

Sampling-based causal inference in cue combination and its neural implementation

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ABSTRACT

Causal inference in cue combination is to decide whether the cues have a single cause or multiple causes. Although the Bayesian causal inference model explains the problem of causal inference in cue combination successfully, how causal inference in cue combination could be implemented by neural circuits, is unclear. The existing method based on calculating log posterior ratio with variable elimination has the problem of being unrealistic and task-specific. In this paper, we take advantages of the special structure of the Bayesian causal inference model and propose a hierarchical inference algorithm based on importance sampling. A simple neural circuit is designed to implement the proposed inference algorithm. Theoretical analyses and experimental results demonstrate that our algorithm converges to the accurate value as the sample size goes to infinite. Moreover, the neural circuit we design can be easily generalized to implement inference for other problems, such as the multi-stimuli cause inference and the same-different judgment.

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1. Introduction

Human brain receives cues from multiple sensory modalities and integrates them in an optimal way [1]. The cues from the outside world are noisy observations of stimuli reflecting uncertainty. It has been demonstrated that, if all cues have the same cause, the optimal process of cue combination is a process of Bayesian inference [2–5]. However, the truth is that, we receive information from various sources simultaneously in our daily life, which means the cues may come from different causes. How to decide whether a single cause or multiple causes is responsible for the cues, known as causal inference in cue combination, is an important problem. This problem is the precondition of cue combination and is quite common in our daily life [6,7]. For example, at a cocktail party, we need to decide whether the face and voice belong to the person who calls our name [8]. Recently, the problem of causal inference in cue combination is partially answered by Kording et al. [9] and Sato et al. [10], who propose the Bayesian causal inference model. Their causal inference model successfully explains the problem of causal inference in cue combination. Yet, how causal inference in cue combination could be implemented by neural circuit, is unclear. Solving this problem

benefits not only theoretical researches but also practical applications. On the one hand, it provides an explanation of how human brain performs causal inference in cue combination, building a bridge between probabilistic models of cognition and neural mechanism. On the other hand, if the causal inference could be implemented by neural circuits, a neural circuit prototype can be easily developed to achieve the function of causal inference, which could be applied in intelligent robots, enabling these robots to perform causal inference.

Over the past decade, several methods with different probability codes have been proposed to perform probability inference with neural circuits. Rao [11–13] establishes the relationship between the dynamic equation of neural circuits and the inference of probabilistic graphical models. He proves that the process that the firing rate of neurons in the recurrent neural circuit varies with respect to time is a process of posterior probabilities inference in a hidden Markov model, under the condition that the firing rate is proportional to the log of posterior probabilities. Ott and Stoop [14] build the relationship between the dynamical equation of continuous Hopfield network and belief propagation on a binary Markov random field. Sampling is another commonly accepted way to perform inference by neural circuits. Based on Monte Carlo sampling, Huang and Rao [15] build a spiking network model to perform approximate inference for any hidden Markov model. Maass et al. [16–18] propose that stochastic networks of spiking neurons could implement inference for graphical models by Markov chain Monte Carlo. Shi and Griffiths [19] apply importance

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sampling to perform inference of chain Bayesian model and design neural circuits to implement it. Another important framework is Probabilistic population coding (PPC), the core idea of which is that the neurons are encoders of distributions, instead of the values of variables [20–22]. Ma et al. [20] present that the inference of cue integration can be conducted simply by linear combinations of each population activity with PPC. The method is exploited thereafter by Beck et al. [23] to realize the Bayesian decision making and the inference of marginalization [24].

To the best of our knowledge, the only work implementing causal inference in cue combination with neural circuits is proposed by Ma et al. [25] in 2013. They calculate the ratio of the posterior probabilities of both situations (a single cause or multiple causes) with variable elimination and design a neural circuit to implement it. This method suffers from three shortcomings. Firstly, the circuit they design is task-specific and only works on two stimuli. If we want to implement multi-stimuli causal inference [26] with the same method, the circuit will be completely different. What is more, the required number of operations increases faster than linear with respect to the number of stimuli, which makes the neural circuit unrealistic [25]. Secondly, it is hard to generalize the circuit to implement a similar task called same-different judgment [27]. Thirdly, since how to implement logarithmic operations with neurons remains unknown, approximations are taken in their neural circuit so that they could only get near-optimal results.

In this paper, different from calculating the posterior ratio with variable elimination in [25], we propose a hierarchical inference algorithm based on importance sampling, which takes advantages of the special structure of the causal inference model. A neural circuit with hierarchical structure is then designed corresponding to the bottom-up inference process. The proposed method has three advantages. Firstly, the neural circuit is simple and easy to be realized by PPC and some simple plausible neural operations. Secondly, it is easy to generalize this neural circuit to implement inference for other problems, such as the multi-stimuli cause inference and the same-different judgment. Thirdly, a theoretical proof is given that the sampling-based method converges to the accurate value with probability one as sample size tends to infinity.

The rest of this paper is organized as follows. Section 2 briefly reviews the causal inference in cue combination. In Section 3 we present a sampling-based inference algorithm and design the corresponding neural circuit. The experimental results are shown in Section 4. We generalize our method to solve other two problems in Section 5 and make a conclusion in Section 6.

2. The causal inference model in cue combination

The problem of causal inference in cue combination is to infer whether cues come from a single or multiple causes. Kording et al.

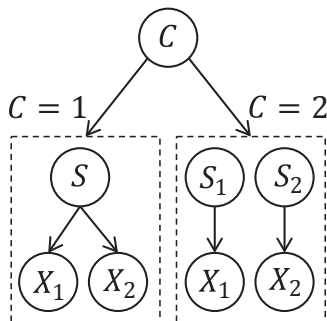


Fig. 1. The causal inference model in cue combination.

[9] and Sato et al. [10] propose a causal inference model of cue combination respectively, which could explain physiological and psychological experiments successfully. Here, we briefly review this model and the stimuli considered here only include visual and auditory ones. The multi-stimuli problem will be explained in Section 5. In Fig. 1, node C represents the common-cause variable, S, S₁, and S₂ express the stimuli. X₁ and X₂ are cues received by the sensory system. The state of cause C is 1 or 2, where C=1 means the cues have the same cause and C=2 means the cues have two different causes. For simplicity, we assume that P(C=1) is equal to P(C=2), both of which have a probability 0.5. When C=1, there is a stimulus S with distribution P(S) corresponding to the common cause, where P(S) is a Gaussian distribution with mean 0 and variance σ_S^2 . Two measurements X₁ and X₂ are generated from two Gaussian distributions with different variances σ_1^2 and σ_2^2 , but with the same mean S. When C=2, there are two different stimuli S₁ and S₂, which are drawn from the same Gaussian distribution with mean 0 and variance σ_S^2 . Then two measurements X₁ and X₂ are drawn from two different Gaussian distributions with their means being S₁ and S₂, and their variances being σ_1^2 and σ_2^2 respectively. Based on the definitions above, the causal inference problem is to decide whether C=1 or C=2 according to the measurements X₁ and X₂.

3. Sampling-based causal inference in cue combination

In this section, we first convert the causal inference model to a three-layer Bayesian network. Then we propose a sampling-based hierarchical inference method and design the corresponding neural circuit. After that we demonstrate that this circuit can be realized by PPC and simple plausible neural operations.

3.1. The three-layer Bayesian network model

In this paper, the problem is to infer the state of node C. In order to simplify inference, we convert the causal inference model above to a three-layer Bayesian model (Fig. 2) with some appropriate prior probabilities and conditional probabilities. In the new model, node C is the common-cause variable, which is similar to that in the causal inference model. S₁ and S₂ refer to two different stimuli, such as visual and auditory stimuli. The conditional probability of S₁ and S₂ under C is expressed as P(S₁, S₂ | C). We define $P(S_1, S_2 | C = 1) = \delta(S_1 - S_2) \frac{1}{\sqrt{2\pi\sigma_S}} \exp\left(-\frac{S_1^2}{2\sigma_S^2}\right)$ and $P(S_1, S_2 | C = 2) = \frac{1}{2\pi\sigma_S^2} \exp\left(-\frac{S_1^2 + S_2^2}{2\sigma_S^2}\right)$, where $\delta(S_1 - S_2)$ is the Dirac Delta distribution. X₁ and X₂ are measurements from S₁ and S₂ respectively. The conditional probability of X₁ under S₁ is defined by P(X₁ | S₁)

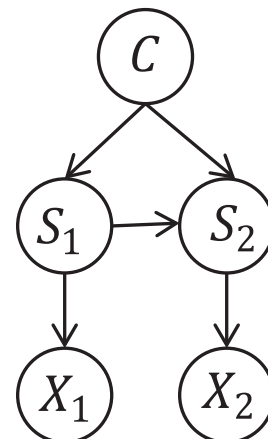


Fig. 2. The three-layer Bayesian network equivalent to the causal inference model in Fig. 1.

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