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Anti-periodic solutions for shunting inhibitory cellular neural networks with time-varying delays in leakage terms

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ABSTRACT

This paper is concerned with the existence and exponential stability of anti-periodic solutions for shunting inhibitory cellular neural networks (SICNNs) with time-varying delays in the leakage terms. Some sufficient conditions are established to ensure that all solutions of the networks converge exponentially to the anti-periodic solution, which are new and complement of previously known results.

1. Introduction

Since Bouzerdoum and Pinter in [1–3] described SICNNs as a new cellular neural networks (CNNs), SICNNs have been extensively applied in psychophysics, speech, perception, robotics, adaptive pattern recognition, vision, and image processing. Moreover, it is well known that SICNNs can be analog voltage transmission, which is often an anti-periodic process. Hence, there have been extensive results on the problem of the anti-periodic solutions of SICNNs with time-varying delays and continuously distributed delays in the literature. We refer the reader to [4–9] and the references cited therein. Recently, considerable effort has been devoted to derive the sufficient conditions on the stability and periodicity of neural networks with constant delays in the leakage (or "forgetting") term (see, e.g., [10–14] and the references therein). In contrast, however, very few results are available on a generic, in-depth, existence and exponential stability of anti-periodic solutions for SICNNs with time-varying delays in the leakage terms. Thus, it is worthwhile to continue to investigate the existence and stability of anti-periodic solutions of SICNNs (1.1) in this case. In this present paper, we shall consider the following SICNNs with time-varying delays in the leakage terms given by

$$x'_{ij}(t) = -a_{ij}(t)x_{ij}(t-\delta_{ij}(t)) - \sum_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t) f(x_{kl}(t-\tau_{kl}(t)))x_{ij}(t) - \sum_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(t)\cdot \int_0^{\infty} K_{ij}(u) g(x_{kl}(t-u))\,du\, x_{ij}(t) + L_{ij}(t),$$
$$i=1,2,\ldots,m, j=1,2,\ldots,n, \tag{1.1}$$

where C_{ij} denotes the cell at the (i,j) position of the lattice. The r-neighborhood $N_r(i,j)$ is given as

$$N_r(i,j) = \{C_{kl} : \max(|k-i|,|l-j|) \le r, 1\le k\le m, 1\le l\le n\},$$

$N_q(i,j)$ is similarly specified. x_{ij} is the activity of the cell C_{ij}, $L_{ij}(t)$ is the external input to C_{ij}, $a_{ij}(t) > 0$ represents the passive decay rate of the cell activity, $C_{ij}^{kl}(t) \ge 0$ and $B_{ij}^{kl}(t) \ge 0$ are the connections or coupling strengths of postsynaptic activity of the cells in $N_r(i,j)$ and $N_q(i,j)$ transmitted to cell C_{ij} depending upon variable delays and continuously distributed delays, respectively, and the activity functions $f(\cdot)$ and $g(\cdot)$ are continuous functions representing the output or firing rate of the cell C_{kl}, $\delta_{ij}(t) \ge 0$ and $\tau_{kl}(t) \ge 0$ correspond to the transmission delays.

The main purpose of this paper is to give the new criteria for the convergence behavior for all solutions of SICNNs (1.1). By applying some new mathematical analysis techniques, we establish some sufficient conditions ensuring that all solutions of SICNNs (1.1) converge exponentially to the zero point, which are new and complement of previously known results. Moreover,

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an example is also provided to illustrate the effectiveness of our results.

Let $u(t): R \longrightarrow R$ be continuous in t. $u(t)$ is said to be T-anti-periodic on R if,

$$u(t+T) = -u(t) \quad \text{for all } t \in R.$$

Throughout this paper, for $ij \in \Lambda = \{11, 12, \ldots, 1n, \ldots, m1, m2, \ldots, mn\}$, it will be assumed that $K_{ij} : [0, +\infty) \to R$ is a continuous function, $|K_{ij}(t)|e^{\kappa t}$ is integrable on $[0, +\infty)$ for some positive constant κ, $t - \delta_{ij}(t) \geq 0$ for all $t \geq 0$, $a_{ij}, \tau_{ij}, C_{ij}$, B_{ij} and δ_{ij} continuous periodic functions with period T,

$$f(-u) = f(u), \quad g(-u) = g(u), \quad L_{ij}(t+T) = -L_{ij}(t) \quad \text{for all } t, u \in R, \tag{1.2}$$

and

$$\overline{B_{ij}^{kl}} = \sup_{t \in R} B_{ij}^{kl}(t), \quad \overline{C_{ij}^{kl}} = \sup_{t \in R} C_{ij}^{kl}(t), \quad a_{ij}^{+} = \sup_{t \in R} a_{ij}(t),$$
$$\delta_{ij}^{+} = \sup_{t \in R} \delta_{ij}(t), \quad L_{ij}^{+} = \sup_{t \in R} |L_{ij}(t)|. \tag{1.3}$$

Set

$$\{x_{ij}(t)\} = (x_{11}(t), \ldots, x_{1n}(t), \ldots, x_{i1}(t), \ldots, x_{in}(t), \ldots, x_{m1}(t), \ldots, x_{mn}(t)) \in R^{m \times n}.$$

We also assume that the following conditions (T_1) and (T_2) hold.

(T_1) There exist constants M_f, M_g, μ_f and μ_g such that

$$|f(u) - f(v)| \leq \mu_f |u - v|, \quad |f(u)| \leq M_f, \quad |g(u) - g(v)| \leq \mu_g |u - v|,$$
$$|g(u)| \leq M_g \quad \text{for all } u, v \in R.$$

(T_2) There exist constants $\eta_{ij} > 0$ such that

$$-\eta_{ij} = \sup_{t \in R} \left\{ -[a_{ij}(t) - a_{ij}(t)\delta_{ij}(t)a_{ij}^{+}] + \sum_{C_{kl} \in N_r(i,j)} [C_{ij}^{kl}(t) + \overline{C_{ij}^{kl}} a_{ij}(t)\delta_{ij}(t)] M_f \right.$$
$$\left. + \sum_{C_{kl} \in N_q(i,j)} [B_{ij}^{kl}(t) + \overline{B_{ij}^{kl}} a_{ij}(t)\delta_{ij}(t)] \int_0^{\infty} |K_{ij}(u)| \, du \, M_g \right\}, \quad ij \in \Lambda \tag{1.4}$$

and

$$-[a_{ij}(t) - a_{ij}(t)\delta_{ij}(t)a_{ij}^{+}] + \sum_{C_{kl} \in N_r(i,j)} [C_{ij}^{kl}(t) + \overline{C_{ij}^{kl}} a_{ij}(t)\delta_{ij}(t)] M_f$$
$$+ \sum_{C_{kl} \in N_r(i,j)} [C_{ij}^{kl}(t) + \overline{C_{ij}^{kl}} a_{ij}(t)\delta_{ij}(t)] \mu_f \frac{(a_{ij}^{+}\delta_{ij}^{+} + 1)L_{ij}^{+}}{\eta_{ij}}$$
$$+ \sum_{C_{kl} \in N_q(i,j)} (B_{ij}^{kl}(t) + \overline{B_{ij}^{kl}} a_{ij}(t)\delta_{ij}(t)) \left(\int_0^{\infty} |K_{ij}(u)| \, du \, M_g \right.$$
$$\left. + \int_0^{\infty} |K_{ij}(u)| \, du \, \mu_g \frac{(a_{ij}^{+}\delta_{ij}^{+} + 1)L_{ij}^{+}}{\eta_{ij}} \right)$$
$$< 0 \quad \text{for all } t > 0, ij \in \Lambda. \tag{1.5}$$

The initial conditions associated with system (1.1) are of the form

$$x_{ij}(s) = \varphi_{ij}(s), \quad s \in (-\infty, 0], \; ij \in \Lambda, \tag{1.6}$$

where $\varphi_{ij}(\cdot)$ denotes real-valued bounded continuous function defined on $(-\infty, 0]$.

2. Preliminary results

The following lemmas will be used to prove our main results in Section 3.

Lemma 2.1. *Let* (T_1) *and* (T_2) *hold. Suppose that* $x(t) = \{x_{ij}(t)\}$ *is a solution of system* (1.1) *with initial conditions*

$$x_{ij}(s) = \varphi_{ij}(s), \; s \in (-\infty, 0], \qquad \sup_{t \in (-\infty, 0]} |\varphi_{ij}(t)| \leq \frac{(a_{ij}^{+}\delta_{ij}^{+} + 1)L_{ij}^{+}}{\eta_{ij}}. \tag{2.1}$$

Then

$$|x_{ij}(t)| \leq \frac{(a_{ij}^{+}\delta_{ij}^{+} + 1)L_{ij}^{+}}{\eta_{ij}} \quad \textit{for all } t \geq 0, \; ij \in \Lambda. \tag{2.2}$$

Proof. Assume, by way of contradiction, that (2.2) does not hold. Then, there exist $ij \in \Lambda$, $\gamma > (a_{ij}^{+}\delta_{ij}^{+} + 1)L_{ij}^{+}/\eta_{ij}$ and $\rho > 0$ such that

$$|x_{ij}(\rho)| = \gamma \quad \text{and} \quad |x_{ij}(t)| < \gamma \quad \text{for all } t \in (-\infty, \rho). \tag{2.3}$$

According to (1.1), we get

$$x_{ij}'(t) = -a_{ij}(t)x_{ij}(t) + a_{ij}(t)[x_{ij}(t) - x_{ij}(t - \delta_{ij}(t))]$$
$$- \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(t) f(x_{kl}(t - \tau_{kl}(t))) x_{ij}(t)$$
$$- \sum_{C_{kl} \in N_q(i,j)} B_{ij}^{kl}(t) \int_0^{\infty} K_{ij}(u) g(x_{kl}(t-u)) \, du \, x_{ij}(t) + L_{ij}(t)$$
$$= -a_{ij}(t)x_{ij}(t) + a_{ij}(t) \int_{t - \delta_{ij}(t)}^{t} x_{ij}'(s) \, ds$$
$$- \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(t) f(x_{kl}(t - \tau_{kl}(t))) x_{ij}(t)$$
$$- \sum_{C_{kl} \in N_q(i,j)} B_{ij}^{kl}(t) \int_0^{\infty} K_{ij}(u) g(x_{kl}(t-u)) \, du \, x_{ij}(t) + L_{ij}(t),$$
$$ij \in \Lambda. \tag{2.4}$$

Calculating the upper left derivative of $|x_{ij}(t)|$, together with (T_1), (T_2), (2.3) and (2.4) , we can obtain

$$0 \leq D^{-}(|x_{ij}(\rho)|)$$
$$\leq -a_{ij}(\rho)|x_{ij}(\rho)| + \left| a_{ij}(\rho) \int_{\rho - \delta_{ij}(\rho)}^{\rho} x_{ij}'(s) \, ds \right.$$
$$- \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(\rho) f(x_{kl}(\rho - \tau_{kl}(\rho))) x_{ij}(\rho)$$
$$\left. - \sum_{C_{kl} \in N_q(i,j)} B_{ij}^{kl}(\rho) \int_0^{\infty} K_{ij}(u) g(x_{kl}(\rho - u)) \, du \, x_{ij}(\rho) + L_{ij}(\rho) \right|$$
$$\leq -a_{ij}(\rho)|x_{ij}(\rho)| + a_{ij}(\rho) \int_{\rho - \delta_{ij}(\rho)}^{\rho} \Big| -a_{ij}(s)x_{ij}(s - \delta_{ij}(s))$$
$$- \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(s) f(x_{kl}(s - \tau_{kl}(s))) x_{ij}(s)$$
$$- \sum_{C_{kl} \in N_q(i,j)} B_{ij}^{kl}(s) \int_0^{\infty} K_{ij}(u) g(x_{kl}(s-u)) \, du \, x_{ij}(s) + L_{ij}(s) \Big| \, ds$$
$$+ \sum_{C_{kl} \in N_r(i,j)} C_{ij}^{kl}(\rho) M_f |x_{ij}(\rho)| + \sum_{C_{kl} \in N_q(i,j)} B_{ij}^{kl}(\rho)$$
$$\times \int_0^{\infty} |K_{ij}(u)| \, du \, M_g |x_{ij}(\rho)| + |L_{ij}(\rho)|$$
$$\leq -[a_{ij}(\rho) - a_{ij}(\rho)\delta_{ij}(\rho)a_{ij}^{+}]|x_{ij}(\rho)|$$
$$+ \sum_{C_{kl} \in N_r(i,j)} [C_{ij}^{kl}(\rho) + \overline{C_{ij}^{kl}} a_{ij}(\rho)\delta_{ij}(\rho)] M_f |x_{ij}(\rho)|$$
$$+ \sum_{C_{kl} \in N_q(i,j)} [B_{ij}^{kl}(\rho) + \overline{B_{ij}^{kl}} a_{ij}(\rho)\delta_{ij}(\rho)] \int_0^{\infty} |K_{ij}(u)| \, du \, M_g |x_{ij}(\rho)|$$
$$+ (a_{ij}^{+}\delta_{ij}^{+} + 1)L_{ij}^{+}$$
$$= \left\{ -[a_{ij}(\rho) - a_{ij}(\rho)\delta_{ij}(\rho)a_{ij}^{+}] + \sum_{C_{kl} \in N_r(i,j)} [C_{ij}^{kl}(\rho) + \overline{C_{ij}^{kl}} a_{ij}(\rho)\delta_{ij}(\rho)] M_f \right.$$