Neurocomputing 111 (2013) 195–203

Contents lists available at SciVerse ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Letters

Further stability criteria on discrete-time delayed neural networks with distributed delay

Ting Wang [a,*], Chun Zhang [a], Shumin Fei [a], Tao Li [b,c]

[a] Key Laboratory of Measurement and Control of CSE (School of Automation, Southeast University), Ministry of Education, Nanjing 210096, China
[b] School of Automation Engineering, Nanjing University of Aeronautics and Astronautics, Nanjing 210016, China
[c] Zhangjiagang Research Institute of Smart Grid, Zhangjiagang 215600, China

ARTICLE INFO

Article history:
Received 12 August 2012
Received in revised form
27 October 2012
Accepted 13 December 2012
Communicated by H. Zhang
Available online 31 January 2013

Keywords:
Discrete-time case
Delayed neural networks (DNNs)
Robust exponential stability
Time-varying delay
LMI approach

ABSTRACT

In this letter, together with some novel Lyapunov–Krasovskii functional (LKF) terms and effective techniques, two novel sufficient conditions can be established to guarantee a class of discrete-time delayed neural networks with distributed delay to be exponentially stable, in which the linear fractional uncertainties are involved and the information on time-delay is fully utilized. Through employing the reciprocal convex technique, some previously ignored terms can be reconsidered when estimating the time difference of LKF and the criteria are presented via linear matrix inequalities (LMIs), whose solvability heavily depends on the information of addressed systems. Finally, three numerical examples are provided to show that the achieved conditions can be less conservative than some existing ones based on comparing results.

1. Introduction

In recent years, neural networks have been applied to various signal processing problems, such as optimization, image processing, associative memory design, etc. In those applications, the key feature of the designed neural network keeps to be convergent. Meanwhile, since there inevitably exists the communication delay which is the main source of oscillation and instability, great efforts have been imposed on delayed neural networks (DNNs) and many elegant results have been reported [1–11,13–34]. Meanwhile, in practical applications, though it is difficult to describe the accurate form of time-delay, the bounds on its range and time variation still can be measured. Since Lyapunov functional approach presented some simple and delay-independent results, the Lyapunov–Krasovskii functional (LKF) method has been widely utilized due to that its analytic procedure can fully cover and utilize the information on time-delay. Thus in past decade, the delay-dependent stability for DNNs has become an important topic of primary significance, in which the main purpose is to derive the time-delay maximum allowable upper bound (MAUB) such that the DNNs keep to be globally stable in different ways [1–11,13–17,19–22,24–34].

In order to implement the continuous-time systems for simulation or computational purposes, it is important to formulate discrete-time systems which are the analogues of the continuous-time ones. It is often to obtain the discrete-time models including neural networks from the continuous-time ones by using a discretization technique. Ideally, the discrete-time analogue should inherit the dynamical behaviors of the continuous-time networks, and maintain functional similarity to the continuous-time ones. Unfortunately, the discretization cannot always preserve the similar dynamics of the continuous-time counterpart even for a small sampling period [12]. Presently, since discrete-time neural networks have already been applied in many fields, such as image processing, quadratic optimization problems, system identification, many elegant results have been reported to study their dynamical behaviors [13–33]. Through using various effective techniques such as free-weighting matrix, LMI, and summation inequality, some easy-to-test results have been given for discrete-time DNNs, in which not only time-varying delay but also distributed delay was involved [13–20]. In [21], the discrete-time BAM DNNs were considered and the restriction $1<\tau(k+1)<1+\tau(k)$ was imposed on time-delay. By utilizing Razumikhin technique [22,23], the stability for impulsive DNNs has been tackled and some results were given via nonlinear matrix inequalities. Meanwhile, some authors have focused on the stability for discrete-time switched DNNs by means of average dwell time method and LMI results were presented [24–26], in which time-delays are variable or constant. Together with Markovian jumping parameters, some sufficient conditions on robust stability were presented in the forms of LMIs [27–29]. In [30–32], by using the stochastic process theory and convex technique, the mean-squared stability was also studied and some

* Corresponding author.
E-mail address: chunchun1010@163.com (T. Wang).

0925-2312/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.neucom.2012.12.017

delay-dependent criteria were obtained. Furthermore, since the delay-partitioning idea has been verified to be very effective in re- ducing the conservatism yet induced the complexities of the derived result [34], which was also improved and extended to discrete-time case [33,35]. Though in [13–33,35], these derived results and techniques were elegant, there still exist two points waiting for the improvements. First, most recently, because the convex technique was a good tool in tackling time-delay system, it was also widely employed in discrete DNNs [19,24,25,27,31,33]. Yet, this technique still needs further improvement owing to that some important terms were ignored during estimating time difference of LKF, which was illustrated in [36]. Second, the constructions of LKFs in [14–17,19–33] seemed simple and they cannot fully cover the information on addressed networks. On the other hand, as for delay $\tau(t)\in[0,\tau_m]$, since the triple integration LKF terms such as

$$\frac{\tau_m^2}{2}\int_{-\tau_m}^{0}\int_{\lambda}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)Q\dot{x}(s)\,ds\,d\theta\,d\lambda$$

were first chosen in continuous-time case [37], it also has been used to tackle continuous-time DNNs since it also could reduce the conservatism greatly [10,11]. Yet, it is worth noting that few present literatures have utilized and extended the triple integration LKF term to tackle the discrete-time DNNs. Meanwhile, we noticed that the analytic techniques need to be improved since the works [10,11] gave the less tighter upper bounds on time difference of the LKFs, which will limit the application areas. Overall, two points above remain challenging and constitute the main focus of this work.

In this work, we will study the robust exponential stability for the uncertain discrete-time DNNs with unbounded distributed delay, in which the state time-delay belongs to an interval and the linear fractional uncertainties are involved. Through constructing one improved Lyapunov–Krasovskii functional and utilizing some most updated techniques for achieving the delay-dependence, two novel conditions can be established. Especially, the derived criteria are presented in terms of LMIs and their feasibility can be easily checked. Finally, three numerical examples show that the proposed conditions are less conservative than some existent ones.

Notations. For the symmetric matrices $X,Y,X>Y$ (respectively, $X\geq Y$) means that $X-Y>0$ $(X-Y\geq 0)$ is a positive-definite (respectively, positive-semidefinite) matrix; $\lambda_{\max}(A)$, $\lambda_{\min}(A)$ denote the maximum eigenvalue and minimum one of matrix A, respectively; I_n denotes the $n\times n$ identity matrix and $0_{m\times n}$ means the $m\times n$ zero matrix; the symmetric term in a symmetric matrix is denoted by $*$.

2. Model descriptions and preliminaries

In this letter, we consider the uncertain discrete-time delayed neural networks (DNNs) described by

$$z(k+1)=C(k)z(k)+A(k)F(z(k))+B(k)F(z(k-\tau(k)))+D(k)\sum_{i=1}^{+\infty}\delta(i)F(z(k-i))+L, \tag{1}$$

for $k=1,2,\ldots,$ where $z(k)=[z_1(k),\ldots,z_n(k)]^T\in\mathbf{R}^n$ is the neural state vector, $F(z(\cdot))=[F_1(z_1(\cdot)),\ldots,F_n(z_n(\cdot))]^T$ represents the neuron activation function, $L=[l_1,\ldots,l_n]^T\in\mathbf{R}^n$ is a constant input vector; $C(k)=C+\Delta C(k)$, $A(k)=A+\Delta A(k)$, $B(k)=B+\Delta B(k)$, and $D(k)=D+\Delta D(k)$ are the uncertain matrices of appropriate dimensions, in which $C=\mathrm{diag}(c_1,\ldots,c_n)$ is a diagonal matrix with $|c_i|<1$ for $i=1,\ldots,n$.

The following assumptions and definition are made throughout the letter.

Assumption 1. The time-varying delay $\tau(k)$ satisfies the condition

$$\tau_0\leq\tau(k)\leq\tau_m, \tag{2}$$

in which τ_0,τ_m are known positive integers. Here we denote $\overline{\tau}_m=\tau_m-\tau_0$.

Assumption 2. For $i=1,2,\ldots,n$, each activation function $F_i(\cdot)$ in (1) satisfies the following condition:

$$\sigma_i^-\leq\frac{F_i(x)-F_i(y)}{x-y}\leq\sigma_i^+,\quad\forall x,\ y\in\mathbf{R},\ x\neq y,$$

and σ_i^+,σ_i^- are any constants. We also introduce the denotations as $\Sigma=\mathrm{diag}(\sigma_1^-,\ldots,\sigma_n^-)$, and

$$\Sigma_1=\mathrm{diag}(\sigma_1^+\sigma_1^-,\ldots,\sigma_n^+\sigma_n^-),\quad\Sigma_2=\mathrm{diag}\left(\frac{\sigma_1^++\sigma_1^-}{2},\ldots,\frac{\sigma_n^++\sigma_n^-}{2}\right). \tag{3}$$

Assumption 3. Here $\Delta C(k)$, $\Delta A(k)$, $\Delta B(k)$, $\Delta D(k)$ represent the time-varying parameter uncertainties and are assumed to satisfy

$$[\Delta C(k)\ \ \Delta A(k)\ \ \Delta B(k)\ \ \Delta D(k)]=H\varDelta(k)[E_c\ \ E_a\ \ E_b\ \ E_d],$$
$$\varDelta(k)=\Lambda(k)(I-J\Lambda(k))^{-1},\quad I-J^TJ>0, \tag{4}$$

in which H, J, E_c, E_a, E_b, E_d are known constant matrices of the appropriate dimensions and $\Lambda(k)$ is an unknown time-varying matrix function satisfying $\Lambda^T(k)\Lambda(k)\leq I$.

Assumption 4. The function $\delta(i)$ is a real-valued non-negative function defined on $i\in\mathbf{Z}^+$, and there exist two constant scalars $\xi>0,\mu>1$ such that

$$\sum_{i=1}^{+\infty}\delta(i)=\xi<+\infty,\quad\sum_{i=1}^{+\infty}\delta(i)i\mu^i=\pi(\mu)<+\infty.$$

Remark 1. From Assumption 3, it is easy to see that the above structured linear fractional form includes the widely used norm-bounded uncertainty as its special case when $J=0$.

It is clear that under Assumptions 1–4 and Proposition in [16], system (1) has one equilibrium point denoted by $z^*=[z_1^*,\ldots,z_n^*]^T$. In the following, the equilibrium point z^* of the system (1) is first shifted to the origin by the transformation $x(\cdot)=z(\cdot)-z^*$, which can convert the system (1) to the following form:

$$x(k+1)=C(k)x(k)+A(k)f(x(k))+B(k)f(x(k-\tau(k)))+D(k)\sum_{i=1}^{+\infty}\delta(i)f(x(k-i)), \tag{5}$$

where $x(k)=[x_1(k),\ldots,x_n(k)]^T$ is the state vector of transformed system (5); $f(x(\cdot))=[f_1(x_1(\cdot)),\ldots,f_n(x_n(\cdot))]^T$, $f_j(x_j(\cdot))=F_j(x_j(\cdot)+z_j^*)-F_j(z_j^*)$, $j=1,\ldots,n$. Note that the function $f_j(\cdot)$ still satisfies the condition

$$\sigma_j^-\leq\frac{f_j(x)-f_j(y)}{x-y}\leq\sigma_j^+,\quad\forall x,\ y\in\mathbf{R},\ x\neq 0. \tag{6}$$

It is easy to check $f_j(0)=0$.

Definition 1 (*Chen [13]*). The delayed neural networks (5) is said to be robustly exponentially stable, if there exist scalars $\alpha>0$ and $\beta\in(0,1)$ such that $\|x(k)\|\leq\alpha\cdot\beta^k\max_{-\infty\leq j\leq 0}\|x(j)\|$ for all $k\geq 0$ and the parameter uncertainties satisfying the admissible condition (4), where $x(k)$ is the solution of model (5) and $\|x(k)\|=[\sum_{i=1}^n x_i^2(k)]^{1/2}$ is the Euclidean norm of $x(k)$.

The problem to be addressed in next section can be formulated as developing a condition ensuring that the discrete-time delayed neural networks (5) is robustly exponentially stable.

3. Delay-dependent stability for discrete-time DNNs

In the section, based on LMI and reciprocal convex approach, a delay-dependent criterion will be first derived for the global