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Robust passivity analysis for neutral-type neural networks with mixed and leakage delays ☆

R. Samidurai [a], S. Rajavel [a], Quanxin Zhu [b], R. Raja [c], Hongwei Zhou [d,*]

[a] Department of Mathematics, Thiruvalluvar University, Vellore 632115, India
[b] School of Mathematical Sciences and Institute of Finance and Statistics, Nanjing Normal University, Nanjing 210023, Jiangsu, China
[c] Ramanujan Centre for Higher Mathematics, Alagappa University, Karaikudi 630004, India
[d] School of Information Engineering, Nanjing Xiaozhuang University, Nanjing 211171, Jiangsu, China

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ABSTRACT

This paper investigates the problem of passivity of neutral-type neural networks with mixed and leakage delays. By establishing a suitable augmented Lyapunov functional and combining a new integral inequality with the reciprocally convex combination technique, we obtain some sufficient passivity conditions, which are formulated in terms of linear matrix inequalities (LMIs). Here, some useful information on the neuron activation function ignored in the existing literature is taken into account. Finally, some numerical examples are given to demonstrate the effectiveness of the proposed method.

1. Introduction

In the past several years, neural networks have been an important issue due to their applications in many areas such as signal processing, pattern recognition, associative memories, fixed-point computations, classification, parallel computation, control theory and optimization see [1–4]. In many engineering problems, the theory of dissipative systems which postulates the energy dissipated inside a dynamic system is less than the energy supplied from external source often links the stability problems. Passivity as special case of dissipativity plays an important role in the analysis. The main idea of the passivity theory is that the passive properties of a system can keep the system internally stable. The passivity theory intimately related to the circuit analysis and has received a lot of attention from the control community since 1970s. Passivity theory has a close relationship with the circuit analysis method and acts as an important role in the analysis of the stability of dynamical systems, complexity, chaos control, signal processing and fuzzy control [5–9]. Many nonlinear systems need to be passive in order to attenuate noises effectively, and the robustness measure (such as robust stability or robust performance) of a system often reduces to a subsystem or a modified system that is passive. Passivity analysis is a major tool for studying stability of uncertain or nonlinear systems especially for high-order systems, and thus the passivity analysis approach has been used in control problems for a long time to deal with robust stability problems for uncertain systems. Recently, increasing attention has been paid to the passivity of neural networks with time-varying delays see [10–18].

On the other hand, in certain physical systems, mathematical models have been described by some functional differential equations of neutral type. Functional differential equation of neutral type depends on the delays of state and state derivative. Practically, neutral type phenomenon always appear in the studies of automatic control, chemical reactors, distributed networks, dynamic process including steam and water pipes, population ecology, heat exchanges, microwave oscillators, systems of turbojet engine, lossless transmission lines, and vibrating masses attached to an elastic bar. Recently, Rakkiyappan et al. [19] studied the new global exponential stability results for neutral type neural networks with distributed time delays. Park et al. [21] discussed the synchronization of cellular neural networks of neutral type via dynamic feedback controller. Samidurai et al. [26] studied the global exponential stability of neutral-type impulsive neural

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* Corresponding author.
E-mail address: hwzhou122@163.com (H. Zhou).

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networks with discrete and distributed delays and reference therein see [19–26]. Hence, passivity analysis for neutral type neural networks have been considered in the recent years. However, so far, very little attention has been paid to stability analysis of neural networks with time delay in leakage (or forgetting) term. In fact, the leakage term has also great impact on the dynamical behavior of neural networks. For instance, it is shown in [32] that time delay in the stabilizing negative feedback term could destabilize a system. Gopalsamy [33] investigated the bidirectional associative memory (BAM) neural networks with constant delay in the leakage term. Recently, Li and Cao [34] discussed the delay-dependent stability of neural networks of neutral type with time delays in the leakage term. Balasubramaniam et al. [35] investigated the passivity analysis for neural networks of neutral-type with Markovian jumping parameters and time delay in the leakage term and the reference therein see [32–37]. Very recently, a new integral inequality based on Wirtinger's inequality was proposed in [31], which is less conservative than other integral inequalities derived by Jensen's inequality in [39]. This motivates our research.

In this paper, new delay-dependent robust passivity analysis for neutral-type neural network with discrete and distributed and leakage delays. By constructing an augmented Lyapunov functional and using a new integral inequality and integral term in the derivative of Lyapunov functional, passivity conditions are derived in terms of linear matrix inequality (LMI). Finally, some numerical examples are given to demonstrate the effectiveness of the proposed result.

Notations: Throughout this paper, the superscript T denotes the transpose, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space; $P>0(P\geq 0)$ means that the matrix P symmetric and positive definite (semi-positive definite); $diag\{\cdots\}$ denotes a block-diagonal matrix; $\|x\|$ is the Euclidean norm of z; I and 0 represent the identity matrix and a zero matrix, respectively.

2. Problem formulation and preliminaries

In this paper, the following neural networks of neutral-type with mixed and leakage delays are considered:

$$\dot{x}(t) = -Ax(t-\sigma)+Wg(x(t))+W_1g(x(t-\tau(t))$$
$$+W_2\int_{t-\tau(t)}^{t} g(x(s))\,ds+W_3\dot{x}(t-h(t))+u(t), y(t)=g(x(t)),$$
$$x(t)=\phi(t),\ -d\leq t\leq 0, \tag{1}$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T\in\mathbb{R}^n$ is the neuron state vector; $u(t)$ and $y(t)$ are the input and output vector, respectively; $g(\cdot)=[g_1(\cdot),g_2(\cdot),\ldots,g_n(\cdot)]^T$ denotes the neuron activation functions; $A=diag(a_1,a_2,\ldots,a_n)>0$, W,W_1,W_2, and W_3 are known interconnection weight matrices, and the delays, $\tau(t)$, $h(t)$, are the time-varying delays with $0\leq\tau(t)\leq\overline{\tau},\dot{\tau}(t)\leq\mu$ and $0\leq h(t)\leq\overline{h}$. The leakage delay $\sigma\geq 0$ is a constant. $\phi(t)$ is the initial condition and $t\in[-d,0]$, where $d=\max[\sigma,\overline{\tau},\overline{h}]$.

(H1) The function $g_i(\cdot)$ in (1) is continuous and satisfies

$$F_i^-\leq\frac{g_i(\alpha_1)-g_i(\alpha_2)}{\alpha_1-\alpha_2}\leq F_i^+ \tag{2}$$

where $g_i(0)=0,\alpha_1,\alpha_2\in\mathbb{R},\alpha_1\neq\alpha_2$, and F_i^- and F_i^+ are known real scalars.

Remark 2.1. Assumption (H1) of this paper is the same as in [29], where the constants are allowed to be positive, negative or zero, that is to say, the activation function under Assumption (H1) may be non-monotonic, non-differentiable, and unbounded. Hence, Assumption (H1) is weaker than the assumption in [10,11,13].

Definition 2.2 (*Li and Liao [9]*). The neural network (1) is said to be passive if there exists a scalar $\gamma\geq 0$ such that for all $t_f\geq 0$

$$2\int_0^{t_f} y(s)^Tu(s)\,ds\geq -\gamma\int_0^{t_f} u(s)^Tu(s)\,ds. \tag{3}$$

under the zero initial condition.

Lemma 2.3 (*Fu et al. [39]*). *For any symmetric positive definite matrix $M>0$, a scalar $\gamma>0$ and a vector function $\omega:[0,\gamma]\rightarrow\mathbb{R}^n$ such that the integrations concerned are well defined, the following inequality holds:*

$$\left(\int_0^{\gamma}\omega(s)\,ds\right)^T M\left(\int_0^{\gamma}\omega(s)\,ds\right)\leq\gamma\left(\int_0^{\gamma}\omega^T(s)M\omega(s)\right). \tag{4}$$

Lemma 2.4 (*Park et al. [30]*). *Given positive integers m and n, a scalar $\beta\in(0,1)$, a given $R>0$, and two matrices $W_1,W_2\in\mathbb{R}^{n\times m}$, define, for all vector $\xi\in\mathbb{R}^m$, the function $\Theta(\beta,R)$ described by:*

$$\Theta(\beta,R)=\frac{1}{\beta}\xi^TW_1^TRW_1\xi+\frac{1}{1-\beta}\xi^TW_2^TRW_2\xi. \tag{5}$$

Then, if there exists a matrix $X\in\mathbb{R}^{n\times n}$ such that $\begin{bmatrix} R & X\\ X^T & R\end{bmatrix}>0$ the following inequality holds:

$$\min_{\beta\in(0,1)}\Theta(\beta,R)\geq\begin{bmatrix} W_1\xi\\ W_2\xi\end{bmatrix}^T\begin{bmatrix} R & X\\ X^T & R\end{bmatrix}\begin{bmatrix} W_1\xi\\ W_2\xi\end{bmatrix}. \tag{6}$$

Lemma 2.5 (*Seuret and Gouaisbaut [31]*). *For any positive matrix Z and for differentiable signal x in $[\alpha,\beta]\rightarrow\mathbb{R}^n$, the following inequality holds:*

$$\int_{\alpha}^{\beta}\dot{x}^T(u)Z\dot{x}(u)\,du\geq\frac{1}{\beta-\alpha}\hat{\Omega}^T\hat{Z}\hat{\Omega} \tag{7}$$

where $\hat{Z}=diag\{Z,3Z\}$ and

$$\hat{\Omega}=\begin{bmatrix} x(\beta)-x(\alpha)\\ x(\beta)+x(\alpha)-\frac{2}{\beta-\alpha}\int_{\alpha}^{\beta}x(u)\,du\end{bmatrix}.$$

Lemma 2.6 (*Petersen and Hollot [27]*). *Let H,E, and $F(t)$ be real matrices of appropriate dimensions with $F(t)$ satisfying $F^T(t)F(t)\leq I$. Then, for any scalar $\varepsilon>0$,*

$$HF(t)E+(HF(t)E)^T\leq\varepsilon^{-1}HH^T+\varepsilon E^TE.$$

3. Main results

Firstly, for simplicity of vector and matrix representation, the following are denoted:

$$\eta_1(t)=\left[x^T(t)\ g^T(x(t))\right]^T,$$
$$\eta_2(t)=\left[x^T(t)\ x^T(t-\tau(t))\ x^T(t-\overline{\tau})\ x^T(t-h(t))\ x^T(t-\overline{h})\ x^T(t-\sigma)\ \int_{t-\sigma}^{t}x^T(s)\,ds\right]^T,$$
$$\eta_3(t)=\left[g^T(x(t))\ g^T(x(t-\tau(t))\ \int_{t-\tau(t)}^{t}g^T(x(s))\ \int_{t-\overline{\tau}}^{t-\tau(t)}g^T(x(s))\ u^T(t)\right]^T,$$
$$\eta_4(t)=\left[\frac{1}{\tau(t)}\int_{t-\tau(t)}^{t}x^T(s)\,ds\ \frac{1}{\overline{\tau}-\tau(t)}\int_{t-\overline{\tau}}^{t-\tau(t)}x^T(s)\,ds\ \frac{1}{h(t)}\int_{t-h(t)}^{t}x^T(s)\,ds\ \frac{1}{\overline{h}-h(t)}\int_{t-\overline{h}}^{t-h(t)}x^T(s)\,ds\right]^T,$$
$$\xi(t)=\left[\eta_2^T(t)\ \eta_3^T(t)\ \eta_4^T(t)\ \dot{x}^T(t)\ \dot{x}^T(t-h(t))\right]^T,$$