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New and improved results for recurrent neural networks with interval time-varying delay ☆

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ABSTRACT

In this paper, the problem of stability analysis for a class of static recurrent neural networks with interval time-varying delay is considered. By constructing a newly augmented Lyapunov–Krasovskii functional containing triple integral terms and utilizing the inverses of first-order and squared reciprocally convex parameters techniques and zero equality, new and improved delay-dependent stability criteria are proposed to guarantee the asymptotic stability of the concerned networks with the framework of linear matrix inequalities (LMIs). Finally, some numerical examples are given to illustrate the effectiveness of the proposed methods.

1. Introduction

Over the past two decades, neural networks have been extensively applied in many areas such as reconstruction of moving image, signal processing, the tasks of pattern recognition, associative memories, fixed-point computations, and so on [1,2]. However, it is well known that for the finite speed limit of information processing and the inherent communication time of neurons, time delay is inevitably encountered in the implementation of networks. And the existence of time delay is often one of the main sources to cause poor performance and even instability of neural networks. As a result, numerous stability analysis criteria of delayed neural networks have been reported. In addition, to characterize the dynamical evolution rule of neural networks, according to the use of the neural states, the model of neural networks can be classified into recurrent neural networks [3]. In the recent years, many results on stability analysis of recurrent neural networks have been obtained in [4–20].

Most of the obtained results are always classified into the delay-independent criteria and delay-dependent one. The delay-independent stability criteria [26,27] can guarantee the stability of the system irrespective of the size of time-delay. Due to unexpected dynamic network behaviors such as oscillation and instability, the delay-dependent stability criteria [25,28-36] are concerned with the size of delay and provide an upper bound of time-delay size which assure the asymptotic stability of the system. Therefore, it is important to analysis the stability of delayed system in the literature.

Referring to recurrent neural networks, this network is also extended to the stability problem with time-delay. Because the lower bounds of time delay are not always restricted to be zero, time delay will appear time-varying property, see Refs. [11,20] and so on. So the interval time-varying delay $h(t)$ appears that time-varying property is a common situation. In addition, the constant time delay is a special case of the interval time-varying delay. In fact, it happens in the real world as we can see in the stock market, the decision making of trade-off is impacted by the information at time t and at time-varying $t-h(t)$. Recently, Zuo et al. [16] investigated the problem of delay-dependent stability for time-varying static neural networks by considering some semipositive-definite free matrices. In [17], the stability and dissipativity problems of static neural networks with time-varying delay were investigated by using the delay partitioning technique. In [18], Sun and Chen proposed the stability criteria for a class of static neural networks by constructing the augmented Lyapunov functional which fully uses the information about the lower of the delay and contains some new double integral and triple-integral terms. Li et al. [19] developed a unified approach for stability analysis of generalized static neural networks with time-varying delays and linear fractional uncertainties by utilizing some novel transformation and discretized scheme. In addition, Lian et al. [28] also derived the

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stability criteria for a class of switched recurrent neural networks with time-varying delays.

Here, for the purpose of conservative reduction, many techniques (such as free weighting matrix, delay decomposition, and Jensen's integral inequalities) have been employed in terms of linear matrix inequalities (LMIs). However, in [13–20], when constructing the Lyapunov–Krasovskii functional, most of the developed approaches in those do not make full use of the information about the time-varying delay $h(t)$ and only consider the low bound of the delay. In addition, previous convex method also only applies the inverses of first-order technique to tackle the time-varying delay. There still exists some conservatism for recurrent neural networks with interval time-varying delay to be further improved.

With this motivation above, in this paper, the problem of stability analysis for a class of static recurrent neural networks with interval time-varying delay is considered. The main contributions of this paper are reflected as follows: First, we construct a newly augmented Lyapunov–Krasovskii functional containing triple integral terms by utilizing the inverses of first-order and squared reciprocally convex parameter techniques and zero equality. Second, based on the reciprocally convex method [21], we directly handle function combinations arising from the triple integral terms. Third, both lower and upper bounds of time delays are taken into consideration, and the variations of the lower bound lemma are also introduced to deal with various kinds of function combinations. As a result, a new and improved less conservative delay-dependent stability criteria are proposed to guarantee the asymptotic stability of the concerned networks with the framework of linear matrix inequalities (LMIs). Numerical examples are given to illustrate the effectiveness of the proposed methods.

Notation: In this presentation, the following notations will be used. $\mathfrak{R}^n$ denotes the n-dimensional Euclidean vector space, and $\mathfrak{R}^{m\times n}$ is the set of all $m\times n$ real matrix. $*$ denotes the symmetric part. For symmetric matrices X and Y, $X>Y$ means that the matrix $X-Y$ is positive definite, whereas $X\geq Y$ means that the matrix $X-Y$ is nonnegative. I_n, 0_n and $0_{m\times n}$ denote $n\times n$ identity matrix, $n\times n$ and $m\times n$ zero matrices, respectively. $X^{\perp}$ denotes a basis for the null-space of X. col $\{x_1,x_2,\ldots,x_n\}$ means $[x_1^T,x_2^T,\ldots,x_n^T]^T$. The subscript '$T$' represents the transpose, diag $\{\cdots\}$ denotes the block diagonal matrix. For any matrix X, Sym $\{X\}$ means $X+X^T$. $X_{[f(t)]}\in\mathfrak{R}^{m\times n}$ means that the elements of matrix $X_{[f(t)]}$ include the scalar value of $f(t)$, i.e., $X_{[f_0]}=X_{[f(t)=f_0]}$.

2. Preliminaries

Consider a class of static recurrent neural network with time-varying delay described as follows:

$$\dot{y}(t)=-Ay(t)+g(Wy(t-h(t))+\mu) \tag{1}$$

where $y(t)=[y_1(t),y_2(t),\ldots,y_n(t)]^T\in\mathfrak{R}^n$ denotes the neuron state vector, $\mu=[\mu_1,\mu_2,\ldots,\mu_n]^T$ is constant input vector, $A=diag\{a_1,a_2,\ldots,a_n\}\in\mathfrak{R}^{n\times n}$ with $a_i>0,\ i=1,2,\ldots,n$, is a positive diagonal matrix, $g(Wy(\cdot))=[g_1(W_1y(\cdot)),g_2(W_2y(\cdot)),\ldots,g_n(W_ny(\cdot))]^T\in\mathfrak{R}^n$ denotes the neuron activation function vector, $W=[W_1^T,W_2^T,\ldots,W_n^T]^T\in\mathfrak{R}^{n\times n}$ is the delayed interconnection weight matrix. $h(t)$ is time-varying delay and satisfies

$$h_1\leq h(t)\leq h_2,\quad h_{12}=h_2-h_1, \tag{2}$$

and

$$\dot{h}(t)\leq u, \tag{3}$$

where h_1 and h_2 are known positive scalars, and u is a constant.

In addition, it is assumed that each neuron activation function in (1), $g_i(\cdot),\ i=1,2,\ldots,n$, satisfies the following condition:

Assumption 2.1. The neuron activation functions $g_i(\cdot),\ i=1,2,\ldots,n$ are continuous, bounded and satisfy

$$k_i^-\leq\frac{g_i(s_1)-g_i(s_2)}{s_1-s_2}\leq k_i^+,\quad \forall s_1,s_2\in\mathfrak{R},\ s_1\neq s_2, \tag{4}$$

where k_i^- and k_i^+ are known real constants.

By using the Brouwers fixed-point theorem, it can be easily proven that there exists one equilibrium point for (1). Assuming that $y^*=[y_1^*,y_2^*,\ldots,y_n^*]^T$ is an equilibrium point of (1) and using the transformation $x(\cdot)=y(\cdot)-y^*$, system (1) can be converted to the following system:

$$\dot{x}(t)=-Ax(t)+f(Wx(t-h(t))) \tag{5}$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T, f(Wx(\cdot))=[f_1(W_1x(\cdot)),f_2(W_2x(\cdot)),\ldots,f_n(W_nx(\cdot))]^T$ with $f(Wx(\cdot))=g(W(x(\cdot)+y^*)+\mu)-g(Wy^*+\mu)$. It is easy to see that $f_i(\cdot),\ i=1,2,\ldots,n$ satisfy the following condition:

$$k_i^-\leq\frac{f_i(s_1)-f_i(s_2)}{s_1-s_2}\leq k_i^+,\ \forall s_1,s_2\in\mathfrak{R},\ s_1\neq s_2,\ f_i(0)=0,\ i=1,2,\ldots,n. \tag{6}$$

From (6), if $s_2=0$, then we have

$$k_i^-\leq\frac{f_i(s_1)}{s_1}\leq k_i^+,\quad \forall s_1\neq 0. \tag{7}$$

And the conditions (6) and (7) are, respectively, equivalent to

$$[f_i(s_1)-f_i(s_2)-k_i^-(s_1-s_2)][f_i(s_1)-f_i(s_2)-k_i^+(s_1-s_2)]\leq 0, \tag{8}$$

$$[f_i(s_1)-k_i^-s_1][f_i(s_1)-k_i^+s_1]\leq 0. \tag{9}$$

The objective of this paper is to study delay-dependent stability conditions for system (5), the following lemmas play an important role in the derivation of the main results.

Lemma 2.1 (Park et al. [21]). *Let $f_1,f_2,\ldots,f_N:\mathfrak{R}^m\longrightarrow\mathfrak{R}$ have positive values in an open subset D of R^m. Then, the reciprocally convex combination of f_i over D satisfies*

$$\min_{\{a_i|\,a_i>0,\sum_i a_i=1\}}\sum_i\frac{1}{a_i}f_i(t)=\sum_i f_i(t)+\max_{g_{i,j}(t)}\sum_{i\neq j}g_{i,j}(t)$$

subject to

$$\left\{g_{i,j}:R^m\longrightarrow R, g_{i,j}(t)\triangleq g_{i,j}(t),\begin{bmatrix} f_i(t) & g_{i,j}(t)\\ g_{i,j}(t) & f_i(t)\end{bmatrix}\geq 0\right\}$$

Lemma 2.2 (Finsler's lemma [22]). *Let $\zeta\in\mathfrak{R}^n$, $\Phi=\Phi^T\in\mathfrak{R}^{n\times n}$, and $B\in\mathfrak{R}^{m\times n}$ such that $rank(B)<0$. Then, the following two statements are equivalent: (a) $\zeta^T\Phi\zeta<0$, $B\zeta=0$, $\zeta\neq 0$, (b) $(B^{\perp})^T\Phi B^{\perp}<0$, where $B^{\perp}$ is a right orthogonal complement of B.*

Lemma 2.3 (Wang et al. [23]). *For the symmetric appropriately dimensional matrices $\Omega>0,\Xi$, matrixΛ, the following two statements are equivalent: (a) $\Xi-\Lambda^T\Omega\Lambda<0$, (b) there exists a matrix of appropriate dimension Ψ such that*

$$\begin{bmatrix}\Xi+\Lambda^T\Psi+\Psi^T\Lambda & \Psi^T\\ \Psi & -\Omega\end{bmatrix}<0.$$

3. Main results

In this section, new stability criteria for the system (5) will be proposed by introducing a new Lyapunov functional and using a new method to estimate the derivative of the Lyapunov functional.

Theorem 3.1. *For given scalars $0<h_1<h_2$ and u, diagonal matrices $K_1=diag\{k_1^-,\ldots,k_n^-\}$, and $K_2=diag\{k_1^+,\ldots,k_n^+\}$, the system (5) is asymptotically stable for any time-varying delay satisfying (2), (3) if there exist positive define matrices $P=[P_{ij}]_{5\times 5}$, $Q=[Q_{ij}]_{3\times 3}$,*