)$. The time sequence $\{t_k\}$ satisfies $0\leq t_0<t_1<\cdots<t_k<t_{k+1}<\cdots$ and $\lim_{k\to+\infty}t_k=+\infty$ with $\tau_k=t_{k+1}-t_k$.

Remark 1. It should be pointed out that the impulsive signal does not have any effect on the consensus of multi-agent systems for $\|B_k\|=1$. When $\|B_k\|<1$, the impulsive signal has positive effect on the consensus. However, it has negative effect on the consensus for $\|B_k\|>1$. In the following, some sufficient conditions will be derived for the consensus of multi-agent systems via impulsive control.

Remark 2. According to (5), for any impulsive signals $\tilde{x}_k$ and $\tilde{v}_k$, one gets $x_i(t_k^+)-\tilde{x}_k=B_k(x_i(t_k^-)-\tilde{x}_k)$ and $v_i(t_k^+)-\tilde{v}_k=B_k(v_i(t_k^-)-\tilde{v}_k)$. That is, $x_i(t_k^+)=B_kx_i(t_k^-)+(I_n-B_k)\tilde{x}_k$ and $v_i(t_k^+)=B_kv_i(t_k^-)+(I_n-B_k)\tilde{v}_k$. Therefore, for each agent, the corresponding position and velocity can be adjusted at any impulsive instant. Moreover, the impulsive signals are not necessary to be identical at different impulsive instants.

Proposition 1. *There exists a nonnegative constant θ satisfying*

$$\|f(v,t)-f(\tilde{v},t)\|\leq\theta\|v-\tilde{v}\|\quad\forall v,\tilde{v}\in R^n.$$

Remark 3. It is easy to verify that some classical chaotic systems satisfy the above assumption, including Lorenz system, Chen system, Lü system, Chua's circuit, and so on.