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Global dynamics of equilibrium point for delayed competitive neural networks with different time scales and discontinuous activations ☆

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ABSTRACT

In this paper, we investigate the global dynamics of equilibrium point for delayed competitive neural networks with different time scales and discontinuous activations. Employing the Leray–Schauder alternative theorem in multivalued analysis, linear matrix inequality technique and generalized Lyapunov-like method, we obtain some new sufficient conditions ensuring the existence, uniqueness and global stability and the global convergence in finite time of the equilibrium point, the results of this paper improve and extend previously known results. Finally, two examples and numerical simulations are conducted to validate the theoretical results.

1. Introduction

In recent years, various neural networks such as Hopfield, cellular, Cohen–Grossberg, and bidirectional associative memory neural networks have been extensively studied, and also have been successfully applied to many fields such as image processing, parallel computing, optimization, associative memory, see [1–8] and the references therein. In these applications, the properties of stability and convergence of the equilibrium point are important in design and application of these neural networks and many researchers have done extensive works on this subject in the literature (see [9–11] and the references therein). Moreover, due to the finite switching speed of the neuron amplifiers and the finite speed of signal propagation, delays are actually unavoidable in the electronic implementation. It is well known, in both biological and artificial neural networks, that the delay is a potential cause of the loss of stability, since it may originate the onset of nonvanishing oscillations [12–15]. Therefore, the study of neural dynamics with consideration of the delayed problem becomes extremely important to manufacture high-quality neural networks.

Over the past decades, the neural network models considered in the dynamics analysis in most papers are the ones with single time scale, which means that in these models only the neuron activity is taken into consideration, there exists only one type of variables, that is, the state variables of the neural neurons. However, in a dynamical neural network, the synaptic weights also vary with respect to time due to the learning process, the variation of connection weights may have influence on the dynamics of neural networks, and so competitive neural networks with different scales introduced by Meyer-Bäse et al. in [16–18], in which the dynamics of neuron states are governed by a set of differential equations as in the usual neural networks. In addition to dynamics of neuron states, connection weights also vary with time under the Hebbian learning law, whose dynamics are described by another set of differential equations. In this model, there are two types of state variables that of the short-term memory (STM) variables describing the fast neural activity and that of the long-term memory (LTM) variables describing the slow unsupervised synaptic modifications. Thus, there are two time scales in these neural networks, in which one corresponds to the fast changes of the neural network states and another corresponds to the slow changes of the synapses by external stimuli. The general neural network equations describing the temporal evolution of the STM and LTM states for the ith neuron of a n-neuron network are

$$\begin{cases} \text{STM}: \varepsilon\dfrac{dx_i(t)}{dt} = -a_i x_i(t) + \sum\limits_{j=1}^{n} D_{ij} f_j(x_j(t)) + \sum\limits_{j=1}^{n} D_{ij}^{\tau} f_j(x_j(t-\tau(t))) + B_i \sum\limits_{k=1}^{P} m_{ik}(t) y_k, \\ \text{LTM}: \dfrac{dm_{ik}(t)}{dt} = -m_{ik}(t) + y_k f_i(x_i(t)), \quad i=1,2,\dots,n, \quad k=1,2,\dots,P, \end{cases} \tag{1.1}$$

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where $x_i>0$ represents the neuron current activity level, $a_i>0$ is a positive function representing the time constant of the neuron, $f_j(x_j(t))$ is the output of the neuron, $m_{ik}(t)$ is the synaptic efficiency, y_k is the constant external stimulus, D_{ij} represents the connection weight between the ith neuron and the jth neuron, and D^{τ}_{ij} denotes the synaptic weight of delayed feedback and B_i is the strength of the external stimulus, $\varepsilon>0$ is the fast time scale of STM state. Recently, considerable effort has been devoted to study the dynamical behaviors of competitive neural networks with different scales, one may refer to [11,19–25] and the references therein.

However, all of the above works were based on the assumption that the activation functions are continuous, even globally Lipschitz. As pointed out by Forti and Nistri in [26,27], a brief review of some common neural network models reveals that neural networks with discontinuous activations are important and frequently arise in practice. Furthermore, the analysis of the ideal discontinuous case is able to reveal crucial features of the dynamics, such as the presence of sliding modes along discontinuity surfaces, the phenomenon of convergence in finite time toward the equilibrium point and the ability to compute the exact global minimum of the underlying energy function, which make these networks especially attractive for the solution of global optimization problems in real world. Thus, considerable attention has been paid on the study of discontinuous neural networks theory and a large body of work has been reported in the literature. In [28], for example, a class of nonsmooth gradient-like systems was considered, by virtue of Lyapunov function and topological degree theory, they investigated the dynamical behaviors of this system. In [29], the authors use the concept of the Filippov solution to study the almost periodic dynamics of a class of delayed dynamical systems with discontinuous right-hand side, we refer to [30–40] and the references therein for other interesting works on neural networks with discontinuous activations.

In addition, according to Nie and Cao [41], when activation functions do not satisfy continuity, we do not know whether the global solution and an equilibrium point of neural networks are existent. Therefore, it is more difficult to study the dynamics of the system (1.1) with discontinuous right-hand side. On the other hand, to the best of our knowledge, a fewer results have been obtained on the global dynamics of equilibrium point for delayed competitive neural networks with different time scales and discontinuous activations.

Motivated by the above discussion, the main contribution of this paper is to investigate the existence, global asymptotic stability and convergence in finite time, global exponential stability of delayed competitive neural networks with different time scales and discontinuous activations. Our analysis is based on the fixed point theorem of differential inclusion theory, linear matrix inequality technique and Lyapunov functional method. The results obtained in the present paper improve and extend previous works in the literature to some extent.

The structure of this paper is outlined as follows. Section 2 discusses the neural network model studied in this paper and presents some preliminaries related to our main results. Section 3 presents the main results on the dynamical behavior for system (2.2). Section 4 gives two examples to demonstrate the effectiveness of the main results. Finally, the study in this paper is concluded in Section 5.

Notations: Given the column vector $x=(x_1,x_2,\dots,x_n)^T$, in which the superscript T denotes the transpose of vector, $\|x\|$ is the Euclidean vector norm, i.e., $\|x\|=(\sum_{i=1}^n x_i^2))^{1/2}$. By $x>0$ (respectively, $x\geq 0$) we mean $x_i>0$ (respectively, $x_i\geq 0$) for all $i=1,2,\dots,n$.

For any matrix A, A^T and A^{-1} denote the transpose and the inverse of A, respectively. If A is a symmetric matrix, $A>0 (A\geq 0)$ means that A is positive definite (nonnegative definite). Similarly, $A<0 (A\leq 0)$ means that A is negative definite (negative semidefinite). $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ represent the minimum and maximum eigenvalues of matrix A, respectively. We also write $\|A\|_2$ to denote the 2-norm of A, i.e., $\|A\|_2=\sqrt{\rho(A^TA)}$, where $\rho(A^TA)$ denotes the spectral radius of A^TA. Finally, E denotes the identity matrix.

Given a set $\Omega\subset\mathbb{R}^n$, $K[\Omega]$ denotes the closure of the convex hull of Ω, $P_{kc}(\Omega)$ denotes the collection of all nonempty, compact and convex subset of Ω.

2. Preliminaries

In this section, we present some definitions and lemmas which will be used throughout the paper.

Firstly, we shall simplify system (1.1) as follows:

Setting

$$S_i(t)=\sum_{k=1}^{P} m_{ik}(t)y_k=\mathbf{y}^T\mathbf{m}_i(t),$$

where

$$\mathbf{y}=(y_1,\dots,y_P)^T,\quad \mathbf{m}_i(t)=(m_{i1}(t),m_{i2}(t),\dots,m_{iP}(t))^T,$$

and summing up the LTM over k, the neural networks (1.1) can be rewritten as the state-space form

$$\begin{cases}\text{STM}:\varepsilon\dfrac{dx_i(t)}{dt}=-a_ix_i(t)+\sum\limits_{j=1}^{n}D_{ij}f_j(x_j(t))+\sum\limits_{j=1}^{n}D^{\tau}_{ij}f_j(x_j(t-\tau(t)))+B_iS_i(t),\\ \text{LTM}:\dfrac{dS_i(t)}{dt}=-S_i(t)+\|\mathbf{y}\|^2f_i(x_i(t)),\quad i=1,2,\dots,n,\end{cases}\tag{2.1}$$

where $\|\mathbf{y}\|^2=y_1^2+y_2^2+\cdots+y_P^2$ is the constant. Without loss of generality, the input stimulus $\mathbf{y}$ is assumed to be normalized with unit magnitude $\|\mathbf{y}\|^2=1$, then the above neural networks are simplified as

$$\begin{cases}\text{STM}:\varepsilon\dfrac{dx_i(t)}{dt}=-a_ix_i(t)+\sum\limits_{j=1}^{n}D_{ij}f_j(x_j(t))+\sum\limits_{j=1}^{n}D^{\tau}_{ij}f_j(x_j(t-\tau(t)))+B_iS_i(t),\\ \text{LTM}:\dfrac{dS_i(t)}{dt}=-S_i(t)+f_i(x_i(t)),\quad i=1,2,\dots,n,\end{cases}$$

or equivalently the vector form

$$\begin{cases}\text{STM}:\varepsilon\dfrac{dx(t)}{dt}=-Ax(t)+Df(x(t))+D^{\tau}f(x(t-\tau(t)))+BS(t),\\ \text{LTM}:\dfrac{dS(t)}{dt}=-S(t)+f(x(t)),\end{cases}\tag{2.2}$$

where $x(t)=(x_1(t),x_2(t),\dots,x_n(t))^T$, $S(t)=(S_1(t),S_2(t),\dots,S_n(t))^T$, $A=\text{diag}(a_1,a_2,\dots,a_n)$, $f(x(t))=(f_1(x_1(t)),f_2(x_2(t)),\dots,f_n(x_n(t)))^T$, $f(x(t-\tau(t)))=(f_1(x_1(t-\tau(t))),f_2(x_2(t-\tau(t))),\dots,f_n(x_n(t-\tau(t))))^T$, $D=(D_{ij})_{n\times n}$, $D^{\tau}=(D^{\tau}_{ij})_{n\times n}$, $B=\text{diag}(B_1,B_2,\dots,B_n)$ and the time-varying delay $\tau(t)$ is differentiable function satisfying

$$0\leq\tau(t)\leq\tau,\quad \dot{\tau}(t)\leq\tau^*<1,\tag{2.3}$$

where τ and τ^* are positive constants.

Definition 2.1 (*Class $\mathcal{F}$ of functions*). We call $f\in\mathcal{F}$, if for all $i=1,2,\dots,n$, $f_i(\cdot)$ satisfies: f_i is continuous except a countable set of points ρ^i_k, where the right and left limits $f_i(\rho^{i+}_k)$ and $f_i(\rho^{i-}_k)$ satisfy $f_i(\rho^{i+}_k)>f_i(\rho^{i-}_k), k=1,2,\dots$. Moreover, f_i has only finite discontinuous points on every compact set of $\mathbb{R}$.

Remark 2.1. If $f\in\mathcal{F}$, then we have

$$K[f(x)]=([f_1(x_1^-),f_1(x_1^+)],[f_2(x_2^-),f_2(x_2^+)],\dots,[f_n(x_n^-),f_n(x_n^+)])^T.$$

Definition 2.2 (*Class $\mathcal{G}$ of functions*). For $f\in\mathcal{F}$, we say $f\in\mathcal{G}$, if $f(\cdot)$ satisfies the growth condition (g.c.), i.e., there exist nonnegative