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Image denoising and enhancement based on adaptive fractional calculus of small probability strategy

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ABSTRACT

This paper presents two methods to deal with the problem that traditional image denoising algorithms may easily neglect image texture details. The first one is global adaptive fractional integral algorithm (GAFIA) which deals with common noises. It selects the optimal integral order of each pixel based on the local average gradient. The second is image denoising and enhancement algorithm based on adaptive fractional calculus of small probability strategy (AFC-SPS) which deals with salt & pepper noise. It regards the appearance of noise points as small probability events, divides them, and segments the image edges and weak textures by the improved two-dimensional Otsu algorithm. Then, the function of adaptive fractional order is constructed. Experimental results show that, both of the methods have good image denoising effect, and the AFC-SPS algorithm has a better effect than other methods in enhancing the edge and preserving the texture.

1. Introduction

A digital image is inevitably affected by noise during acquisition and transmission, which results in certain difficulties in subsequent image processing, such as image segmentation, and image recognition. Therefore, research on image denoising has become an important component of digital image processing. Related studies have proposed several classic image denoising algorithms, such as mean, median, low-pass, Wiener, and other filtering methods derived from the preceding methods [1–6]. Although these methods can reduce image noise to various degrees, they may cause loss of image details, including edge and texture while image denoising. That was because of these algorithms adopting integer-order integral directly or indirectly while constructing a denoising model.

On the other hand, the introduction of fractional calculus [7–9] to image processing has recently become a novel research direction. Fractional differential algorithm could preserve the information of weak texture, while enhancing the edge of image [10–15]. In particular, fractional integral algorithm based on R–L definition (FIA-1) and G–L definition (FIA-2) are used for image denoising in [16,17]. Recently, Ref. [18] proposed a new mathematical method by using fractional Alexander polynomials for image denoising, and Ref. [19] proposed an improved fractional order differential masks which result in a 45° isotropic rotation and appropriately smooth the image. However, since these algorithms adopted the same fractional order specified manually for global image, they are only based on the frequency response curve of the same order while processing different intensity noise points and hard to achieve a satisfactory denoising effect. Till now, to the author's knowledge, image denoising based on adaptive fractional calculus has not been explored fully with small probability strategy.

This paper presents two image denoising methods: global adaptive fractional integral algorithm (GAFIA) (applied to several types of common image noises) and adaptive fractional calculus of small probability strategy (AFC-SPS) (only applied to salt & pepper noise with low density). GAFIA uses the fractional integral to process every pixel of image and finds the optimal orders according to the average gradient of the pixels (in eight directions). Then, the pixels are processed by the frequency response curves of the optimal orders to get a relatively good denoising effect. Since the experimental results of GAFIA show a good performance in salt & pepper denoising, this paper further puts forwards the AFC-SPS method which regards the appearance of the salt & pepper noise points as small-probability events and divides them, and meanwhile the image edges and weak textures are segmented by the improved two-dimensional Otsu algorithm. Then, the function of adaptive fractional order is constructed based on the area feature of image. AFC-SPS algorithm is composed of the adaptive fractional integral algorithm (AFIA) and the adaptive fractional differential algorithm (AFDA). According to the characteristics of G–L definition mask, when the fractional order is adaptive and negative, the mask has the function of fractional integral (we call it AFIA); when the fractional order is adaptive and

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positive, the mask has the function of fractional differential (we call it AFDA). The divided noise points are processed by AFIA while the edges and weak textures are enhanced by AFDA, so the good effect of image denoising and enhancement is achieved.

2. Associated theoretical guidance and analysis

2.1. Effect of fractional calculus on signals and images

For an arbitrary square-integrable energy signal $s(t)\in L^2(R)$, the v-order differential operator D^v is the multiplicative operator of the v-order differential multiplier function while $v>0$, and its v-order fractional differential is:

$$D^v s(t)=\frac{d^v s(t)}{dt^v} \tag{1}$$

According to the basic theory of signal processing, its Fourier transform is:

$$\begin{aligned}D^v s(t) \overset{FT}{\Leftrightarrow} (\hat{D}^v s)(w)&=(iw)^v \hat{s}(w)=|w|^v \exp[i\theta(w)]\,\hat{s}(w)\\&=|w|^v \exp\left[\frac{v\pi i}{2}\operatorname{sgn}(w)\right]\hat{s}(w)\,,v>0\end{aligned} \tag{2}$$

Based on fractional order operator theory, fractional differential and fractional integral are inverse operations. $I=D^-$ denotes fractional integral operator. Suppose $v'=-v$ and achieve Fourier transform of fractional integral from (2):

$$\begin{aligned}I^{v'} s(t) \overset{FT}{\Leftrightarrow} (\hat{I}^{v'} s)(w)&=(iw)^{v'} \hat{s}(w)=|w|^{v'} \exp[i\varphi(w)]\,\hat{s}(w)\\&=|w|^{v'} \exp\left[\frac{v'\pi i}{2}\operatorname{sgn}(w)\right]\hat{s}(w), v'<0\end{aligned} \tag{3}$$

The frequency response of each fractional calculus order can be drawn according to the above formulas. Fig. 1a shows that the fractional differential operation increases the high-frequency (HF) portion of signal. With the increasing order and frequency of fractional differential, the increase rate shows a nonlinear rapid growth. Meanwhile, fractional differential preserves the low-frequency (LF) portion of signal to a certain degree non-linearly. Fig. 1b shows that the fractional integral operation attenuates the HF portion of signal. With the increasing order and frequency of fractional differential, the attenuation gradually increases. Therefore, fractional integral has image denoising effect, whereas fractional differential enhances and preserves image edge and texture.

2.2. Definitions of fractional calculus

No uniform expression of fractional calculus has been identified. Mathematicians have analyzed the problem from their different points of view and then proposed different definitions of fractional calculus. For example, calculus has three classical expressions, namely, G–L, R–L, and Capotu [19–22]. The G–L expression is deduced from the expression of integer-order differential, whereas the R–L and Capotu expressions are derived from integer-order integral Cauchy formula.

Gamma function is used to extend differential order from integeral to fractional. $[\cdot]$ represents the integer part. When order $v<0$, the fractional integral with v-order defined by G–L is

$${}_aD_t^v f(t)=\lim_{h\to 0} h^{-v}\sum_{j=0}^{[(t-a)/h]}(-1)^j\frac{\Gamma(v+1)}{j!\Gamma(v-j+1)}f(t-jh) \tag{4}$$

v-order fractional integral defined by R–L is

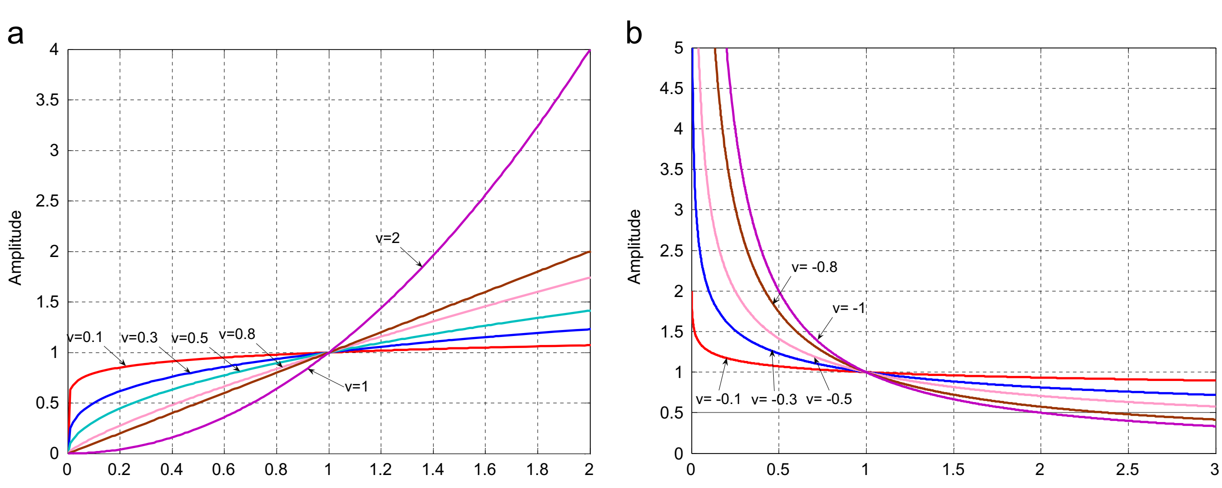
$${}_aD_t^v f(t)=\frac{1}{\Gamma(-v)}\int_a^t (t-\xi)^{-v-1}f(\xi)d\xi \tag{5}$$

2.3. The realization of fractional differential masks

When the duration of signal $f(t)$ is $t\in[a,t]$ divided into equal parts by the interval $h=1$, then $n=[\frac{t-a}{h}]=[t-a]$. When $v<0$, the

ξ_n	0	$\cdots$	0	ξ_n	0	$\cdots$	0	ξ_n
0	$\ddots$	$\ddots$	$\vdots$	$\vdots$	$\vdots$	$\iddots$	$\iddots$	0
$\vdots$	$\ddots$	ξ_3	0	ξ_3	0	ξ_3	$\iddots$	$\vdots$
0	$\cdots$	0	ξ_2	ξ_2	ξ_2	0	$\cdots$	0
ξ_n	$\cdots$	ξ_3	ξ_2	$8\xi_1$	ξ_2	ξ_3	$\cdots$	ξ_n
0	$\cdots$	0	ξ_2	ξ_2	ξ_2	0	$\cdots$	0
$\vdots$	$\iddots$	ξ_3	0	ξ_3	0	ξ_3	$\ddots$	$\vdots$
0	$\iddots$	$\iddots$	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\ddots$	0
ξ_n	0	$\cdots$	0	ξ_n	0	$\cdots$	0	ξ_n

Fig. 2. The superposition of partial integral mask by 8 directions.



Frequency response of each fractional differential order

Frequency response of each fractional integral order

Fig. 1. Frequency response of each fractional calculus order.