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Understanding neurodynamical systems via Fuzzy Symbolic Dynamics

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ABSTRACT

Neurodynamical systems are characterized by a large number of signal streams, measuring activity of individual neurons, local field potentials, aggregated electrical (EEG) or magnetic potentials (MEG), oxygen use (fMRI) or activity of simulated neurons. Various basis set decomposition techniques are used to analyze such signals, trying to discover components that carry meaningful information, but these techniques tell us little about the global activity of the whole system. A novel technique called Fuzzy Symbolic Dynamics (FSD) is introduced to help in understanding of the multidimensional dynamical system's behavior. It is based on a fuzzy partitioning of the signal space that defines a non-linear mapping of the system's trajectory to the low-dimensional space of membership function activations. This allows for visualization of the trajectory showing various aspects of observed signals that may be difficult to discover looking at individual components, or to notice otherwise. FSD mapping can be applied to raw signals, transformed signals (for example, ICA components), or to signals defined in the time–frequency domain. To illustrate the method two FSD visualizations are presented: a model system with artificial radial oscillatory sources, and the output layer (50 neurons) of Respiratory Rhythm Generator (RRG) composed of 300 spiking neurons.

1. Introduction

Neuroimaging data and simulated neurodynamical systems are characterized by multiple streams of non-stationary data, and thus may be represented only in high-dimensional signal spaces. For example, functional magnetic resonance imaging (fMRI) provides thousands of streams corresponding to the changing activity of voxels, with sampling rate of a few hertz, and electroencephalographic (EEG) recordings hundreds of streams with sampling frequency of hundreds of hertz. High data volumes that quickly change in time make such signals very hard to understand. Popular signal processing techniques remove artifacts by various filtering techniques, involving waveform analysis, morphological analysis, decomposition of data streams into meaningful components using Fourier or Wavelet Transforms, Principal and Independent Component Analysis (PCA, ICA), etc. (Rangayyan, 2001; Sanei & Chambers, 2008). Interesting events are then searched for using processed signal components, with time–frequency–intensity maps calculated for each component.

Such techniques are very useful, but do not show global properties of processes in the high-dimensional signal spaces. Simulation of complex dynamics is usually described in terms of attractors, but precise characterization of their basins and possible transitions between them is rarely attempted. A mapping that separates interesting segments of the trajectory could help to categorize such events. Global analysis is needed to characterize different types of system's behavior, see how attractors trap dynamics, notice partial synchronization and desynchronization events, or filter the high frequency noise. For many applications (including brain–computer interfaces) a snapshot of the whole trajectory helping to understand its main characteristics, would be very useful. This is the goal of our paper, presenting a global approach to the high-dimensional signal analysis (to focus attention we shall talk about neurodynamics, although any dynamical system can be analyzed in this way).

Two inspirations have been important in the development of our approach. First, an observation that different brain areas probably "understand" and collaborate with each other by filtering the main properties of their large-scale activity, reacting to specific activations that may be roughly characterized in a symbolic way. The second inspiration comes from the successes of the symbolic dynamics (Hao & Zheng, 1998) in understanding and simplifying the description of dynamical systems. Symbolic dynamics may be used as an approximation to brain processes if hard partitioning of the activity of various brain regions is done and labeled by a finite set of symbols. However, such a discretization may for most applications be either too rough or require too many symbols to be useful. The Fuzzy Symbolic Dynamics (FSD) introduced in

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this paper is based on a few membership functions rather than a large set of symbols. To see the trajectory $\{x(t)\}$ of the whole system, localized membership functions, or "probes" that are activated by the trajectories that pass near their center, are placed in the signal space. Using k such membership functions $y_i(x(t))$, strategically placed in important points of the signal space, a non-linear reduction of dimensionality suitable for visualization of trajectories is achieved. Inevitably a lot of details will be lost but with a proper choice of parameters the information that correlates with observed behavior or an experimental task may be preserved, while irrelevant information will be suppressed.

A long-term goal of this research is to find the brain–mind transformation that maps the trajectory representing measured neural activity to the psychological space with dimensions that represent perceptions, intentions and other inner events that are part of our mental life. Many important properties of neurodynamics should be reflected in such relatively low-dimensional psychological spaces (Duch & Diercksen, 1995). The next section introduces the Fuzzy Symbolic Dynamics approach and defines the FSD mapping that captures some interesting properties of system's trajectories. To illustrate how to set up mapping parameters and how to interpret resulting images a very simple model of EEG sources is analyzed in Section 3, with radial and plain wave sources placed in a few points on a mesh, and sensors that record the amplitude of incoming waves in nodes of this mesh. As an example of real application in Section 4 visualization of trajectories of the neural Respiratory Rhythm Generator model (RRG) are analyzed. The final section contains a brief discussion with a list of many open questions.

The purpose of the visualization is to gain insight into general behavior of neurodynamical systems. For example, changing parameters of neurons will change the landscape of attractors that may potentially be reached. Although automatic quantization of some properties along the lines of recurrence plots, may be quite useful, this is beyond the topic of the present paper.

2. Fuzzy Symbolic Dynamics

Assume that some unknown sources create a multidimensional signal that is changing in time, for example an EEG signal measured by n electrodes:

$$x(t) = \{x_i(t)\} \quad i = 1, \ldots, n \; t = 0, 1, 2, \ldots. \tag{1}$$

Vectors $x(t)$ represent the state of the dynamical system at time t, forming a trajectory in the signal space. Observing the system for a longer time should reveal the landscape created by this trajectory, areas of the signal space where the state of the system is found with the highest probability, and other areas where it never wonders. Recurrence maps (Marwan, Romano, Thiel, & Kurths, 2007) and other techniques may be used to view some aspects of such trajectories, but do not capture many important properties that it reflects.

In the symbolic dynamics (Hao & Zheng, 1998) the signal space is partitioned into regions that are labeled with different symbols, emitted every time the trajectory is found in one of these regions. The sequence of symbols gives a coarse-grained description of dynamics that can be analyzed using statistical tools. Dale and Spivey (2005) and Spivey (2007) argue that symbolic dynamics gives an appropriate framework for cognitive representations, although discretization of continuous dynamical states loses the fluid nature of cognition. Symbols obviously reduce the complexity of dynamical description because the partitioning of highly-dimensional signal spaces into regions with sharply defined boundaries is highly artificial. However, the symbolic approach may help to simplify the dynamics and make it more understandable. In fact the common practice of showing the differences that contrast two experimental conditions using averaged fMRI activations is an extremely simplified version of symbolic labeling that loses all dynamical information.

The notion of the symbolic dynamics is generalized in a natural way to a Fuzzy Symbolic Dynamics (FSD). Instead of discrete partitioning of the signal space leading to a set of symbols, interesting regions are determined analyzing probability density $p(x)$ of finding the trajectory $x(t)$ at some point x, averaging the results over time with an appropriate smoothing kernel, $p(x) = \sum_i K(x; x(t_i))$ (Duda & Hart, 1973). Local maxima of this probability define quasi-stable states around which trajectories tend to cluster. Such maxima may serve as centers μ_k of prototypes associated with fuzzy membership functions $y_k(x; \mu_k)$ that measure the degree to which the $x(t)$ state belongs to the prototype μ_k. Membership functions may be defined using knowledge-based clustering (Pedrycz, 2005), or as prototype-based rules with context-based clustering techniques (Blachnik, Duch, & Wieczorek, 2006). Context is defined by questions that are of interest, for example discrimination between different experimental conditions, or searching for invariants in one of these condition. Such methods will improve upon naive clusterization by automatically finding optimal parameters of membership functions that should reveal differences between various conditions.

For visualization Gaussian membership functions are quite useful:

$$y_k(x; \mu_k, \Sigma_k) = \exp\left(-(x-\mu_k)^{\mathrm{T}} \Sigma_k^{-1} (x-\mu_k)\right). \tag{2}$$

Diagonal dispersions Σ_k are frequently sufficient, suppressing irrelevant signals, but in general covariance matrices (used in Mahalanobis distance) may extract more meaningful combinations of signals that correlate with experimental conditions, or with features of mapped signals that correspond to qualities of mental experience that are subjectively felt. Such a brain–mind mapping will be closer to the idea of cognitive representations than the symbolic dynamics that Dale and Spivey (2005) and Spivey (2007) advocate. They also stress "the continuity of mind", based on distributed patterns of neural activation. Such patterns may be approximated by fuzzy dynamics in a much better way than the purely symbolic description, generated by thresholding strongly activated prototypes. For example, sensorimotor actions cannot certainly be well approximated by symbolic labels.

Selecting only two or three prototypes is sufficient to visualize trajectories $x(t)$ in a two-dimensional space $y_i(t), y_j(t)$. For visualization each pair of functions should have sufficiently large dispersions σ_i and σ_j to cover the space between them, for example $\sigma_i = \sigma_j = \left\|\mu_i - \mu_j\right\|/2$. Visualizations in three dimensions require plotting transformed points for three clusters, one for each dimension. Dispersions should then be set to the largest among the 3 pairs. Pairwise plots can be used to observe the trajectory from different points of view. Normalization of vectors in the signal space is assumed. To distinguish several experimental conditions optimization of parameters of membership functions should be done using context-based clustering techniques, creating clear differences in corresponding maps. Adding more localized functions in some area where dynamics is complex will show fine structure of the trajectory.

An alternative to fuzzy membership functions is to define reference points R_i in the signal space, and measure the distance between the trajectory and these points $\|x(t) - R_i\|$, using some metric function. Non-linear metric functions should have some advantage in analysis of neurodynamics, as the influence of the trajectory on prototypes decreases sharply to zero with the distance, reflecting non-linear properties of neurons. We shall not consider here the problem of adaptation of membership function parameters, concentrating instead on the interpretation of global