Neurocomputing 106 (2013) 31–41

Contents lists available at SciVerse ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Robust adaptive fuzzy output feedback control for stochastic nonlinear systems with unknown control direction ☆

Tong Wang, Shaocheng Tong *, Yongming Li

Department of Mathematics, Liaoning University of Technology, Jinzhou, Liaoning 121000, China

ARTICLE INFO

Article history:
Received 19 June 2012
Received in revised form
14 September 2012
Accepted 18 October 2012
Communicated by H. Zhang
Available online 16 November 2012

Keywords:
Stochastic nonlinear systems
Fuzzy adaptive backstepping control
Changing supply function
Nussbaum function
K-filters

ABSTRACT

This paper discusses the problem of adaptive fuzzy output feedback control for a class of uncertain stochastic nonlinear strict-feedback systems. The concerned systems have certain characteristics, such as unknown nonlinear functions, dynamical uncertainties, unknown control direction and unmeasured state variables. In this paper, the fuzzy logic systems are used to approximate the unknown nonlinear functions, and a filter state observer is developed for estimating the unmeasured states. To solve the problems of the dynamical uncertainties and the unknown control direction, the changing supply function and Nussbaum function techniques are incorporated into the backstepping recursive design technique, and a new robust adaptive fuzzy output feedback control approach is constructed. It is proved that the proposed control approach can guarantee that all the signals of the resulting closed-loop system are bounded in probability, and also that the observer errors and the output of the system can be regulated to a small neighborhood of the origin by choosing design parameters appropriately. A simulation example is provided to show the effectiveness of the proposed approach.

1. Introduction

In the past decades, many approximation-based adaptive backstepping control approaches have been developed to control uncertain nonlinear strict-feedback systems with unstructured uncertainties via fuzzy-logic-systems (FLSs) or neural-networks (NNs), see for example [1–19]. Works in [1–9] are for single-input and single-output (SISO) nonlinear systems, works in [10–12] are for multiple-input and multiple-output (MIMO) nonlinear systems, and works in [11–19] for SISO or MIMO nonlinear systems with immeasurable states, respectively. In general, these adaptive fuzzy or neural network backstepping control approaches provide a systematic methodology of solving control problems of unknown nonlinear systems, where fuzzy systems or neural networks are used to model the uncertain nonlinear systems, and then an adaptive fuzzy or neural network controller is developed based on the backstepping design principle. Two of the important features of these adaptive approaches include (i) they can be used to deal with those nonlinear systems without satisfying the matching conditions, and (ii) they do not require the unknown nonlinear functions being linearly parameterized. Therefore, nowadays, the approximation-based adaptive fuzzy backstepping control has become one of the most popular design approaches in nonlinear control field.

It is well known that stochastic systems have been used in a variety of fields, such as chemical process, biology, ecology, and also can be found in control and information systems. The investigation on stochastic systems have received considerable attention in the past years, and a great number of the results have been reported in the literature, see for example [20–30]. Adaptive backstepping controllers are proposed in [20,21] for stochastic nonlinear systems in strict-feedback form. Adaptive backstepping controllers are developed in [22–27] for stochastic nonlinear systems with time delays or stochastic jump systems. Moreover, several adaptive output-feedback controllers are investigated in [28–30] for strict-feedback stochastic nonlinear systems by using linear state observer. However, the above mentioned results are only suitable for those nonlinear systems with nonlinear dynamics models being known exactly or with the unknown parameters appearing linearly with respect to known nonlinear functions, they can not be applied to those stochastic systems with structured uncertainties.

In order to deal with the structured uncertainties included in the stochastic nonlinear systems in strict-feedback form, by combining the fuzzy logic systems and neural networks with the backstepping design technique, several adaptive NN or fuzzy backstepping control schemes have been developed. For example, [31–33] proposed adaptive fuzzy output feedback control approach for a class of SISO stochastic nonlinear systems, while [34 and 35] extended the above results to a class of stochastic large-scale

☆ This work was supported by the National Natural Science Foundation of China (Nos.61074014, 61203008), and Program for Liaoning Innovative Research Team in University (LT2012013).

* Corresponding author.
E-mail address: jztsc@sohu.com (S. Tong).

0925-2312/$ - see front matter © 2012 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.neucom.2012.10.013

nonlinear systems. However, the aforementioned adaptive fuzzy or NN controllers have two main limitations as follows: One is that they do not consider the problem of unmodeled dynamics. The other is that they assume that the control directions are known. As stated in [15,18], the unmodeled dynamics and the unknown signs of control directions often exist in many practical nonlinear system, they are also the major source of resulting in the instability of the control systems. Therefore, to study the stochastic nonlinear systems with consideration of dynamical uncertainties and the unknown control directions is very important in control theory and applications. To handle the unknown control directions, Authors in [36,37] proposed adaptive fuzzy and NN backstepping control methods for stochastic nonlinear systems by using the Nussbaum function technique, and the stabilization properties of the control systems are achieved. However, the adaptive control schemes in [36,37] are restricted to a class of stochastic nonlinear systems without the unmodeled dynamics. Moreover, they need the assumption that all the states are available for the controllers design.

Motivated by the above observation, in this paper, an adaptive fuzzy output feedback control scheme is investigated for a class of stochastic nonlinear systems without satisfying the matching condition. The considered stochastic nonlinear systems include unknown nonlinear functions, dynamical uncertainties and unknown control direction, and unmeasured states. In the control design, fuzzy logic systems are first employed to model the uncertain nonlinear systems, and then a fuzzy filters is developed estimate the unmeasured states. To solve the problems of the unknown control direction and unmodeled dynamics, by introducing the Nussbaum functions and the changing supply function technique into the backstepping recursive design, a new robust adaptive fuzzy backstepping output feedback control scheme is constructed. It is demonstrated that all the variables of the resulting closed-loop system are bounded in probability, and that the observer errors and the output of the system converge to a small neighborhood of the origin by choosing design parameters appropriately.

2. System descriptions and preliminary results

2.1. System descriptions and basic assumptions

Consider the following uncertain stochastic nonlinear system in strict feedback form:

$$d\zeta = q_1(\zeta,y)dt + q_2(\zeta,y)dw$$

$$dx_i = [x_{i+1} + f_i\left(\underline{x}_i\right) + \Delta_i(x,\zeta)]dt + g_i(x)dw$$

$$i = 1,\ldots,n-1,$$

$$dx_n = [b_0\eta(y)u + f_n\left(\underline{x}_n\right) + \Delta_n(x,\zeta)]dt + g_n(x)dw \; y = x_1 \tag{1}$$

where $\underline{x}_i = [x_1,x_2,\ldots,x_i]^T \in R^i, i = 1,2,\ldots,n$ $(x = \underline{x}_n)$ are the states, u and y are the control and output of the system, ζ is unmodeled dynamics and $\Delta_i(x,\zeta)$ are the dynamic disturbances. $f_i\left(\underline{x}_i\right)$, $i = 1,2,\ldots,n$ are unknown smooth nonlinear functions. $q_1(\zeta,y)$, $q_2(\zeta,y)$, $\Delta_i(x,\zeta)$ and $g_i(x)$ are uncertain functions; $\eta(y) \neq 0$is a known smooth nonlinear function; $b_0 \neq 0$ is unknown constant and the sign of b_0 is unknown; $w \in R$ is an independent standard Wiener process defined on a complete probability space. In this paper, it is assumed that the functions$f_i\left(\underline{x}_i\right)$,$g_i(x)$,$q_i(\zeta,y)$ and $\Delta_i(x,\zeta)$ satisfying the locally Lipschitz condition, and only the output$y$is available for measurement.

Assumption 1. ([30,31]): For each $1 \le i \le n$, there exist unknown positive constantsp_i^* such that

$$|\Delta_i(x,\zeta)| \le p_i^*\psi_{i1}(y) + p_i^*\psi_{i2}(|\zeta|)$$

$$|g_i(x)| \le p_i^*\psi_{i3}(y)$$

where$\psi_{i1}(y)$, $\psi_{i2}(|\zeta|)$ and $\psi_{i3}(y)$ are known nonnegative smooth functions with $\psi_{i1}(0) = \psi_{i2}(0) = \psi_{i3}(0) = 0$.

Assumption 2. **([30,31]):** For each ζ -subsystem in (1), there exist function $V_\zeta(\zeta)$ and known k_∞ functions $\underline{\alpha}(|\zeta|)$, $\overline{\alpha}(|\zeta|)$, $\alpha(|\zeta|)$, $\gamma(|y|)$, ψ_ζ and $\psi_0$such that

$$\underline{\alpha}(|\zeta|) \le V_\zeta(\zeta) \le \overline{\alpha}(|\zeta|) \text{ and } \ell V_\zeta \le \gamma(|y|) - \alpha(|\zeta|)$$

$$|\partial V_\zeta/\partial\zeta| \le \psi_\zeta(|\zeta|) \;\text{ and }\; \|q_2(\zeta,y)\| \le \psi_0(|\zeta|)$$

Control objective: The control task is to design an adaptive output feedback controller using the output y and state estimations $\hat{x}_i$ so that all the variables of the closed-loop system are bounded in probability and the outputs of the system can be regulated to a small neighborhood of the origin in probability.

In order to cope with the unknown control direction, the Nussbaum gain technique is employed in this paper.

Definition 1. **([36,37]).** A function $N(\varsigma)$ is called a Nussbaum-type function if it has the following properties:

$$\lim_{s\to\infty} \sup \frac{1}{s} \int_0^s N(\varsigma)d\varsigma = \infty$$

$$\lim_{s\to\infty} \inf \frac{1}{s} \int_0^s N(\varsigma)d\varsigma = -\infty$$

From the above definition, we know that the Nussbaum functions have infinite gains and infinite switching frequencies. There are many functions satisfying the above conditions, for example,exp $(x^2)\cos((\pi/2)x)$, $x^2\cos(x)$ and $x^2\sin(x)$.

Let us give a useful lemma on stochastic differential equation as the following

$$dx = f(t,x)dt + h(t,x)dw \tag{2}$$

where the definition of x and w are the same as in (1). Denote $C^{2,1}: R^n \times R_+; R_+$ as the family of all nonnegative functions $V(x,t) \in R^n \times R_+$, which are continuously twice differentiable in x and one differentiable in t, and written as $C^{2,1}$ for simplification.

Lemma 1. ([36]). Consider the stochastic system (2), assume that there exists functions $V(x,t) \in C^{2,1}$, smooth function $\varsigma_1: R_+ \to R$ and Nussbaum type even function $N(\cdot)$; let χ be a nonnegative random variable, $M(t)$ be a real valued continuous local martingale with $M(0) = 0$ such that

$$V(x,t) \le \chi + e^{-ct}\int_0^t (b_0N(\varsigma_1)\dot{\varsigma}_1 + \dot{\varsigma}_1)e^{c\tau}d\tau + M(t)$$

Then the functions $V(x,t)$, $\varsigma_1(t)$ and $\int_0^t (b_0N(\varsigma_1)\dot{\varsigma}_1 + \dot{\varsigma}_1)d\tau$ must be bounded in probability.

2.2. Fuzzy logic systems

A fuzzy logic system (FLS) consists of four parts: the knowledge base, the fuzzifier, the fuzzy inference engine, and the defuzzifier. The knowledge base is composed of a collection of fuzzy. If-then rules of the following form:

R^l: If x_1 is F_1^l and x_2 is F_2^l and … and x_n is F_n^l,

Then y is $G^l, l = 1,2,\ldots,N$ where $x = (x_1,x_2\ldots x_n)^T$ and y are FLS input and output, respectively,$\mu_{F_i^l}(x_i)$ and $\mu_{G^l}(y)$ are the membership function of fuzzy sets F_i^l and G^l, N is the number of inference rules. Through singleton fuzzifier, center average defuzzification