Neurocomputing 175 (2016) 223–232

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

A novel stabilization condition for a class of T–S fuzzy time-delay systems

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ARTICLE INFO

Article history:
Received 3 June 2015
Received in revised form
5 September 2015
Accepted 20 October 2015
Communicated by Hak Keung Lam
Available online 31 October 2015

Keywords:
Fuzzy time-delay systems
Linear matrix inequality (LMI)
Pólya's theorem
Homogeneous polynomials
Takagi–Sugeno (T–S) fuzzy model

ABSTRACT

In this paper, a relaxed stabilization problem for a class of Takagi and Sugeno (T–S) fuzzy time-delay systems is explored. By utilizing homogeneous polynomials scheme and Pólya's theorem, a relaxed delay-dependent stabilization condition is proposed. In addition, a novel slack matrix scheme is presented for stabilization condition of a class of T–S fuzzy time-delay systems in terms of linear matrix inequalities (LMIs). Lastly, a well-know numerical example and a truck-trailer example are given to demonstrate that the proposed stabilization condition can provide a longer allowable delay time than some existing studies.

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1. Introduction

Takagi–Sugeno (T–S) fuzzy-model-based method is widely adopted to investigate the control synthesis and stabilization problems of nonlinear systems. For system modeling, the T–S fuzzy model provides an effective and systematic framework to represent the nonlinear system as an average weighted sum of some linear-type subsystems. By examination of the controller design problem, the fuzzy controller can be designed via the parallel distributed compensation (PDC) scheme [1,2] to stabilize T–S fuzzy systems. The output of the overall fuzzy controller is a fuzzy blending of each individual linear controller. Moreover, linear matrix inequalities (LMIs) have been widely adopted to solve the stability and stabilization problems of T–S fuzzy control system.

In past few decades, T–S fuzzy time-delay systems have been successfully applied in chemical engineering and other industrial processes [3]. These results can be roughly classified into two types: one is independent of the size of the delay, i.e., the so-called "delay-independent" condition [4,5], while the other is concerned with the delay size, which is called as the "delay-dependent" condition. It has been recognized that the latter is less conservative than the former. Therefore, many various delay-dependent stabilization conditions are proposed in recent years. For example, a previous study [6] proposed a delay partition method for T–S fuzzy time-delay systems to reduce the conservatism. The free-weighting matrix approach has been employed to obtain the delay-dependent stabilization condition for discrete T–S fuzzy time-delay system in [7]. In [8], the small-gain theorem is adopted to convert discrete-time system and the less conservative stability and stabilization conditions are thus obtained. By combining delay-decomposition with state vector augmentation, a novel Lyapunov–Krasovskii function is proposed and a novel stability condition is formulated in the form of LMI in [9]. Although these studies proposed worth results, there leaves much room to further extend the delay time of T–S fuzzy time-delay systems. Hence, how to further reduce the conservatism for T–S fuzzy time-delay systems is still an important problem.

Recently, there are some studies investigating the relaxed stability and stabilization problems by using polynomial technique [10,11]. For example, in [12], homogeneous polynomial Lyapunov functions are proposed to explore the stability problem of linear systems with time-varying structured uncertainties. By utilizing homogeneous polynomial Lyapunov function and Pólya's theorem, a relaxed asymptotically necessary and sufficient condition is proposed to reduce the conservatism of the stabilization condition for T–S fuzzy systems in [13]. In [14], the local stability and stabilization conditions for discrete-time T–S fuzzy system are explored via homogeneous polynomial parameter-dependent matrix.

In addition to homogeneous polynomial technique, right-hand-side (RHS) technique was another technique which can reduce the

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http://dx.doi.org/10.1016/j.neucom.2015.10.054
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conservatism of the stability/stabilization condition [15–17]. Tanaka et al. [18] firstly proposed a RHS slack variable matrix to reduce the conservatism of the stabilization condition for T–S fuzzy systems. By collecting the interaction between each subsystem into a single matrix and adopting the *S*-procedure, a relaxed quadratic stability condition for fuzzy control system was proposed in [19]. Instead of two constant slack variable matrices in [19] by two transport constant matrices, a novel relaxed stabilization condition was proposed in [20]. Furthermore, by the property of $\sum_{i=1}^{r} h_i = 1$, [21] proposed a LMI-based quadratic relaxed stabilization condition. The information of the lower and upper membership functions are employed and novel slack matrices are proposed to obtain the membership function dependent stability conditions for the interval type-2 fuzzy systems in [17]. However, few studies explore T–S fuzzy time-delay systems by using polynomial technique and RHS technique. Therefore, in this study, we will adopt polynomial technique and RHS technique to explore the stabilization problem of T–S fuzzy time-delay systems.

Considering the advantages of homogeneous polynomial and RHS techniques, this paper explores the relaxed stabilization condition of T–S fuzzy time-delay systems. The main contributions of this paper includes (i) a relaxed Pólya stabilization condition for T–S fuzzy time-delay systems in LMIs; and (ii) a novel RHS relaxation scheme for further relaxation.

The rest of this paper is organized as follows. In Section 2, the delay-dependent stabilization problem for T–S fuzzy time-delay systems is introduced. Based on homogeneous polynomial technique and Pólya's theorem, the stabilization conditions for T–S fuzzy time-delay systems are presented in LMI form. Besides, a novel RHS polynomial slack matrix is proposed in Section 3 to reduce the conservativeness. In Section 4, a numerical example and a truck-trailer example are given to demonstrate that the proposed stabilization condition provides longer allowable delay times than existing ones. Finally, conclusions are given in Section 5.

2. Preliminaries

The following notations are used throughout this paper. The symbol $\star$ indicates transposed elements in an LMI, which can be obtained via transpose operation, denoted by ^{T}. $H(t) = \sum_{i=1}^{r} h_i(\xi(t))H_i$, where $h_i(\xi(t)) > 0$ and $\sum_{i=1}^{r} h_i(\xi(t)) = 1$. *Diag* denotes a block diagonal matrix. *I* is the identity matrix with a compatible dimension. ! denotes factorial for combinatoric expression. $\otimes$ is Kronecker product. $K(h)$ be the set of *r*-tuples defined as [22]:

$$K(h) = \{(k_1 k_2 \cdots k_r) : k_1 + k_2 + \ldots + k_r = h,\ \forall k_i \in I^+ (\text{positive integers}),\quad i = 1, 2, \ldots, r\}$$

where *h* is the total polynomial degree. Since the number of fuzzy rules is *r*, the number of elements in $K(h)$ is expressed by $J(h) = (r+h-1)!/(h!(r-1)!)$. For example, $r = 2$, $h = 4$

$$J(4) = (2+4-1)!/(4!(2-1)!) = 5$$

$$K(4) = \{(40), (31), (22), (13), (04)\} = \{t(1),\ t(2),\ t(3),\ t(4),\ t(5)\} = \left\{(h_1^4 h_2^0), (h_1^3 h_2^1), (h_1^2 h_2^2), (h_1^1 h_2^3), (h_1^0 h_2^4)\right\}.$$

For clarity, the following notations are adopted:

$$k = k_1 k_2 \cdots k_r,\ h^k = h_1^{k_1} h_2^{k_2} \cdots h_r^{k_r},\ e_i = 0 \cdots \underbrace{1}_{i^{th}} \cdots 0$$

$$k - e_i = k_1 k_2 \cdots (k_i - 1) \cdots k_r,\ \pi(k) = (k_1!)(k_2!) \cdots (k_r!)$$

$$C_{ii}^k(h) = \frac{(h!) k_i (k_i - 1)}{\pi(k)}, \quad C_{ij}^k(h) = \frac{(h!) k_i k_j}{\pi(k)}.$$

The T–S fuzzy model is expressed in terms of fuzzy IF-THEN rules. To begin, consider the following *i*th rule of a T–S fuzzy time-delay system.

Rule i : **IF** $\xi_1(t)$ is $M_1(t)$ and $\cdots$ and $\xi_p(t)$ is $M_{ip}(t)$

THEN $\dot{x}(t) = (A_i + \Delta A_i)x(t) + (A_{di} + \Delta A_{di})x(t - \sigma(t)) + (B_i + \Delta B_i)u(t)$ (1)

where $x(t) = \phi(t), t \in [-\max\{\sigma_M\}, 0]$, $\xi_1(t),\ \xi_2(t),\ \cdots,\ \xi_p(t)$ are premise variables, $M_{ij}(t)$, $i, j = 1, 2, \cdots, r$ are fuzzy sets, where r is the number of fuzzy rules. $x(t) \in R^{n \times 1}$ is the state, $u(t) \in R^{m \times 1}$ is the input, and σ_M is the upper bound of delay time. The matrices A_i, $A_{di} \in R^{n \times n}$ and $B_i \in R^{n \times m}$ are system matrices, and the initial vector $\phi(t)$ belongs to the set of continuous functions. ΔA_i, ΔA_{di} and ΔB_i are unknown matrices representing time-varying parameter uncertainties and are assumed to be of the following matched form, stated in Assumption 1. $\sigma(t)$ is the time-varying delay in the state and satisfies that

$$0 \le \sigma(t) \le \sigma_M, \quad 0 \le \dot{\sigma}(t) \le \sigma_D \tag{2}$$

where σ_M and σ_D are constants. That is, the time-varying delay is differentiable and its derivative is bounded.

Assumption 1 (*Tsai [3]*). The parameter uncertainties considered here are norm-bounded and presented in the form

$$\Delta A_i = M_{ai} G_i(t) N_{ai}, \quad \Delta A_{di} = M_{di} G_i(t) N_{di}, \quad \Delta B_i = M_{bi} G_i(t) N_{bi}$$

where M_{ai}, M_{di}, M_{bi}, N_{ai}, N_{di}, N_{bi} and $G_i(t)$ are unknown matrix functions with Lebesgue-measurable elements and satisfy $G_i^T(t) G_i(t) \le I$.

For convenience, the following notations are defined. $A_{ci} = A_i + \Delta A_i$, $A_{cdi} = A_{di} + \Delta A_{di}$ and $B_{ci} = B_i + \Delta B_i$. Similar to [1], using a center average defuzzifier, product inference, and a singleton fuzzifier, the overall uncertain T–S fuzzy time-delay system can be expressed as in the following equation:

$$\begin{aligned}
\dot{x}(t) &= \sum_{i=1}^{r} \mu_i(\xi(t))(A_{ci}x(t) + A_{cdi}x(t - \sigma(t)) + B_{ci}u(t)) \Big/ \sum_{i=1}^{r} \mu_i(\xi(t)) \\
&= \sum_{i=1}^{r} h_i(\xi(t))(A_{ci}x(t) + A_{cdi}x(t - \sigma(t)) + B_{ci}u(t)) \\
&= A_c(t)x(t) + A_{cd}(t)x(t - \sigma(t)) + B_c(t)u(t)
\end{aligned} \tag{3}$$

where $\mu_i(\xi(t)) = \prod_{j=1}^{p} M_{ij}(\xi(t))$, $h_i = \mu_i(\xi(t)) / \sum_{i=1}^{r} \mu_i(\xi(t))$, $\sum_{i=1}^{r} h_i(\xi(t)) A_{ci} = A_c(t)$, $\sum_{i=1}^{r} h_i(\xi(t)) A_{cdi} = A_{cd}(t)$, and $\sum_{i=1}^{r} h_i(\xi(t)) B_{ci} = B_c(t)$. $M_{ij}(\xi(t))$ is the membership degree of $\xi(t)$, and $\xi(t) = [\xi_1(t), \ldots, \xi_n(t)]$. Two basic properties of $\mu_i(\xi(t))$ are $\mu_i(\xi(t)) \ge 0$ and $\sum_{i=1} \mu_i(\xi(t)) > 0$. It is clear that $\sum_{i=1}^{r} h_i(\xi(t)) \ge 0$, and $\sum_{i=1}^{r} h_i(\xi(t)) = 1$.

The state feedback fuzzy controller for T–S fuzzy time-delay system (3) is represented as follows

$$u(t) = \sum_{k \in K(r-1), r \ge 2} h^k F_k x(t) = F(t)x(t) \tag{4}$$

where $F_k \in R^{m \times n}$ is the controller gain matrix.

By substituting (4) into (3), the closed-loop system can be obtained as

$$\dot{x}(t) = (A_c(t) + B_c(t)F(t))x(t) + A_{cd}(t)x(t - \sigma(t)) \tag{5}$$

3. Main results

Before discussing the proof of the theorems, the following results, which are used in the proof of theorems, are given.

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