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Global μ-stability analysis of discrete-time complex-valued neural networks with leakage delay and mixed delays

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ABSTRACT

In this paper, the problem of μ-stability for discrete-time complex-valued neural networks with three kinds of time-delays including leakage delay, discrete delay and distributed delay is considered. Based on contraction mapping theorem and homeomorphism mapping theorem in complex domain, some sufficient conditions are proposed for the existence and uniqueness of the equilibrium point of the addressed neural networks. By constructing an appropriate Lyapunov–Krasovskii functional, and employing the matrix inequality techniques, several delay-dependent criteria for checking the global μ-stability of the complex-valued neural networks are established in linear matrix inequalities (LMIs), which can be checked numerically using the effective YALMIP Tool in MATLAB. As direct applications of these results, we get some criteria on the exponential stability, power-stability and log-stability of the neural networks.

1. Introduction

The nonlinear dynamic systems are ubiquitous in the real world [1–8]. As one type of the most important nonlinear systems, the complex-valued neural networks (CVNNs) have attracted an increasing research interest in recent years. This is mainly due to their widespread applications in physical systems dealing with electromagnetic, light, ultrasonic, and quantum waves. For example, see [9–12] and the references therein. It has been shown that such applications strongly depend on the stability of CVNNs. Therefore, stability analysis of CVNNs has received much attention and various stability conditions have been obtained [13–26].

In general, the dynamical systems can be classified into two types, such as continuous systems and discrete systems. In [13–20], the continuous-time CVNNs with delays were considered, and the boundedness, complete stability and exponential stability were investigated. In [21–24], the discrete-time CVNNs were investigated, and some conditions for the boundedness, global attractivity, complete stability, global exponential stability as well as global exponential periodicity of the considered neural networks were derived. In [25–27], the global stability was investigated for CVNNs on time scales, which is useful to unify the continuous-time and discrete-time CVNNs under the same framework.

Recently, the authors proposed the power-rate global stability of the equilibrium of neural networks in [28]. In addition, a new concept of global μ-stability was proposed to unify the exponential stability, power-rate stability and log-stability of neural networks in [29]. Thereafter, papers about μ-stability of neural networks emerged in large numbers consistently. In [30], the authors investigated the global robust μ-stability in the mean square for a class of stochastic neural networks. In [31], the delayed neural systems with impulsion were considered, and the μ-stability criteria were derived by using Lyapunov–Krasovskii functional method. In [32], the multiple μ-stability of delayed neural networks was investigated, and several criteria for the co-existence of equilibrium points and their local μ-stability were derived. In [33], the CVNNs with unbounded time-varying delays were considered, and several delay-dependent criteria for checking the global μ-stability of the addressed complex-valued neural networks were established in linear matrix inequality. In [34], the authors

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investigated the μ-stability of impulsive CVNNs with leakage delay and mixed delays, and obtained several sufficient conditions to ensure the global μ-stability. In [35], the global μ-stability was investigated for the CVNNs with leakage time delay and unbounded time-varying delays, and several sufficient and delay-dependent criteria were established to ensure the global μ-stability of the neural networks. To the best of our knowledge there is no results on μ-stability of discrete-time CVNNs with the three kinds of time-delays including leakage delay, discrete delay and distributed delay in the literature, and remains as a topic for further investigation.

Motivated by the above discussions, in this paper, we will consider the problem of μ-stability for discrete-time CVNNs with leakage delay, discrete delay and distributed delay. Based on contraction mapping theorem and homeomorphism mapping theorem in complex domain, some sufficient conditions for the existence and uniqueness of the equilibrium point of the addressed CVNNs are proposed. Several delay-dependent criteria for checking the global μ-stability of the discrete-time CVNNs are obtained by constructing an appropriate Lyapunov–Krasovskii functional, and employing the matrix inequality techniques. The obtained results can also be applied to several special cases and we obtain some criteria on the exponential stability, power-stability and log-stability of the CVNNs, correspondingly. Finally, two illustrative examples are provided to show the effectiveness of the proposed criteria.

Notations: The notations are quite standard. Throughout this paper, let $\mathbb{Z}^+$ denote the set of positive integers. Let i denote the imaginary unit, i.e. $i=\sqrt{-1}$. $\mathbb{C}^n$, $\mathbb{R}^{m\times n}$ and $\mathbb{C}^{m\times n}$ denote, respectively, the set of n-dimensional complex vectors, $m\times n$ real matrices and $m\times n$ complex matrices. The subscripts T and $*$ denote matrix transpose and matrix conjugate transpose, respectively. For complex vector $z\in\mathbb{C}^n$, let $|z|=(|z_1|,|z_2|,\ldots,|z_n|)^T$ be the module of the vector z, and $\|z\|=\sqrt{\sum_{k=1}^{n}|z_k|^2}$ be the norm of the vector z. I denotes the identity matrix with appropriate dimensions. The notation $X\ge Y$ (respectively, $X>Y$) means that $X-Y$ is positive semi-definite (respectively, positive definite). $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ are defined as the largest and the smallest eigenvalue of positive definite matrix P, respectively. The notation $\star$ always denotes the conjugate transpose of block in a Hermitian matrix. In addition, for $x\in\mathbb{R}$, the notation $\lfloor x\rfloor$ denotes the largest integer not greater than x, while the notation $\lceil x\rceil$ denotes the smallest integer not less than x.

2. Problems formulation and preliminaries

Consider the following complex-valued neural networks with leakage delay and mixed delays described by a non-linear difference equation of the form

$$z_i(k+1)=a_iz_i(k-\delta)+\sum_{j=1}^{n}b_{ij}f_j(z_j(k))+\sum_{j=1}^{n}c_{ij}f_j(z_j(k-\tau(k)))+\sum_{j=1}^{n}d_{ij}\sum_{l=1}^{+\infty}\rho_lf_j(z_j(k-l))+J_i,\quad i=1,2,\ldots,n, \tag{1}$$

or, in an equivalent vector form

$$z(k+1)=Az(k-\delta)+Bf(z(k))+Cf(z(k-\tau(k)))+D\sum_{l=1}^{+\infty}\rho_lf(z(k-l))+J, \tag{2}$$

where $z(k)=(z_1(k),z_2(k),\ldots,z_n(k))^T\in\mathbb{C}^n$ is the state vector of the neural networks; $A=\text{diag}\{a_1,a_2,\ldots,a_n\}\in\mathbb{R}^{n\times n}$ is the self-feedback connection weight matrix with $a_i>0$ $(i=1,2,\ldots,n)$; B, C and $D\in\mathbb{C}^{n\times n}$ are, respectively, the connection weight matrix, the discretely delayed connection weight matrix and distributively delayed connection weight matrix; $f(z(t))=(f_1(z_1(t)),f_2(z_2(t)),\ldots,f_n(z_n(t)))^T\in\mathbb{C}^n$ represents the neuron activation function; $J=(J_1,J_2,\ldots,J_n)^T\in\mathbb{C}^n$ is the external input vector; $\delta\ge 0$, $\tau(k)\ge 0$ and $l>0$ are the leakage time delay, the discrete time delay and the distributed time delay, respectively.

Remark 1. The term $z(k-\delta)$ in system (2) corresponds to a stabilizing negative feedback of the system which acts instantaneously with time delay. The term is variously known as leakage (or "forgetting") term. As pointed out by Gopalsamy in [36], leakage time delay in the stabilizing negative feedback term has a tendency to destabilize a system.

Remark 2. The term $f(k-\tau(k))$ represents the discrete delay where the time-varying delay $\tau(k)$ might not be bounded. Time delays describe a kind of ubiquitous phenomenon present in real systems where the rate of change of the state depends on not only the current state of the system but also its state at some time in history [37].

Remark 3. The delay term $\sum_{l=1}^{+\infty}\rho_lf(z(k-l))$ in system (2) is the so-called infinitely distributed delay in the discrete-time setting, which can be regarded as the discretization of the infinite integral form

$$\int_{-\infty}^{t}k(t-s)f(z(s))\,ds$$

for the continuous-time system. It is noted that the inclusion of such a distributed delay term will bring additional difficulties in the stability analysis and some special inequalities will need to be developed.

In the analysis of complex-valued neural networks, it is usually assumed that the activation functions are differentiable in [13,15]. However, in this paper, we adopt the following assumption (**H1**) on the activation functions in which the differentiability is not be required. Meanwhile, we give the convergence assumption (**H2**) on the parameters in the distributed delay term.

(**H1**) For each $j=1,2,\ldots,n$, the neuron activation function f_j satisfies

$$|f_j(z_1)-f_j(z_2)|\le\gamma_j|z_1-z_2|, \tag{3}$$

for any $z_1,z_2\in\mathbb{C}$, where γ_j is a real constant. Moreover, we define $\Gamma=\text{diag}\{\gamma_1^2,\gamma_2^2,\ldots,\gamma_n^2\}$.

(**H2**) The real constants $\rho_l\ge 0$ and satisfy the following convergence conditions:

$$\sum_{l=1}^{+\infty}\rho_l<+\infty\quad\text{and}\quad\sum_{l=1}^{+\infty}l\rho_l<+\infty. \tag{4}$$