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Exponential passivity analysis of stochastic neural networks with leakage, distributed delays and Markovian jumping parameters ☆

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ABSTRACT

This paper deals with the problem of exponential passivity of Markovian jumping stochastic neural networks with leakage and distributed delays. By constructing a proper Lyapunov–Krasovskii functional, utilizing the free-weighting matrix method and some stochastic analysis techniques, we deduce new delay-dependent sufficient conditions that ensure the passivity of the proposed model. These sufficient conditions are computationally efficient and they can be solved numerically by linear matrix inequality (LMI) toolbox in Matlab. Finally, numerical examples are given to verify the effectiveness and the applicability of the proposed results.

1. Introduction

During the last two decades, neural networks (NNs) have been applied successfully in many areas, because of their massive potential applications in modern culture of science and technology. Its potential applications are optimization, data mining, fixed-point computations and mechanics of structures and materials [1–3]. During the implementation of neural networks, the occurrence of time delays are unavoidable due to its finite switching speed of amplifiers in electronic networks. It is often encountered in many practical systems, for example, aircraft stabilization, biological systems, chemical systems, hydraulic systems, metallurgical processing systems, nuclear reactor and electrical networks. It is well known that the existence of time delays in a system may cause instability or bad system performance. Therefore, the study of neural network with time delays has received considerable attention [4–10].

The passivity theory has long been a nice tool for analyzing the stability of systems, which has been applied in diverse areas such as stability, complexity, signal processing, chaos control and synchronization, and fuzzy control [11]. Recently, the problem of passivity analysis for delayed neural networks has been addressed in [12–17]. On the other hand, in implementation of many practical systems such as aircraft, chemical, biological systems and electric circuits, there exist occasionally stochastic perturbations. It is no less important than the time-delay as a considerable factor affecting dynamics in the systems. Therefore, stochastic modeling with time-delays plays an important role in many fields of science and engineering applications. For this reason, various approaches to passivity criteria for stochastic system with time-delays have been investigated in the literature [18,19a,19b,20]. Also, neural networks usually have a spatial extent due to the presence of a multitude of parallel pathways with a variety of axon sizes and lengths. Thus, there will be a distribution of conduction velocities along these pathways and a distribution of propagation delays [21]. In this circumstance the signal propagation is not instantaneous and cannot be modeled with discrete delays. And a more appropriate way is to incorporate continuously distributed delays in neural network models. Recently, there has been a growing interest in the study of neural networks with discrete and distributed delays [22–24].

Recently, the effect of leakage delay in dynamical neural networks is one of the research topics and it has been studied by many researchers in the literature. As pointed out in the literature [25], the time delay in stabilizing negative feedback term has a tendency to destabilize the system. Furthermore, sometimes it has

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more significant effect on dynamics of neural networks than other kinds of delays. Hence, the leakage term also has great impact on the dynamical behavior of neural networks. Therefore, there are many authors considering the problem of neural networks with leakage delay in the literature [26–31].

On the other hand, as discussed in [32], a neural network sometimes has finite modes that switch from one to another at different times, and such a switching (or jumping) can be governed by a Markovian chain. Applications of this kind of stochastic systems can be found in modelling production systems, economic systems, and other practical systems. To deal with this situation, neural networks with Markovian jumping parameters, which are also called Markovian jump neural networks, have been widely used to model the above complex systems. Generally speaking, a Markovian jump neural network is a hybrid system with a state vector that has two components $x(t)$ and $r(t)$, whereas the first component $x(t)$ is referred to as the state, and the second component $r(t)$ is a continuous-time Markov chain with a finite state space $S=\{1,2,\dots,N\}$, which is usually regarded as the mode. In its operation, this class of neural networks will switch from one mode to another in a random way, which is determined by a continuous-time Markov chain $r(t)$. Notably, the control and stability problems for dynamical systems with Markovian jumping parameters have already been widely studied, see e.g. [33–37]. In [17], passivity issue has been studied first time for delayed recurrent neural networks with Markovian jumping parameters. Recently, the exponential passivity problem of neural networks has been studied in [38], where some sufficient conditions have been achieved for the considered neural networks to be exponential passivity. It is worth pointing out that exponential passivity implies passivity, but the converse does not necessarily hold. Unfortunately, due to its mathematical complexity, the exponential passivity criteria have not yet been addressed in the literature for stochastic Markovian jumping neural network with the effects of leakage and distributed time-delays. This situation motivates our present investigation.

Based on the above discussion, we consider the problem of robust exponential passivity for Markovian jumping stochastic neural networks with leakage and distributed time-delays. By employing a proper Lyapunov–Krasovskii functional and stochastic analysis approach as well as the LMI technique, delay-dependent passivity criteria have been established to guarantee the global exponential passivity of the suggested system. Finally numerical examples are provided to show the merits of the proposed theoretical results.

Notations: Throughout this paper, $\mathcal{R}^n$ and $\mathcal{R}^{n\times m}$ denote the n-dimensional Euclidean space and the set of all $n\times m$ real matrices, respectively. The notation $X\ge Y$ (respectively, $X>Y$), means that the matrix $X-Y$ is positive semi-definite (respectively, positive definite). Let $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\ge 0},\mathcal{P})$ be the complete probability space with a filtration $\{\mathcal{F}_t\}_{t\ge 0}$ satisfying the usual conditions (i.e., the filtration contains all $\mathcal{P}-$null sets and is right continuous). $\omega(t)$ be a scalar Brownian motion defined on the probability space. $\mathcal{E}[\cdot]$ is the mathematical expectation operator with respect to the given probability measure $\mathcal{P}$. $\|\cdot\|$ denotes the Euclidean norm of a vector and its induced norm of a matrix. $\lambda_{max}(W)$ and $\lambda_{min}(W)$ mean its largest and smallest eigenvalues, respectively. The notation $*$ always denotes the symmetric block in one symmetric matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation

Let $\{r(t),t\ge 0\}$ be a right-continuous Markov chain on the probability space taking values in a finite state space $S=\{1,2,\dots,N\}$ with transition probabilities given by

$$\mathcal{P}\{r(t+\Delta t)=j|r(t)=i\}=\begin{cases}\pi_{ij}\Delta t+o(\Delta t), & i\ne j,\\ 1+\pi_{ii}\Delta t+o(\Delta t), & i=j,\end{cases}$$

where $\Delta t\ge 0$, $\lim_{\Delta t\to 0}(o(\Delta t)/\Delta t)=0$, and $\pi_{ij}\ge 0$, for $j\ne i$, is the transition rate from mode i at time t to mode j at time $t+\Delta t$ and $\pi_{ii}=-\sum_{j=1,j\ne i}^{N}\pi_{ij}$; consider the following stochastic neural networks with leakage delays and Markovian jumping parameters

$$\begin{aligned}dx(t)&=\Big[-A(r(t))x(t-\delta)+B(r(t))g(x(t))+C(r(t))g(x(t-\tau(t)))\\&\quad+D(r(t))\int_{t-r(t)}^{t}g(x(s))\,ds+u(t)\Big]\,dt+\sigma(x(t),x(t-\delta),x(t-\tau(t)),\\&\qquad x(t-r(t)),t,r(t))\,d\omega(t),\\ z(t)&=g(x(t))+g(x(t-\tau(t)))+u(t),\end{aligned}\tag{1}$$

Remark 1. The constant time delay δ which is known as leakage delay exists in the negative feedback term of stochastic neural network (1), which is different from the existing works [28]. This kind of time delay has a quick tendency to destabilize a system. Moreover, sometimes it has more significant effect on dynamics of neural networks than other kinds of delays. The main feature of our newly constructed system is leakage delays, which are considered within the stochastic effects of the corresponding neural network. To the best of authors knowledge there is no works having such kind of system model and it is very challenging. On the other hand, considering the general neural networks, stochastic jumping neural networks is an advanced model. We have noted that noise disturbances and Markovian jumping parameters play an essential role in the passivity analysis of stochastic jumping neural networks. The passivity criteria in [28] fail to apply the system (1) because they have ignored the effects of noise disturbances and Markovian jump parameters. This motivates us to consider the leakage delay effects on the Markovian jumping stochastic neural network with distributed delays.

For convenience, in the neural networks (1) each possible value of $r(t)$ is denoted by i, $i\in S$ in the sequel. Then, we have

$$A(r(t))=A_i,\quad B(r(t))=B_i,\quad C(r(t))=C_i,\quad D(r(t))=D_i,$$

in system (1). $x(t)=[x_1(t),x_2(t),\dots,x_n(t)]^T\in\mathcal{R}^n$ is the state vector associated with the neurons. $g(x(t))=[g_1(x_1(t)),g_2(x_2(t)),\dots,g_n(x_n(t))]^T$ is the activation function, $u(t)=[u_1(t),u_2(t),\dots,u_n(t)]^T$ is an external input vector to neurons, $z(t)$ be the output vector of neuron networks. $A_i=diag\{a_{1i},a_{2i},\dots,a_{ni}\}$ is a positive diagonal matrix. B_i,C_i,D_i are known matrices. $\omega(t)$ is a scalar zero mean Gaussian white noise process.

In the neural network (1), $\delta\ge 0$ denotes the constant leakage delay. $\tau(t)$ and $r(t)$ denote discrete and distributed time varying delays respectively, and it is assumed to satisfy

$$0\le\tau(t)\le\tau,\quad 0\le r(t)\le r,\quad \dot{\tau}(t)\le\mu,\quad \dot{r}(t)\le\eta,\tag{2}$$

where τ,r,μ and η are constants. The initial condition associated with model (1) is given by

$$x(t)=\phi(t),\quad \forall t\in[-\max\{\delta,\tau,r\},0].$$

Throughout this paper, we make the following assumptions and definition.

Assumption (H1). For any $j\in 1,2,\dots,n$, $g_j(0)=0$, there exist constants l_j^- and l_j^+ such that

$$l_j^-\le\frac{g_j(x_1)-g_j(x_2)}{x_1-x_2}\le l_j^+,\tag{3}$$

for any $x_1,x_2\in\mathcal{R}$, $x_1\ne x_2$.

Assumption (H2). Assume that $\sigma:\mathcal{R}^n\times\mathcal{R}^n\times\mathcal{R}^+\times S\to\mathcal{R}^n$ is locally Lipschitz continuous and satisfies the linear growth condition