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A hybrid quantum-inspired neural networks with sequence inputs

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ABSTRACT

To enhance the performance of classical neural networks, a quantum-inspired neural networks model based on the controlled-Hadamard gates is proposed. In this model, the inputs are discrete sequences described by a matrix where the number of rows is equal to the number of input nodes, and the number of columns is equal to the sequence length. This model includes three layers, in which the hidden layer consists of quantum neurons, and the output layer consists of classical neurons. The quantum neuron consists of the quantum rotation gates and the multi-qubits controlled-Hadamard gates. A learning algorithm is presented in detail according to the basic principles of quantum computation. The characteristics of input sequence can be effectively obtained from both *breadth* and *depth*. The experimental results show that, when the number of input nodes is closer to the sequence length, the proposed model is obviously superior to the BP neural networks.

1. Introduction

In many applications, the system input is a temporal processes, such as the chemical reaction process and the stock market volatility process [1,2]. Many neurophysiological experiments indicate that the information processing character of the biological nerve system mainly includes the following eight aspects: the spatial aggregation, the multi-factor aggregation, the temporal cumulative effect, the activation threshold characteristic, self-adaptability, exciting and restraining characteristics, delay characteristics, conduction and output characteristics [3]. From the definition of the M-P neuron model, classical ANN preferably simulates voluminous biological neurons' characteristics such as the spatial weight aggregation, self-adaptability, conduction and output, but it does not fully incorporate temporal cumulative effect because the outputs of ANN depend only on the inputs at the moment regardless of the prior moment. In the process of practical information processing, the memory and output of the biological nerve not only depend on the spatial aggregation of multidimensional input information, but also depend on the temporal cumulative effect.

Since Kak firstly proposed the concept of quantum-inspired neural computation [4] in 1995, quantum neural networks (QNN) have attracted a great attention by the international scholars during the past decade, and a large number of novel techniques have been studied for quantum computation and neural networks. For example, Ref. [5] proposed the model of quantum neural networks with multilevel hidden neurons based on the superposition of quantum states in the quantum theory. In Ref. [6], an attempt was made to reconcile the linear reversible structure of quantum evolution with nonlinear irreversible dynamics of neural networks. In 1998, a new neural networks model with quantum circuit was developed for quantum computation, and was proven to exhibit a powerful learning capability [7]. Matsui et al. developed a quantum neural networks model using the single bit rotation gate and two-bit controlled-not gate. They also investigated its performance in solving the four-bit parity check and the function approximation problems [8]. Altaisky suggested that a quantum neural networks can be built using the principles of quantum information processing [9]. In his model, the input and output qubits in the QNN were implemented by optical modes with different polarization, the weights of the QNN were implemented by optical beam splitters and phase shifters. Ref. [10] proposed a completely different kind of networks from the mainstream works. In his model neurons are states, connected by gates. In our previous work [11], we proposed a quantum BP neural networks model with learning algorithm based on the single-qubit rotation gates and two-qubit controlled-not gates. Ref. [12] proposed a wave probabilities resonance principle describing quantum entanglement, and demonstrated the possible applications of the theory. Ref. [13] presented models of quasi-non-ergodic probabilistic systems that are defined through the theory of wave probabilistic functions, and showed two illustrative examples of applications of introduced theories and models. Ref. [14] proposed a weightless model based on quantum circuit, it is not only quantum-inspired but it is

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actually a quantum NN. This model is based on Grover's search algorithm, and it can perform both quantum learning and simulate the classical models. However, like M-P neurons, it also do not fully incorporate temporal cumulative effect because a single input sample is either irrelative to time or relative to a moment instead of a period of time.

In order to fully simulate biological neuronal information processing mechanisms, and to enhance the approximation and generalization ability of ANN, in this paper, a novel quantum-behaved neural networks based on the controlled-Hadamard gates, called CHQNN, is proposed. Our networks are a three-layer model with a hidden layer, which employs the gradient descent principle for learning. The input/output relationship of this model is derived, based on the physical meaning of the quantum gates. The experimental results show that, under a certain condition, the CHQNN is obviously superior to the common BP neural networks.

2. The qubits and quantum gates

2.1. Qubits

In quantum computing, a qubit is a two-level quantum system, described by a two-dimensional complex Hilbert space. From the superposition principles, any state of the qubit may be written as

$$|\varphi\rangle = \cos\frac{\theta}{2}|0\rangle + e^{\mathrm{i}\phi}\sin\frac{\theta}{2}|1\rangle, \tag{1}$$

where $0 \le \theta \le \pi$, $0 \le \phi \le 2\pi$.

Therefore, unlike the classical bit, which can only be set equal to 0 or 1, the qubit resides in a vector space parametrized by the continuous variables θ and ϕ. Thus, a continuum of states is allowed. The Bloch sphere representation is useful in thinking about qubits since it provides a geometric picture of the qubit and of the transformations that one can operate on the state of a qubit. Owing to the normalization condition, the qubit's state can be represented by a point on a sphere of unit radius, called the Bloch Sphere. This sphere can be embedded in a three-dimensional space of Cartesian coordinates ($x = \cos\phi\sin\theta$, $y = \sin\phi\sin\theta, z = \cos\theta$). By definition, a Bloch vector is a vector whose components (x,y,z) single out a point on the Bloch sphere. We can say that the angles θ and ϕ define a Bloch vector, as shown in Fig. 1(a), where the points corresponding to the following states are shown: $|A\rangle = [1,0]^{\mathrm{T}}$, $|B\rangle = [0,1]^{\mathrm{T}}$, $|C\rangle = |E\rangle = [\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}]^{\mathrm{T}}$, $|D\rangle = [\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}]^{\mathrm{T}}$, $|F\rangle = [\frac{1}{\sqrt{2}}, -\mathrm{i}/\sqrt{2}]^{\mathrm{T}}$, $|G\rangle = [\frac{1}{\sqrt{2}}, \mathrm{i}/\sqrt{2}]^{\mathrm{T}}$.

For convenience, in this paper, we represent the qubit's state by a point on a circle of unit radius as shown in Fig. 1(b). The corresponding relations between Fig. 1(a) and (b) can be written as

$$\begin{cases} \alpha : 0 \longrightarrow \pi/2 \Leftrightarrow \phi = 0 \quad \text{and} \quad \theta : \pi/2 \longrightarrow 0 \\ \alpha : \pi/2 \longrightarrow \pi \Leftrightarrow \phi = \pi \quad \text{and} \quad \theta : 0 \longrightarrow \pi/2 \\ \alpha : \pi \longrightarrow 3\pi/2 \Leftrightarrow \phi = \pi \quad \text{and} \quad \theta : \pi/2 \longrightarrow \pi \\ \alpha : 3\pi/2 \longrightarrow 2\pi \Leftrightarrow \phi = 0 \quad \text{and} \quad \theta : \pi \longrightarrow \pi/2 \end{cases}. \tag{2}$$

At this time, any state of the qubit may be written as

$$|\varphi\rangle = \cos\alpha|0\rangle + \sin\alpha|1\rangle. \tag{3}$$

An n qubits system has 2^n computational basis states. For example, a 2 qubit system has basis $|00\rangle, |01\rangle, |10\rangle, |11\rangle$. Similar to the case of a single qubit, the n qubits system may form the superpositions of 2^n basis states

$$|\phi\rangle = \sum_{x \in \{0,1\}^n} a_x |x\rangle, \tag{4}$$

where a_x is called probability amplitude of the basis states $|x\rangle$, and $\{0,1\}^n$ means the set of strings of length two with each letter being either zero or one. The condition that these probabilities can sum to one is expressed by the normalization condition

$$\sum_{x \in \{0,1\}^n} |a_x|^2 = 1. \tag{5}$$

2.2. Quantum rotation gate

A *quantum gate* is the analogue of a logic gate in a classical computer, and quantum gates are basic units of quantum algorithms. The difference between the classical and quantum context is that a quantum gate has to be implemented reversibly and, in particular, must be a unitary operation. The definition of a single qubit rotation gate is given by

$$R(\theta) = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}. \tag{6}$$

Let the quantum state $|\phi\rangle = [\begin{smallmatrix}\cos\theta_0 \\ \sin\theta_0\end{smallmatrix}]$, then $|\phi\rangle$ can be transformed by $R(\theta)$ as follows:

$$R(\theta)|\phi\rangle = \begin{bmatrix} \cos(\theta_0 + \theta) \\ \sin(\theta_0 + \theta) \end{bmatrix}. \tag{7}$$

It is obvious that $R(\theta)$ shifts the phase of $|\phi\rangle$.

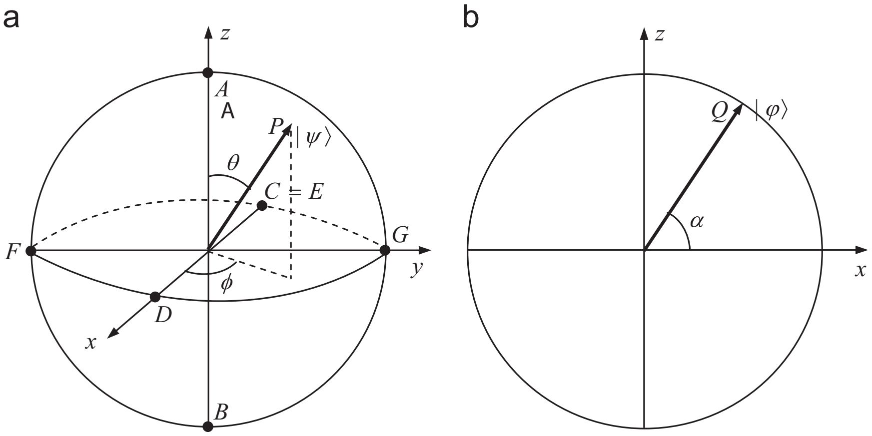

Fig. 1. A qubit description. (a) A qubit description on Bloch sphere. (b) A qubit description on unit circle.