Neurocomputing 118 (2013) 237–244

Contents lists available at SciVerse ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

New passivity conditions with fewer slack variables for uncertain neural networks with mixed delays ☆

Cheng-De Zheng [a],*, Chao-Ke Gong [a], Zhanshan Wang [b]

[a] Department of Mathematics, Dalian Jiaotong University, 116028, China
[b] School of Information Science and Engineering, Northeastern University, 110004, China

ARTICLE INFO

Article history:
Received 27 September 2012
Received in revised form
14 December 2012
Accepted 13 February 2013
Communicated by D. Liu
Available online 3 April 2013

Keywords:
Passivity
Cellular neural networks
Augmented Lyapunov–Krasovskii functional (LKF)
Time-varying delays

ABSTRACT

This paper introduces an effective approach to studying the passivity of neutral-type neural networks with discrete and continuous distributed time-varying delays. By employing a novel Lyapunov–Krasovskii functional based on delay partitioning, several improved delay-dependent passivity conditions are established to guarantee the passivity of uncertain neural networks by applying the Jensen integral inequality. These criteria are expressed in the framework of linear matrix inequalities, which can be verified easily by means of standard Matlab software. One special case of the obtained criteria turns out to be equivalent to some existing result with same reduced conservatism but including fewer slack variables. As the present passivity conditions involve fewer free-weighting matrices, the computational burden is largely reduced. Three examples are provided to demonstrate the advantage of the theoretical results.

1. Introduction

Neural networks have been extensively studied over the past few decades and have found many applications in a variety of areas, such as signal processing, pattern recognition, static image processing, associative memory, and combinatorial optimization. Although considerable effort has been devoted to analyzing the stability of neural networks without a time delay, in recent years, the stability of delayed neural networks has also received attention [5,8,18–26,28] since time delay is frequently encountered in neural networks, and it is often a source of instability and oscillation in a system. Generally speaking, the stability criteria for delayed neural networks can be classified into two categories, namely, delay-independent [18,19,21–25] and delay-dependent [8,20,26,28]. Since delay-independent criteria tend to be conservative, especially when the delay is small, much attention has been paid to the delay-dependent type.

The passivity theory has long been a nice tool for analyzing the stability of systems, which has been applied in diverse areas such as stability, complexity, signal processing, chaos control and synchronization, and fuzzy control [12]. Recently, the problem of passivity analysis for delayed neural networks has been addressed in [6,10], where sufficient conditions for passivity were established. Considering that the passivity criteria in both [6] and [10] are delay-independent, several delay-dependent passivity conditions for delayed neural networks were proposed in [11,14,17,27], which are based on linear matrix inequalities (LMIs) techniques and Jensen integral inequality or free-weighting matrix method. For neural networks with discrete and bounded distributed time-varying delays, delay-dependent passivity results were obtained in [2] in terms of LMIs techniques and free-weighting matrix method. However, all these criteria are based on the assumption that the activation functions are monotonic non-decreasing. Furthermore, the conditions of [6,10,11,14] are established with the assumption that the derivatives of time-vary delays are less than 1. On the other hand, neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths. Therefore there will be a distribution of conduction velocities along these pathways and a distribution of propagation be modeled with discrete delays and a more appropriate way is to incorporate continuously distributed delays. Hence, it is our intention in this paper to tackle such an important yet challenging problem.

Motivated by aforementioned discussion, in this paper we will relax the constraint on the monotonicity of the activation function and the assumption that the derivatives of time-vary delays are

* Corresponding author. Tel.: +86 15566913851.
E-mail addresses: chd4211853@163.com, zhengchengde@dl.cn (C.-D. Zheng), chaoke886@163.com (C.-K. Gong), zhanshan_wang@163.com (Z. Wang).
☆This work was supported by the National Natural Science Foundation of China under Grants 61074073, 61034005, 61273022, Program for New Century Excellent Talents in University of China (NCET-10-0306), and the Fundamental Research Funds for the Central Universities under Grant N110504001.

0925-2312/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.neucom.2013.02.032

less than 1, and study the passivity of neutral-type neural networks with discrete and continuous distributed time-varying delays. Based on delay partitioning, a new Lyapunov–Krasovskii functional is constructed to obtain several improved delay-dependent passivity conditions which guarantee the passivity of uncertain neural networks by applying the Jensen integral inequality. These criteria are expressed in the framework of linear matrix inequalities, which can be verified easily by means of standard Matlab software. By Finsler's Lemma, one special case of the obtained criteria turns out to be equivalent to some existing result with same reduced conservatism but including fewer slack variables [7]. As the present passivity conditions involve fewer free-weighting matrices, the computational burden is largely reduced. Finally three examples are provided to verify the effectiveness of the proposed criteria.

Notation: Throughout this paper, let W^T, W^{-1} denote the transpose and the inverse of a square matrix W, respectively. Let $W>0$ (<0) denote a positive (negative) definite symmetric matrix, I_n, 0_n denote the identity matrix and the zero matrix of n-dimension respectively, $0_{m\times n}$ denotes the $m\times n$ zero matrix, the symbol "$*$" denotes a block that is readily inferred by symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem description

Considering the following uncertain neutral-type neural networks with discrete and distributed time-varying delays:

$$\dot{x}(t) = -A(t)x(t) + W_0(t)g(x(t)) + W_1(t)g(x(t-\tau(t))) + W_2(t)\int_{t-\sigma(t)}^{t} g(x(s))\,\mathrm{d}s + W_3(t)\dot{x}(t-\varpi) + u(t),$$

$$y(t) = g(x(t)),$$
$$x(t) = \phi(t), \quad t\in[-\max\{\overline{\tau},\overline{\sigma},\varpi\},0] \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T\in\mathcal{R}^n$ is a neural state vector, $u(t)$ is an external input vector, and $g(x(t)) = (g_1(x(t)),\ g_2(x_2(t)), \ldots, g_n(x_n(t)))^T\in\mathbb{R}^n$ denotes the neural activation function, continuous function $\phi(t) = (\phi_1(t), \phi_2(t), \ldots, \phi_n(t))^T\in\mathbb{R}^n$ is the initial condition. $0\le\underline{\tau}\le\tau(t)\le\overline{\tau}$, $0\le\sigma(t)\le\overline{\sigma}$, $\varpi\ge0$ are time-varying delays, where $\underline{\tau}$, $\overline{\tau}$, $\overline{\sigma}$, ϖ are constant scalars. $A(t) = A + \Delta A(t)$, $W_\iota(t) = W_\iota + \Delta W_\iota(t)$ $(\iota = 0,1,2,3)$. $A = \mathrm{diag}\{a_1, a_2, \ldots, a_n\}$ is a positive diagonal matrix, $W_\iota = (w_{ij}^\iota)_{n\times n}$ is known constant matrix, $\Delta W_\iota(t)$ is the parametric uncertainty. It is assumed that $g_j(x_j(t))$ $(j = 1,\ldots,n)$ is bounded and satisfies the following conditions:

Assumption 1 (*Liu et al. [9]*). There exist constants k_j^-, k_j^+, such that $k_j^- < k_j^+$ and

$$k_j^- \le \frac{g_j(s_1)-g_j(s_2)}{s_1-s_2} \le k_j^+, \quad g_j(0) = 0, \tag{2}$$

for any $s_1, s_2\in\mathcal{R}$, $s_1\ne s_2$.

For notational simplicity, we denote

$$K_1 = \mathrm{diag}\{k_1^-k_1^+, k_2^-k_2^+, \ldots, k_n^-k_n^+\},$$
$$K_2 = \mathrm{diag}\{k_1^- + k_1^+, k_2^- + k_2^+, \ldots, k_n^- + k_n^+\},$$
$$K_3 = \mathrm{diag}\{k_1^-, k_2^-, \ldots, k_n^-\}.$$

Remark 1. As pointed out by Liu et al. [9], the constants k_j^-, k_j^+ in (2) are allowed to be positive, negative, or zero. Hence, the resulting activation functions may be non-monotonic, and more general than the usual sigmoid functions in [2,6,14,17,27].

Suppose that the time-varying uncertain matrices $\Delta A(t)$, $\Delta W_\iota(t)$ $(\iota = 0,1,2,3)$ are linear fractional norm-bounded, which are in the form of

$$[\Delta A(t)\ \ \Delta W_\iota(t)] = H\Delta(t)[G\ \ G_\iota], \tag{3}$$

where H, G, G_ι $(\iota = 0,1,2,3)$ are known real constant matrices with appropriate dimensions. The uncertainty $\Delta(t)$ is defined as

$$\Delta(t) = (I_n - F(t)J)^{-1}F(t), \tag{4}$$

where J is also a known real constant matrix satisfying $J^TJ < I_n$ and $F(t)$ is an unknown time-varying matrix satisfying

$$F^T(t)F(t)\le I_n. \tag{5}$$

Remark 2. The aforementioned structured linear fractional form includes the norm-bounded uncertainty as a special case when $J=0$. Note also that condition $J^TJ < I_n$ and (5) guarantee that $I-F(t)J$ is invertible.

We now introduce the following definition of passivity.

Definition 1 (*Li and Liao [6]*). The system in (1) is said to be passive if there exists a scalar $\gamma > 0$ such that for all $t_f\ge0$

$$2\int_0^{t_f} y^T(s)u(s)\,\mathrm{d}s \ge -\gamma\int_0^{t_f} u^T(s)u(s)\,\mathrm{d}s,$$

under the zero initial condition.

In order to obtain the results, we need the following lemmas.

Lemma 1 (*See Gu [4]*). *For any positive symmetric constant matrix $M\in\mathcal{R}^{n\times n}$, scalars $r_1 < r_2$ and vector function $\omega:[r_1,r_2]\to\mathcal{R}^n$ such that the integrations concerned are well defined, then*

$$\left(\int_{r_1}^{r_2}\omega(s)\,\mathrm{d}s\right)^T M\left(\int_{r_1}^{r_2}\omega(s)\,\mathrm{d}s\right) \le (r_2-r_1)\int_{r_1}^{r_2}\omega^T(s)M\omega(s)\,\mathrm{d}s.$$

Lemma 2 (*See Li et al. [8]*). *Suppose that $\Delta(t)$ is given by (4). Given matrices $Q = Q^T, H$ and G with compatible dimensions, the matrix inequality*

$$Q + S\Delta(t)N^T + N\Delta^T(t)S^T < 0$$

holds for any $F(t)$ satisfying $F^T(t)F(t)\le I_n$ if and only if for any positive scalar ε, the following matrix inequality holds:

$$\begin{bmatrix} Q & S & \varepsilon N \\ * & -\varepsilon I_n & \varepsilon J^T \\ * & * & -\varepsilon I_n \end{bmatrix} < 0.$$

Lemma 3 (*See de Oliveir [13]*). *Let $y\in\mathbb{R}^n, L\in\mathbb{R}^{n\times n}$ and $B\in\mathbb{R}^{m\times n}$ such that $\mathrm{rank}(B) < n$ and $L = L^T$. Then the following statements are equivalent*:

(i) *$y^TLy < 0$ holds for any $y\ne0$, $By = 0$;*
(ii) *$(B^\perp)^TLB^\perp < 0$;*
(iii) *There exists a matrix $X\in\mathbb{R}^{n\times m}$ such that $L + XB + B^TX^T < 0$;*

where $B^\perp$ is a matrix whose columns form the bases of the right null space of B.

3. Main results

In the sequel, we will establish several passivity results by employing the so-called delay-partitioning approach introduced in Zhang et al. [26]. In order to estimate the upper bound of the delay for passivity, we partition delay interval $[\underline{\tau},\overline{\tau}]$ into several components, that is, $0\le\underline{\tau} = \tau_0 < \tau_1 < \cdots < \tau_r = \overline{\tau}$, where r is a positive integer.

Before introducing the main results for system (1) with $\underline{\tau} > 0$, following notations are defined for simplicity:

$$\Omega_{11} = -(1-\eta)Q_1 - 2T_+K_1,$$
$$\Omega_{1,r+7} = -(1-\eta)Q_2 + T_+K_2,$$