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Letters

Comparison principle and stability of stochastic delayed neural networks with Markovian switching ☆

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ABSTRACT

This paper deals with the stability issue for a class of stochastic delayed neural networks with Markovian switching. The jumping parameters are determined by a continuous-time, discrete-state Markov chain. Different from the usual Lyapunov–Krasovskii functional and linear matrix inequality method, we first introduce and study a new comparison principle in the field of stochastic delayed neural networks. Then, we apply this new comparison principle to obtain several novel stability criteria of the suggested system. Moreover, an example is given to illustrate the theoretical results well.

1. Introduction

As is well known, an artificial neural network consists of an interconnected group of artificial neurons, and it processes information depending on a computation approach. During the past decades, artificial neural networks received a great deal of attention owing to the fact that neural networks can be applied to many areas, such as robotics, aerospace, associative memory, pattern recognition, signal processing, automatic control engineering, fault diagnosis, telecommunications, parallel computation and combinatorial optimization. Such applications depend on the existence and uniqueness of equilibrium points and the qualitative properties of stability, and so the stability analysis is important in the practical design and applications of neural networks. As a consequence, a large number of results have appeared in the literature, see e.g. [1–32] and references therein.

On the other hand, noises and delays are two main factors of affecting the stability of neural networks. In fact, a real nervous system is usually affected by the external perturbation which in many cases is of great uncertainty and hence may be treated as random. Also, time delays are unavoidable in neural networks because of various reasons such as the finite switching speed of amplifiers in circuit implementation of a neural network. Moreover, time delays could change a network from stable to unstable. Hence, noises and delays should be taken into consideration in modeling neural networks. As in [22–32], neural networks with noises and delays are called stochastic delayed neural networks.

With the rapid development of the social and economic changes, there often appears the phenomenon of information latching, and the abrupt phenomena such as random failures or repairs of the components, sudden environmental changes, changing subsystem interconnections. To cope with such phenomena, a class of stochastic delayed neural networks with Markovian switching has been recognized to be the best system to model them. Generally speaking, this class of neural networks is a hybrid system with a state vector that has two components $x(t)$ and $r(t)$, where $x(t)$ denotes the state and $r(t)$ is a continuous-time Markov chain with a finite state space $S=\{1,2,\ldots,N\}$, which is usually regarded as the mode. In its operation, this class of neural networks will switch from one mode to another in a random way, which is determined by a continuous-time Markov chain $r(t)$. Therefore, it is significant and challenging to investigate the stability of stochastic delayed neural networks with Markovian switching.

Recently, a large amount of results on the stability of stochastic delayed neural networks with Markovian switching have been

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reported in the literature, for instance, see [25–32] and references therein. In [25], Balasubramaniam and Rakkiyappan discussed the globally asymptotical stability problem for a class of Markovian jumping stochastic Cohen–Grossberg neural networks with discrete interval and distributed delays; Zhu and Cao studied the exponential stability for several new classes of Markovian jump stochastic neural networks with mixed time delays with/without impulse control in [26–28], and they investigated the asymptotical stability for a class of stochastic neural networks of neutral type with both Markovian switching and mixed time delays in [29]. In [30], Zhang et al. dealt with the asymptotical stability analysis of neutral-type impulsive neural networks with mixed time-varying delays and Markovian switching. It should be mentioned that the methods and techniques used in the previous literature (see [25–32]) mainly depended on the Lyapunov–Krasovskii functional and linear matrix inequality method, and the comparison principle had not been applied in the previous literature. This situation motivates our present research.

Inspired by the above discussion, in this paper we investigate the stability issue for a class of stochastic delayed neural networks with Markovian switching. Without using the usual Lyapunov–Krasovskii functional and linear matrix inequality method, instead, we first introduce and study a new comparison principle in the field of stochastic delayed neural networks. Then, we apply this new comparison principle to obtain several novel stability criteria of the suggested system. Moreover, an example is given to illustrate the theoretical results well.

The rest of the paper is arranged as follows. In Section 2, we introduce the model of stochastic delayed neural networks, notations and several definitions. Section 3 presents our main results: we first obtain a new comparison principle, and then apply this new comparison principle to obtain several novel stability criteria of the suggested system, which includes the stability in probability, pth moment stability and pth moment exponential stability. In Section 4, an example is provided to illustrate the effectiveness of the proposed results. Finally, we conclude the paper with some general remarks in Section 5.

2. Model description and problem formulation

Throughout this paper, unless otherwise specified, let $(\Omega, \mathcal{F}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\ge 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathcal{F}_t$ contains all P-null sets). Let $w(t)=(w_1(t),\ldots,w_m(t))^T$ be an m-dimensional Brownian motion defined on the probability space. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote the m-dimensional Euclidean space and the set of all $n\times m$ real matrix, respectively. $\mathbb{R}_+=[0,\infty)$ and the superscript "T" denotes the transpose of a matrix or vector. Let $|\cdot|$ denote the Euclidean norm in $\mathbb{R}^n$. If A is a vector or matrix, its transpose is denoted by A^T. If A is a matrix, its trace norm is denoted by $|A|=\sqrt{\mathrm{tr}(A^TA)}$ while its operator norm is denoted by $\|A\|=\sup\{|Ax|: |x|=1\}$. If A is a symmetric matrix, we use $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ to denote its largest and smallest eigenvalue, respectively.

Let $\{r(t), t\ge 0\}$ be a right-continuous Markovian chain on the probability space taking values in a finite state space $S=\{1,2,\ldots,N\}$ with generator $Q=(Q_{ij})_{N\times N}$ given by

$$P\{r(t+\Delta)=j|r(t)=i\}=\begin{cases} q_{ij}\Delta t+o(\Delta) & \text{if } i\neq j\\ 1+q_{ij}\Delta+o(\Delta) & \text{if } i=j,\end{cases}$$

where $\Delta>0$. Here, $q_{ij}\ge 0$ is the transition rate from i to j if $i\neq j$ while $q_{ii}=-\sum_{j\neq i}q_{ij}$.

In this paper, we consider the following stochastic delayed neural network with Markovian switching:

$$dx(t)=[-C(r(t))x(t)+A(r(t))f(x(t))+B(r(t))g(x(t-\tau(t)))]\,dt \\ +\sigma(x(t),x(t-\tau(t)),t,r(t))\,dw(t). \quad (1)$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T$ is the state vector associated with the n neurons, the diagonal matrix $C(r(t))=diag(c_1(r(t)),c_2(r(t)),\ldots,c_n(r(t)))$ has positive entries $c_i(r(t))\ge 0$ $(i=1,2,\ldots,n)$. $A(r(t))$ denotes the feedback matrix, and $B(r(t))$ represents the delayed feedback matrix. Both $f(x(t))=[f_1(x_1(t)),f_2(x_2(t)),\ldots,f_n(x_n(t))]^T$ and $g(x(t))=[g_1(x_1(t)),g_2(x_2(t)),\ldots,g_n(x_n(t))]^T$ are the neuron activation functions. The noise perturbation $\sigma:\mathbb{R}^n\times\mathbb{R}^n\times\mathbb{R}_+\times S\to\mathbb{R}^{n\times m}$ is a Borel measurable function. $\tau(t)$ is the time-varying delay such that $0\le\tau(t)\le\tau$.

As usual, we assume that the Markovian chain $r(t)$ is independent of the Brownian motion $w(t)$. It is known that almost every sample path of $r(t)$ is a right-continuous step function with a finite number of $\mathbb{R}_+$. The matrices $C(r(t))$, $A(r(t))$ and $B(r(t))$ will be simply written by C_i, A_i and B_i, respectively, if $r(t)$ takes a value $i\in S$, then we can rewrite (1) as the following form:

$$dx(t)=[-C_ix(t)+A_if(x(t))+B_ig(x(t-\tau(t)))]\,dt \\ +\sigma(x(t),x(t-\tau(t)),t,i)\,dw(t). \quad (2)$$

Let $C([-\tau,0];\mathbb{R}^n)$ denote the family of continuous functions ϕ from $[-\tau,0]$ to $\mathbb{R}^n$ with the uniform norm $\|\phi\|=\sup_{-\tau\le\theta\le 0}|\phi(\theta)|$. For any $p>0$, denote by $L^p_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$ the family of all $\mathcal{F}_0$ measurable, $C([-\tau,0];\mathbb{R}^n)$-valued stochastic variables $\phi=\{\phi(s):-\tau\le s\le 0\}$ such that $\int_{-\tau}^0 \mathbf{E}|\phi(s)|^p\,ds<\infty$, where $\mathbf{E}[\cdot]$ stands for the correspondent expectation operator with respect to the given probability measure P.

The main aim of this paper is to establish a new comparison principle to study the stability of system (1) (or (2)). To this end, we assume that σ, f and g satisfy the local Lipschitz and linear growth conditions. Then, it is clear that for every initial data $x(s)=\phi(s)$ on $-\tau\le s\le 0$ in $L^p_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$, system (1) (or (2)) has a unique solution, which is denoted by $x(t;\phi)$. Furthermore, we assume that $f(0)=g(0)=0$, $\sigma(0,0,t,i)\equiv 0$ for all $i\in S$. Then system (1) (or (2)) admits a trivial solution or zero solution $x(t;0)\equiv 0$ corresponding to the initial data $\phi=0$.

Let $C^{2,1}(\mathbb{R}^n\times\mathbb{R}_+\times S;\mathbb{R}_+)$ denote the family of all nonnegative functions V on $\mathbb{R}^n\times\mathbb{R}_+\times S$ which is continuously twice differential in x and once differential in t, then we define an operator $\mathcal{L}V$ from $\mathbb{R}^n\times\mathbb{R}_+\times S$ to $\mathbb{R}$ by

$$\mathcal{L}V(x,t,i)=V_t(x,t,i)+V_x(x,t,i)[-C_ix+A_if(x)+B_ig(y)] \\ +\frac{1}{2}\mathrm{tr}[\sigma^T(x,y,t,i)V_{xx}(x,t,i)\sigma(x,y,t,i)]+\sum_{j=1}^N q_{ij}V(x,t,j),$$

where

$$V_t(x,t,i)=\frac{\partial V(x,t,i)}{\partial t},$$
$$V_x(x,t,i)=\left(\frac{\partial V(x,t,i)}{\partial x_1},\ldots,\frac{\partial V(x,t,i)}{\partial x_n}\right),$$
$$V_{xx}(x,t,i)=\left(\frac{\partial^2 V(x,t,i)}{\partial x_j\partial x_k}\right)_{n\times n}.$$

Definition 1. A function $\varphi(u)$ is said to belong to the class $\mathcal{K}$ if φ is a continuous function such that $\varphi(0)=0$ and $\varphi(u)$ is strictly increasing in u. A function is said to belong to the class $\mathcal{VK}$, if φ belongs to $\mathcal{K}$ and φ is convex.

Definition 2. The trivial solution of (1) (or (2)) is said to be

(1) Stable in probability, if for each $\varepsilon>0,\eta>0$, there exists $\delta=\delta(\varepsilon,\eta)>0$ such that $P\{\|x(t,\phi)\|\ge\varepsilon\}<\eta$, $t\ge-\tau$ when $\mathbf{E}(\|\phi\|)\le\delta$.