



# Bifurcation control of complex networks model via PD controller

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## ABSTRACT

In this paper, we investigate the problem of bifurcation control for a complex networks in a small-world networks model with time delay. By choosing the nonlinear interactive parameter as the bifurcation parameter, we present a Proportional-Derivative (PD) feedback controller to control Hopf bifurcation which inherently happens due to the networks topology. Stability analysis shows that the onset of Hopf bifurcation can be delayed or advanced via a PD controller by setting proper control parameters. Therefore the Hopf bifurcation of the model became controllable to achieve desirable behaviors which is applicable in certain circumstances. Meanwhile, the direction and stability of bifurcating periodic solutions are determined by using the normal form theory and the center manifold theorem. Finally, numerical simulations confirm the effectiveness of the control strategy in controlling the Hopf bifurcation for the complex network model.

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## 1. Introduction

Dramatic advances in the field of complex networks have been made in the past few years. The current study of complex networks is pervading all kinds of sciences, ranging from physical to biological, and to social sciences [1–6]. In order to capture the global properties of real world systems, scientists use mathematical tools to construct simple and abstract model whose nodes represent the dynamical units and whose links stand for the interactions between units. Numerous theory and applications of complex networks have been established since the works of Albert, Barabasi, Watts, Strogatz [7,8].

In 1998, Watts and Strogatz found a simple model of networks in order to describe the transition from a regular lattice to a random graph. These coupled systems were so called small-world networks [8]. The discovery of small-world phenomena has led to many fascinating common problems in the research of complex networks. A prevailing research of modeling the spreading (propagation) of viruses, diseases and even disasters has made great progress in practical [9–13]. Among those models, small-world networks were argued to be more practical in epidemic spreading research for it presents a much faster propagation than reaction–diffusion models based on regular lattices. Recently, a variety of small-world networks models have been established for epidemic spreading in which the transient behaviors are affected by topology structure and typical dynamical

behaviors [14–18]. One of these models is famous N–W model which was investigated in [19,20]. The presented governing equations of the model is linear and the model is simplified as a react–instantly networks by neglecting the communicating delay. Yang [21] extends the model by considering the effects of time delay and nonlinear topology on the N–W model, while the nonlinear invariant interaction in Yang's model failed to incorporate the topologic difference in the N–W model. To overcome this disadvantage, a more general nonlinear spreading model was developed by Li and Chen [22]. In their model, the nonlinear interaction is variable to the topology which incorporates the topologic difference in the N–W model. The model is described as:

$$\frac{dV(t)}{dt} = 1 + 2pV(t - \delta) - \mu(1 + 2p)V^2(t - \delta) \quad (1)$$

where  $V$  is the total influenced volume and  $\delta$  is the time delay. By varying  $p$  from the 0 to 1, the topology structure varies from a regular networks when  $p = 0$  to a completely random networks when  $p = 1$ .

The stability analysis and bifurcation behaviors in model (1) have been studied a lot to investigate the effect of topology on network dynamics. It is found that the occurrence of Hopf bifurcation behaviors is inevitable due to the characteristics of the small-world networks topology [4]. Under certain circumstances, bifurcations can be very useful or harmful. For example, when analyzing and controlling the infection and the spread of viruses and diseases, it is vital to predict the equilibrium and the dynamics of the equilibrium for adjusting impact parameters effectively. Bifurcation control can be useful when trying to achieve desirable behaviors, therefore some control strategies such as hybrid control

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[24], predictive control [25] and delayed feedback control [26] have been applied to N–W model. Proportional-Integral-Derivative (PID) controller is a widely used control method for dynamics control in nonlinear system for its superior performance [27–29]. A PID-controller consists of a PI control portion connected in serial

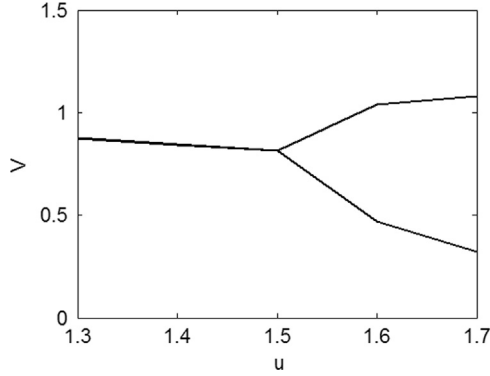


Fig. 1. Bifurcation diagram of uncontrolled model (1) corresponds to  $\mu$ .

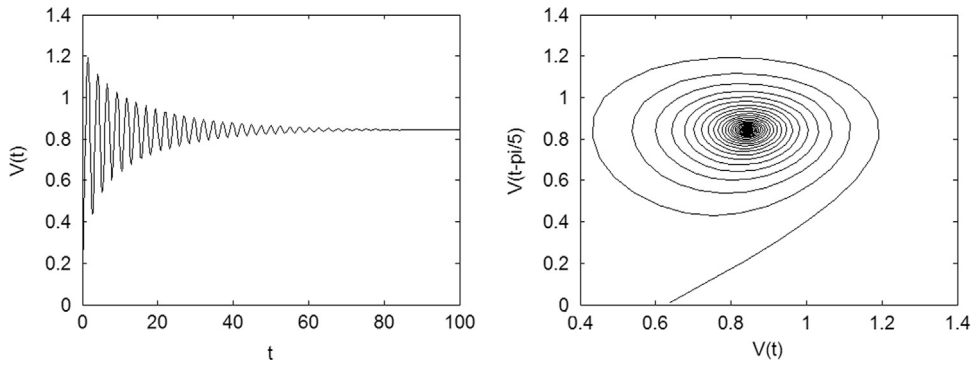


Fig. 2. Waveform plot and phase portrait of uncontrolled system (1) with  $\mu = 1.4$ .

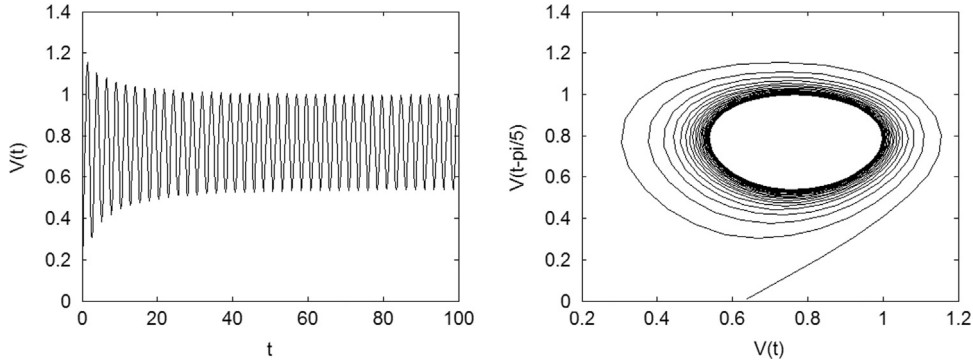


Fig. 3. Waveform plot and phase portrait of uncontrolled system (1) with  $\mu = 1.6$ .

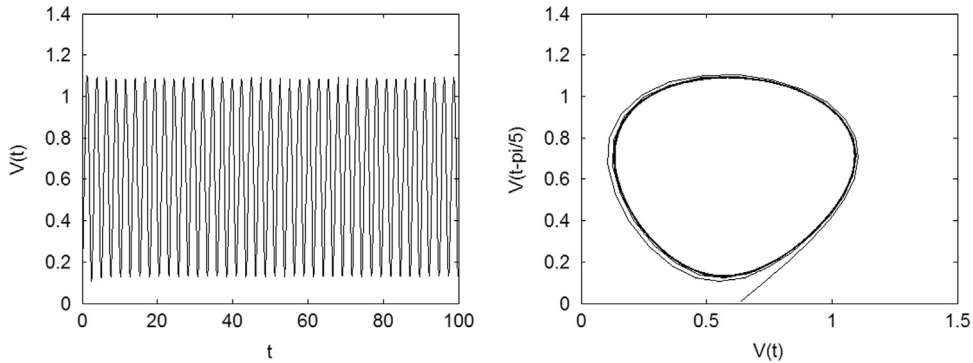


Fig. 4. Waveform plot and phase portrait of uncontrolled system (1) with  $\mu = 1.95$ .

with a PD control portion. PI control can improve the steady-state error at the expense of an increasing response time while PD control can improve the speed of response of a control system [29]. In order to consider the small-world feature of model (1) and not lose the transmission speed, we decide to apply a PD controller to the model (1) in this paper. Common control law of a PD controller is defined as follows:

$$u(t) = k_p e(t) + k_d \frac{d}{dt} e(t) \quad (2)$$

where  $k_p$  and  $k_d$  are proportional control parameter and derivative control parameter respectively.  $e(t)$  stands for the input deviation and  $u(t)$  stands for the control law.

The rest of this paper is arranged as follows. In Section 2, a PD controller is applied to the model, then the bifurcation behavior in the controlled model is analyzed. In Section 3, the direction, stability and the period of the bifurcating periodic solutions of the model are analyzed by using the normal form theory and the center manifold theorem. In Section 4, the effectiveness of the PD control strategy is verified through numerical simulation. Conclusions are given in Section 5.

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