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Passivity-based stabilization and passive synchronization of complex nonlinear networks

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ABSTRACT

This paper investigates the passive stabilization and passive synchronization problems for complex nonlinear networks. Two classes of complex networks are considered with or without time delays (including the state time-varying delays, feedback time-varying delays and interconnection time-varying delays). The purpose of passive stabilization and passive synchronization is to design a state feedback controller such that the nonlinear dynamical network and the corresponding error dynamical network are passive and stable asymptotically. Delay-independent sufficient conditions for complex nonlinear networks without delays and delay-dependent sufficient conditions for complex nonlinear networks with time-varying delays are provided, respectively, to solve the passive stabilization and passive synchronization problems for the given networks. Two numerical examples are employed to illustrate the effectiveness of the proposed results.

1. Introduction

Passivity, since it was introduced by Willems in 1972 [1], has drawn great attention in the control area mainly because of its relation with stability. As an important system property, passivity offers a good method and an effective tool for solving the problem of stability analysis and synthesis [2,3].

Complex nonlinear networks, owing to their wide applications and cross-disciplinary nature, have penetrated into many fields, such as engineering, mathematical subject, life sciences and so on. Recently, people have devoted to the study of network topology structure and model, propagation, cascading failure, algorithm research, the control and synchronization of complex networks. The problems of synchronization and bifurcation for complex dynamical networks with a somewhat general coupling matrix are investigated in [4]. In [5], the synchronization problem of complex dynamical networks is studied via impulsive control. In [6], the problem of cascading failure for weighted complex networks is studied based on the relationship between capacity and load. The problem of community detection algorithm for complex network in order to discover its local community structure is effectively given in [7].

Generally, the information flow in complex networks is not instantaneous. Moreover, in networks, the finite speed of signal transmission over a distance and information processing gives rise to a finite time delay. As is well known, the existence of time delays may cause instability of systems and usually lead to unsatisfactory performance. So it is very necessary to research how to reduce or eliminate the bad effects from time delay. Recently, the dynamics of networks with time delays have been extensively studied, and many results are reported [8–14]. The fractal problem of small-world networks with time delay is investigated in [8]. The problem of stability for the synchronous state in networks of identical phase oscillators with delayed coupling is studied in [9]. In [10–13], the synchronization problem of complex dynamic networks with different time delays is investigated. The problem of synchronization between two general delayed dynamical complex networks using the Lyapunov stability theory and the adaptive periodical pinning intermittent control technique is studied in [14]. It is noted that the research on passive synchronization and passive stabilization of complex networks is relatively few, especially for complex nonlinear networks with time-varying delays, which motivates us to study this problem.

In this paper, the problems of passive stabilization and passive synchronization are addressed for complex nonlinear networks with or without state time-varying delays, feedback time-varying delays and communication time-varying delays. The objective is to find a state feedback controller such that the nonlinear dynamical network and the corresponding error dynamical network are passive and asymptotically stable. By constructing a proper Lyapunov function and using the matrix inequality approach, sufficient conditions for complex nonlinear networks without delays and delay-dependent sufficient conditions for complex nonlinear

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networks with time-varying delays are established, respectively, to achieve the expected passive stabilization and passive synchronization. All these conditions are cast into linear matrix inequalities for the convenience of computation. The desired state feedback controllers are derived by solving such linear matrix inequalities efficiently. Finally, two numerical examples illustrate the effectiveness of the proposed results.

Notations: Let $\mathbb{N}$ denote the set of nonnegative integers, and $J=[t_0,\infty)(t_0\ge 0)$. $\mathbb{R}^n$ and $\mathbb{R}^{m\times n}$ stand for the usual n-dimensional Euclidean space and the set of all $m\times n$ real matrices, respectively. X^{-1} and X^T represent the inverse and transpose of matrix X, respectively.

2. Complex nonlinear networks without time delays

2.1. Mathematical models of networks

In this section, we consider the following complex dynamical network:

$$\dot{x}_i(t)=A_ix_i(t)+B_iu_i(t)+E_i\omega_i(t)+f_i(t,x_i(t))+\sum_{j=1}^{m}M_{ij}(x_j(t)-x_i(t)),$$
$$z_i(t)=C_ix_i(t),\quad t\ge t_0\ge 0,\ i=1,2,\dots,m, \tag{1}$$

where $x_i=(x_{i1},x_{i2},\dots,x_{in})^T\in\mathbb{R}^n, u_i\in\mathbb{R}^m$ are the state and the control input of node i respectively; $\omega=(\omega_1^T,\omega_2^T,\dots,\omega_m^T)^T, z=(z_1^T,z_2^T,\dots,z_m^T)^T$ with $\omega_i\in\mathbb{R}^q, z_i\in\mathbb{R}^q$ are the exogenous input and the output of the whole network, respectively; $f_i:\mathbb{R}^n\mapsto\mathbb{R}^n$ are nonlinear vector-valued functions with $f_i(t,0)\equiv 0, t\ge t_0\ge 0$; A_i,B_i,C_i,E_i are known matrices with appropriate dimensions; $M_{ij}=M(i,j)$ is the i-th row j-th column element of matrix M, which is the coupling configuration matrix representing the coupling strength and topological structure of the whole network. If there is a connection between node i and node j $(i\ne j)$, then $M_{ij}=M_{ji}>0$, otherwise, $M_{ij}=M_{ji}=0$ $(i\ne j)$, and $M_{ii}=0$ $(i=1,2,\dots,m)$.

Design a state feedback control law for each subsystem as

$$u_i(t)=-L_ix_i(t),\quad t\in[t_0,\infty),\ i=1,2,\dots,m, \tag{2}$$

where $L_i\in\mathbb{R}^{m\times n}$ is the feedback gain matrix to be determined. Submitting (2) into (1) gives

$$\dot{x}_i(t)=(A_i-B_iL_i)x_i(t)+E_i\omega_i(t)+f_i(t,x_i(t))+\sum_{j=1}^{m}M_{ij}(x_j(t)-x_i(t)),$$
$$z_i(t)=C_ix_i(t),\quad t\in[t_0,\infty),\ i=1,2,\dots,m. \tag{3}$$

2.2. Passivity of complex networks without time delays

Firstly, we introduce the following definitions.

Definition 1. A system with input $\omega\in\mathbb{R}^q$ and output $z\in\mathbb{R}^q$ is said to be passive if there are constants $\epsilon\ge 0,\delta\ge 0$, and functions $\alpha(t)\le 0$, $\int_{t_0}^t\beta(s)\,ds\ge 0$ such that

$$\int_{t_0}^t\omega^T(s)z(s)\,ds\ge\alpha(t_0)+\epsilon\int_{t_0}^t\omega^T(s)\omega(s)\,ds+\delta\int_{t_0}^t z^T(s)z(s)\,ds+\int_{t_0}^t\beta(s)\,ds \tag{4}$$

for all $t\ge t_0\ge 0$. Besides, the system is input strictly passive if $\epsilon>0$, and output strictly passive if $\delta>0$.

Definition 2. If we can design a control law (2) such that every closed-loop subnetwork is asymptotically stable, and output $z(t)$ and input $\omega(t)$ satisfy inequality (4), then the dynamical network (1) is said to be passively stabilized under feedback control law (2).

In the following, we will present the passive stabilization of system (1).

Theorem 1. *If there are continuous functions $\varphi_i(t)\ge 0$, symmetric positive-definite matrix P and constant $a<0$ such that*

$$\Lambda_i=A_i^TP+PA_i-L_i^TB_i^TP-PB_iL_i+2\varphi_i(t)P\le aC_i^TC_i, \tag{5}$$

$$f_i^T(t,x_i)Px_i\le\varphi_i(t)x_i^T(t)Px_i(t),\quad i=1,2,\dots,m, \tag{6}$$

$$C_i=E_i^TP,\quad i=1,2,\dots,m, \tag{7}$$

then system (1) is passively stabilized under controller (2).

Proof. By the definition of passive stabilization, we know what we need is to find L_i $(i=1,2,\dots)$ such that the closed-loop system (3) is asymptotically stable and passive under controller (2). Choose a candidate Lyapunov function as

$$V(t)=V(x(t))=\sum_{i=1}^{m}x_i^T(t)Px_i(t), \tag{8}$$

then for $t\ge t_0$, we have

$$\begin{aligned}
\dot{V}(t)|_{(3)}=&\sum_{i=1}^{m}\Big\{x_i^T(t)(A_i-B_iL_i)^TPx_i(t)+x_i^T(t)P(A_i-B_iL_i)x_i(t)\\
&+2f_i^T(t,x_i(t))Px_i(t)+\sum_{j=1}^{m}(x_i^T(t)PM_{ij}(x_j(t)-x_i(t))+(x_j(t)\\
&-x_i(t))^TM_{ij}Px_i(t))+2x_i^T(t)PE_i\omega_i(t)\Big\}\\
\le&\sum_{i=1}^{m}\Big\{x_i^T(t)(A_i^TP+PA_i-L_i^TB_i^TP-PB_iL_i+2\varphi_i(t)P)x_i(t)\\
&+2x_i^T(t)PE_i\omega_i(t)\Big\}-\sum_{i,j}(x_j(t)-x_i(t))^TPM_{ij}(x_j(t)-x_i(t))\\
=&\sum_{i=1}^{m}\{x_i^T(t)\Lambda_ix_i(t)+2x_i^T(t)PE_i\omega_i(t)\}\\
&-\sum_{i,j}(x_j(t)-x_i(t))^TPM_{ij}(x_j(t)-x_i(t))\\
\le&\sum_{i=1}^{m}\{ax_i^T(t)C_i^TC_ix_i(t)+2x_i^T(t)PE_i\omega_i(t)\}.
\end{aligned}$$

When $\omega_i(t)=0$, one gets $\dot{V}(t)|_{(3)}\le az^T(t)z(t)<0$, thus the closed-loop system (3) is asymptotically stable.

Moreover, for any given $t\ge t_0\ge 0$, one has

$$\begin{aligned}
2\int_{t_0}^t\omega^T(s)z(s)\,ds=&\,2\int_{t_0}^t\sum_{i=1}^{m}\omega_i^T(s)z_i(s)\\
ds=&\sum_{i=1}^{m}\int_{t_0}^t\Bigg\{\Bigg(\frac{dx_i(s)}{ds}-(A_i-B_iL_i)x_i(s)\\
&-f_i(s,x_i(s))-\sum_{j=1}^{m}M_{ij}\big(x_j(s)-x_i(s)\big)\Bigg)^TPx_i(s)\\
&+x_i^T(s)P\Bigg(\frac{dx_i(s)}{ds}-(A_i-B_iL_i)x_i(s)-f_i(s,x_i(s))\\
&-\sum_{j=1}^{m}M_{ij}\big(x_j(s)-x_i(s)\big)\Bigg)\Bigg\}\,ds\\
\ge&\int_{t_0}^t\frac{dV(s)}{ds}\,ds\\
&-\sum_{i=1}^{m}\int_{t_0}^t x_i^T(s)\Lambda_ix_i(s)\,ds\\
&+\int_{t_0}^t\sum_{i,j}(x_j(s)-x_i(s))^TPM_{ij}\big(x_j(s)-x_i(s)\big)\,ds\\
\ge&\,V(t)-V(t_0)-\int_{t_0}^t\sum_{i=1}^{m}ax_i^T(s)C_i^TC_ix_i(s)\,ds,
\end{aligned}$$