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Synchronization of delayed discrete-time neural networks subject to saturated time-delay feedback

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ABSTRACT

This paper considers the synchronization problem of delayed discrete-time neural networks under saturated time-delay feedback. Based on the drive-response concept, and using generalized delay-dependent sector condition, appropriate Lyapunov–Krasovskii (L–K) functionals, and some finite-sum inequalities, two delay-dependent local synchronization conditions are well established in terms of linear matrix inequalities (LMIs). In addition, global synchronization conditions are also proposed on the basis of classical sector condition. Finally, numerical examples and simulations illustrate the effectiveness of the proposed synchronization conditions in this paper.

1. Introduction

Since the pioneering work of Pecora and Carroll [1], synchronization problem of chaotic systems has attracted considerable attention over the past decades due to its potential applications for image processing, secure communication, and harmonic oscillation generation, and several important synchronization schemes were well proposed [2–7]. As special dynamical systems, delayed neural networks have also been found to exhibit some complex and unpredictable behaviors including stable equilibria, periodic oscillations, bifurcation and chaotic attractors [8,9]. Therefore, much effort has also been made to the synchronization of delayed neural networks during the past decade [10–20]. In [10,11], adaptive control schemes were proposed to investigate the synchronization problem of chaotic delayed neural networks. By introducing stochastic perturbed response systems, the problems of exponential synchronization were respectively considered in [12] and [13] for continuous-time and discrete-time delayed neural networks. For the synchronization problems of stochastic delayed neural networks and neural networks with mixed delays, the corresponding conditions were well proposed in [14,15], respectively. Considering the external disturbances, H_∞ synchronization conditions were respectively established in [16] and [17] for continuous-time and discrete-time delayed neural networks. In [18] and [19], sampled-data controller and stochastic controller were respectively proposed to achieve global synchronization, and in [20], a nonlinear controller was designed for achieving lag synchronization of delayed neural networks in finite time.

It should be pointed out that the synchronization controllers proposed in [10–20] are effected under the ideal assumption of linear input. Generally, the control schemes for synchronization of neural networks can be realized by physical actuators. Therefore, it is possible that the practical control input may be nonlinear because of limitations of physical actuators. On the other hand, input nonlinearity can lead to serious degradation of system performance, and even cause system failure. Based on such reasons, it is of great importance to take the effects of input nonlinearity into account when designing some controllers to achieve synchronization of neural networks [21–24]. In [21] and [22], global synchronization conditions were respectively proposed for delayed neural networks subject to dead-zone nonlinearity and sector nonlinearity. In [23], Tang et al. investigated the exponential synchronization of stochastic jumping neural networks with mixed delays and sector nonlinearity. Recently, local synchronization was considered in [24] for chaotic neural networks with sampled-data and saturating actuators.

For the synchronization of neural networks subject to input nonlinearity, it is observed that the references [21–24] are concerned only with the continuous-time case. To the best of our knowledge, synchronization of delayed discrete-time neural

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networks subject to input nonlinearity has not been well investigated. In fact, when implementing and applications of neural networks, discrete-time neural networks play a more important role than their continuous-time counterparts in today's digital world [13,17,19,25]. Moreover, the discrete-time neural networks have already been applied in a wide range of areas, such as image processing, time series analysis, quadratic optimization problems, and system identification. On the other hand, saturation nonlinearity, which can be seen as a special kind of sector nonlinearity, is often inevitable in practice since physical actuators cannot deliver unlimited signals to the controlled plants [26–29]. Generally speaking, two main approaches are adopted to deal with saturation nonlinearity. One approach is to take actuator saturation into accounted at the beginning of control design [26–28], and another approach is to first ignore the actuator saturation when designing a controller, and then design an anti-windup compensator to weaken the influence caused by actuator saturation [29].

Based on the above discussions, this paper investigates the synchronization problem of delayed discrete-time neural networks subject to saturation nonlinearity, where the controller is the time-delay state feedback. Based on the drive-response concept, and using generalized delay-dependent sector condition, appropriate L–K functionals, and some finite-sum inequalities, delay-dependent LMI-based local synchronization conditions are well proposed for discrete-time neural networks with time-varying and constant delays. As the by-products, global synchronization conditions are also obtained by using classical sector condition. Finally, numerical examples and simulations are given to show the effectiveness of the proposed conditions in this paper. The main contributions of this paper can be summarized as follows: (1) the generalized delay-dependent sector condition is proposed to deal with the saturated time-delay feedback; (2) delay-dependent local synchronization conditions are first time established for delayed discrete-time neural networks subject to saturated time-delay feedback; (3) global synchronization conditions are proposed for delayed discrete-time neural networks subject to saturated time-delay feedback.

Notation: The superscript "T" stands for the matrix transposition. R^n and $R^{n\times n}$ denote the n-dimensional Euclidean space and set of all $n\times n$ real matrices, respectively. $\lambda_M(P)$ denotes the maximum eigenvalue of matrix P. A real symmetric matrix $P>0$ (≥ 0) means that P is positive definite (positive semi-definite). I denotes an identity matrix with proper dimension. The symmetric terms in a symmetric matrix are denoted by $*$. $\|\cdot\|$ denotes the Euclidean vector norm. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation

Consider the following discrete-time neural network with interval time-varying delay

$$x(k+1)=Ax(k)+Bg(x(k))+Cg(x(k-\tau(k))), \tag{1}$$

$$x(k)=\phi_1(k),\quad k\in[-\tau_2,0], \tag{2}$$

where $x(k)=[x_1(k),x_2(k),\ldots,x_n(k)]^T\in R^n$ is the neuron state vector; $g(x(k))=[g_1(x_1(k)),g_2(x_2(k)),\ldots,g_n(x_n(k))]^T\in R^n$ denotes the activation function; $A=diag\{a_1,a_2,\ldots,a_n\}$ is a diagonal matrix, B and C are the connection weight matrix and the delayed connection weight matrix, respectively; $\tau(k)$ denotes the time-varying transmission delay satisfying $\tau_1\le\tau(k)\le\tau_2$, where τ_1 and τ_2 are positive integers; $\phi_1(k)$ is the initial condition.

It is assumed that the activation function $g(x)$ in model (1) satisfies the following condition:

$$0\le\frac{g_i(\zeta_1)-g_i(\zeta_2)}{\zeta_1-\zeta_2}\le\sigma_i,\quad \forall\zeta_1,\zeta_2\in R,\zeta_1\ne\zeta_2,\ i=1,2,\ldots,n, \tag{3}$$

where $\sigma_i, i=1,2,\ldots,n$, are some positive scalars.

In this paper, we consider the model (1)–(2) as the drive system, and the response system is given as follows

$$y(k+1)=Ay(k)+Bg(y(k))+Cg(y(k-\tau(k)))+sat(u(k)), \tag{4}$$

$$y(k)=\phi_2(k),\quad k\in[-\tau_2,0], \tag{5}$$

where $\phi_2(k)$ denotes the initial condition, and $sat(u)\in R^n$ is the standard vector-valued saturation function, which represents that the actuator is subject to amplitude limitation and is described by $sat(u)=[sat(u_1)\ sat(u_2)\ \cdots\ sat(u_n)]^T$ with $sat(u_l)=\text{sgn}(u_l)\min\{|u_l|,\overline{u}_l\},\overline{u}_l>0,\ \ i=1,2,\ldots,n.$

Let $e(k)=y(k)-x(k)$ be the error state, then the synchronization error system can be described as follows

$$e(k+1)=Ae(k)+Bf(e(k))+Cf(e(k-\tau(k)))+sat(u(k)), \tag{6}$$

$$e(k)=\phi_2(k)-\phi_1(k)\triangleq\phi(k),\quad k\in[-\tau_2,0], \tag{7}$$

where $f(e(k))=g(y(k))-g(x(k))$. It is obvious from (3) that the function $f_i(e_i(k))=g_i(y_i(k))-g_i(x_i(k))$ satisfies $f_i(0)=0$ and the following condition

$$0\le\frac{f_i(e_i(k))}{e_i(k)}=\frac{g_i(y_i(k))-g_i(x_i(k))}{y_i(k)-x_i(k)}\le\sigma_i,\quad e_i(k)\ne0,\ i=1,2,\ldots,n. \tag{8}$$

As in most literature [12–17], this paper employs the following time-delay state feedback controller

$$u(k)=Ke(k)+K_\tau e(k-\tau(k)), \tag{9}$$

where $K,K_\tau\in R^{n\times n}$ are the gain matrices to be designed.

Introduce the dead-zone nonlinearity $\psi(u(k))=u(k)-sat(u(k))$, then from (6) and (9), one can obtain the following error system

$$e(k+1)=(A+K)e(k)+K_\tau e(k-\tau(k))+Bf(e(k))+Cf(e(k-\tau(k)))-\psi(u(k)). \tag{10}$$

Let $v(k)=Ux(k)+U_\tau x(k-\tau(k))$, where $U,U_\tau\in R^{n\times n}$, and assume that the following inequalities hold

$$|u_l(k)-v_l(k)|=|(K_l-U_l)x(k)+(K_{\tau l}-U_{\tau l})x(k-\tau(k))|\le\overline{u}_l,\quad l=1,2,\ldots,n, \tag{11}$$

where u_l,v_l are the lth elements of the vectors u,v, respectively, and $K_l,U_l,K_{\tau l},U_{\tau l}$ are the lth rows of the matrices K,U,K_τ,U_τ, respectively, then for any diagonal and positive definite matrix $H\in R^{n\times n}$, similar to the proof of Lemma 1 in [29], it can be concluded that the following generalized delay-dependent sector condition holds

$$\psi^T(u(k))H[\psi(u(k))-v(k)]=\psi^T(u(k))H[\psi(u(k))-Ux(k)-U_\tau x(k-\tau(k))]\le0. \tag{12}$$

Remark 1. The time-delay controller (9) was widely used in existing literature (see [12–17]). The main benefit of introducing the delayed feedback term $K_\tau e(k-\tau(k)$ in (9) is that the design invariable K_τ will be introduced in the proposed conditions, and as a result, the proposed conditions may be more slack. On the other hand, if one sets $K=0$ in (9), the resulted controller $u(k)=K_\tau e(k-\tau(k))$ is more practical in some engineering applications.

Remark 2. The proposed generalized sector condition (12) is delay-dependent, which utilizes the information of time-varying delay $\tau(k)$ and is different from that in [29]. Due to the introduction of the time-delay term $U_\tau x(k-\tau(k)$ in (12), it can be seen that