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New approaches on stability criteria for neural networks with two additive time-varying delay components

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ABSTRACT

In this paper, we study the stability problem for neural networks with two additive time-varying delay components. By constructing the Lyapunov–Krasovskii functional and considering the relationship between time-varying delays and their upper delay bounds, delay-dependent stability criteria are obtained by using reciprocally convex method and convex polyhedron method, respectively. More information of the lower and upper delay bounds of time-varying delays is used to derive the stability criteria, which can lead less conservative results. All the obtained criteria are in terms of Linear Matrix Inequalities (LMIs). Numerical examples are given to show the effectiveness and less conservativeness of the proposed method.

1. Introduction

Neural networks have attracted many researcher's attention during the past decades and have found successful applications in many engineering and scientific areas such as signal processing, pattern recognition, model identification and optimization problem. On the other hand, time delay is a common phenomenon that often occur in neural networks, and the existence of time delay can cause system's instability and sometimes degrade system's performance. Since stability is an important property to many systems, much effort has been done to the stability analysis problem for neural networks with time delay in recent years [1–19].

The stability criteria for delayed neural networks can be classified into two types: delay-independent stability criteria [1–4,6] and delay-dependent stability criteria [5,8–14,16,17]. It is generally known that the later one has less conservatism than the former one, especially when the time-delay is small. Much effort has been done to the research of delay-dependent stability problem for delayed neural networks, and various methods are utilized for getting less conservative results, such as free weighting matrix method [7,9,13], delay-decomposing method [15,18], augmented Lyapunov functional method [15,19], the consideration of more useful terms [13] and the use of new lower bounds lemma [19]. The literature that research the stability problem for delayed neural networks mentioned above mainly based on the following basic mathematical model:

$$\dot{z}(t) = -Cx(t) + Ag(x(t)) + Bg(x(t-d(t))) + u, \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T$ is the neural state vector, $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots, g_n(x_n(t))]^T$ denotes the neuron activation function with $g(0) = 0$, $C = \mathrm{diag}\{c_1, c_2, \ldots, c_n\} > 0$, A and B are the connection weight matrix and the delayed connection weight matrix, respectively, $u = [u_1, u_2, \ldots, u_n]^T$ is constant input vector, and $d(t)$ is a continuous time-varying function which satisfies

$$0 \le d(t) \le d, \tag{2}$$

$$\dot{d}(t) \le \mu, \tag{3}$$

where d and μ are constants.

The neuron activation functions $g_i(\cdot)$ in (1) are bounded and satisfy

$$0 \le \frac{g_i(x) - g_i(y)}{x - y} \le \sigma_i, \quad x \ne y, \ \ i = 1, 2, \ldots, n, \tag{4}$$

where σ_i $(i = 1, 2, \ldots, n)$ are positive constants.

In this model, the time delay in state $x(t)$ is assumed to appear in singular form. However, Zhao et al. [20] point out that in some practical situations, signals transmissions may experience a few segments of networks. Since the conditions of network transmission may be different, it can possibly induce successive delays with different properties. So the method of using models (1)–(4) to study the stability problem for such type of neural networks will get conservative results. In [20] the following model of neural

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networks with time-varying delays is proposed:

$$\dot{z}(t) = -Cx(t) + Ag(x(t)) + Bg(x(t-d_1(t)-d_2(t))) + u,$$

and it proposes a new type of delay-dependent stability criterion that depend on the upper bounds of time-varying delays $d_1(t)$ and $d_2(t)$. However, in [20] some integrals that appeared in the derivative of Lyapunov functional are over bounded, and this will lead to conservative results. For example, the term $-\int_{t-d_1}^{t} \dot{z}(s)Z_1\dot{z}(s)\,ds$ is enlarged as $-\int_{t-d_1(t)}^{t} \dot{z}(s)Z_1\dot{z}(s)\,ds$, the term $-\int_{t-d_1}^{t-d_1(t)} \dot{z}(s)Z_1\dot{z}(s)\,ds$ is omitted. By using a convex polyhedron method, Shao and Han [21] get stability criteria that have less conservatism than [20]. Tian and Zhong [22] do further research to this problem by constructing augmented Lyapunov functional and using reciprocally convex method which is proposed by Park et al. [23]. When researching this problem, we find that the existing results still have room for improvement, by using the relationship between the time-varying delays and their upper delay bounds, we can appropriate enlarge some terms appeared in the derivative of Lyapunov functional, and obtain less conservative delay-dependent stability criteria.

Enlightened by the above ideals, we investigate the stability problem for neural networks with two additive time-varying delay components in this paper. By considering the relationship between time-varying delays and their upper bounds, we enlarge the derivative of Lyapunov functional appropriately according to different value of time-varying delay $d(t)$, and get two new delay dependent stability criteria: one is obtained based on Lyapunov stability theory and reciprocally convex approach, the other is obtained by using free-weighting matrix method and convex polyhedron method. Since more information of the time-varying delays is used, our method can get less conservative results compared to some existing literature. All the obtained criteria are expressed in terms of LMIs that can be solved by using Matlab Toolbox. Finally, we give numerical examples to show the effectiveness of the obtained criteria.

Notation: In this paper, the notation $P>0$ (≥ 0) means that P is real symmetric and positive definite (semi-definite). The superscripts T denotes matrix transposition. The length of a line PQ is denoted by $|PQ|$. The notation $x \rightleftharpoons y$ means to swap the value of x and y. $\|\cdot\|$ stands for the Euclidean vector norm. A symmetric term in a symmetric matrix is denoted by $\star$, diag$\{\cdots\}$ stands for a block-diagonal matrix. I and $\mathbf{0}$ denote the identity matrix and zero matrix with proper dimensions, respectively.

2. Problem statement and preliminaries

Consider the following delayed neural network with two additive time-varying delay components:

$$\dot{z}(t) = -Cx(t) + Ag(x(t)) + Bg(x(t-d_1(t)-d_2(t))) + u, \tag{5}$$

where $d_1(t)$ and $d_2(t)$ are continuous time-varying functions that represent the two delay components in the state which satisfy

$$0 \leq d_1(t) \leq d_1, 0 \leq d_1(t) \leq d_2, \tag{6}$$

$$\dot{d}_1(t) \leq \mu_1, \dot{d}_2(t) \leq \mu_2, \tag{7}$$

where $d_i \geq 0$, $\mu_i \geq 0$ $(i=1,2)$ are constants, and we denote $d(t) = d_1(t) + d_2(t)$, $d = d_1 + d_2$, $\mu = \mu_1 + \mu_2$.

To simplify the model, we assume that x^* is an equilibrium point for system (5), by using the transformation $z(t) = x(t) - x^*$, (5) is converted to the following system:

$$\dot{z}(t) = -Cz(t) + Af(z(t)) + Bf(z(t-d_1(t)-d_2(t))), \tag{8}$$

where

$$z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T,$$
$$f(z(\cdot)) = [f_1(z_1(\cdot)), f_2(z_2(\cdot)), \ldots, f_n(z_n(\cdot))]^T,$$
$$f_i(z_i(\cdot)) = g_i(z_i(\cdot) + x_i^*) - g_i(x_i^*), \quad i = 1, 2, \ldots, n,$$

and according to the inequality (4) we can get

$$0 \leq \frac{f_i(z_i(t))}{z_i(t)} \leq \sigma_i, \quad f_i(0) = 0,\ i = 1, 2, \ldots, n. \tag{9}$$

Note that the origin $z(t)=0$ of (8) is stable means the equilibrium point $x(t)=x^*$ of the neural network (5) is asymptotically stable. In this paper, we use (8) to study the delay dependent stability problem for neural network (5) with time-varying delays satisfying (6) and (7) and neuron activation function satisfying (9). Before deriving our main results, the following lemmas are given.

Lemma 1 (*Gu [24]*). *For any constant matrix $P = P^T > 0$ and $0 \leq h_1 < h_2$ such that the following integrations are well defined, then*

$$-h_{12} \int_{t-h_2}^{t-h_1} x^T(s)Px(s)\,ds \leq -\left(\int_{t-h_2}^{t-h_1} x(s)\,ds\right)^T P \left(\int_{t-h_2}^{t-h_1} x(s)\,ds\right),$$

where $h_{12} = h_2 - h_1$.

Lemma 2 (*Park et al. [23]*). *For $k_i(t) \in [0,1]$, $\sum_{i=1}^{N} k_i(t) = 1$, and vectors η_i which satisfy $\eta_i = 0$ with $k_i(t) = 0$, matrices $R_i > 0$, there exist matrix S_{ij} $(i = 1, \ldots, N-1, j = i+1, \ldots, N)$, satisfies*

$$\begin{bmatrix} R_i & S_{ij} \\ \star & R_j \end{bmatrix} \geq 0,$$

such that the following inequality holds

$$\sum_{i=1}^{N} \frac{1}{k_i(t)} \eta_i^T R_i \eta_i \geq \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_N \end{bmatrix}^T \begin{bmatrix} R_1 & \cdots & S_{1,N} \\ \star & \ddots & \vdots \\ \star & \star & R_N \end{bmatrix} \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_N \end{bmatrix}.$$

Proof. For $i=2$, notice that the following inequality:

$$\begin{bmatrix} \sqrt{\frac{k_2(t)}{k_1(t)}}\eta_1 \\ -\sqrt{\frac{k_1(t)}{k_2(t)}}\eta_2 \end{bmatrix}^T \begin{bmatrix} R_1 & S_{12} \\ \star & R_2 \end{bmatrix} \begin{bmatrix} \sqrt{\frac{k_2(t)}{k_1(t)}}\eta_1 \\ -\sqrt{\frac{k_1(t)}{k_2(t)}}\eta_2 \end{bmatrix} \geq 0$$

always holds, then we have

$$\begin{aligned}
&\frac{1}{k_1(t)}\eta_1^T R_1 \eta_1 + \frac{1}{k_2(t)}\eta_2^T R_2 \eta_2 \\
&\quad = \frac{1}{k_1(t)}\eta_1^T (k_1(t)+k_2(t)) R_1 \eta_1 + \frac{1}{k_2(t)}\eta_2^T (k_1(t)+k_2(t)) R_2 \eta_2 \\
&\quad = \eta_1^T R_1 \eta_1 + \eta_2^T R_2 \eta_2 + \frac{k_2(t)}{k_1(t)}\eta_1^T R_1 \eta_1 + \frac{k_1(t)}{k_2(t)}\eta_2^T R_2 \eta_2 \\
&\quad \geq \eta_1^T R_1 \eta_1 + \eta_2^T R_2 \eta_2 + \eta_1^T S_{12} \eta_2 + \eta_2^T S_{12}^T \eta_1 \\
&\quad = \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}^T \begin{bmatrix} R_1 & S_{12} \\ \star & R_2 \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \end{bmatrix}.
\end{aligned}$$

The hold for $i=N$ is similar to proof and is omitted here.

Lemma 3. *Let $R \geq 0$ and $g(s)$ be appropriate dimensional symmetric matrix and vector, respectively. Then for scalar functions $f_1(t) \geq f_2(t)$, there exist positive matrix $M \geq 0$, matrix S and vector $\zeta(t)$ with appropriate dimensions, such that*

$$\begin{bmatrix} M & S \\ \star & R \end{bmatrix} \geq 0,$$