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New sufficient conditions for global stability of neutral-type neural networks with time delays ☆

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ABSTRACT

This paper studies the equilibrium and stability properties of the class of neutral-type neural network model with discrete time delays. By employing a Lyapunov functional and examining the time derivative of the Lyapunov functional, we obtain some delay independent sufficient conditions for the existence, uniqueness and global asymptotic stability of the equilibrium point for this class of neutral-type systems. The obtained conditions can be easily verified as they can be expressed in terms of the network parameters only. We also compare our results with the previous corresponding results derived in the literature by giving some numerical examples.

1. Introduction

In recent years, various classes of neural networks such as Hopfield neural networks, Cohen–Grossberg neural networks, cellular neural networks, bidirectional associative memory neural networks have been widely used in solving some signal processing, optimization, image processing problems. When designing a neural network for such applications, it is crucial to know the equilibrium and stability of properties of the designed neural network that depends on the network parameters on the neural system. On the other hand, in the hardware implementation of dynamical neural networks, due to the finite switching speed of amplifiers and the transmission delays during the communication between neurons, some time delays inevitably occur in the network, which affects the dynamical behavior of the neural systems. Therefore, equilibrium and stability properties of various classes of neural networks at the presence of time delays have received a great deal of attention in the recent literature [1–34]. In the classical neural network models such as Hopfield neural networks, Cohen–Grossberg neural networks, cellular neural networks, bidirectional associative memory neural networks, the time delays are in the states of the neural system. However, since the time derivatives of the states are the functions of time, in order to completely determine the stability properties of equilibrium point, some delay parameters must be introduced into the time derivatives of states of the system. The neural network model having time delays in the time derivatives of states is called delayed neutral-type neural networks. This class of neutral systems has been used in many areas such as population ecology [11], distributed networks with lossless transmission lines [11], propagation and diffusion models [12] and VLSI systems [12]. In the recent literature, many researchers have studied the equilibrium and stability properties of neural networks of neutral type with a single delay and presented various sufficient conditions for the global asymptotic stability of the equilibrium point [1–32].The results obtained in these papers are basically expressed in the linear matrix inequality (LMI) forms. The LMI approach to the stability problem of neutral-type neural networks involves some difficulties with determining the constraint conditions on the network parameters as it requires to test positive definiteness of high dimensional matrices. In the current paper, by employing a suitable Lyapunov functional, we will present new delay-independent sufficient conditions for the existence, uniqueness and global asymptotic stability of the equilibrium point for the class of neutral-type neural networks with many delays. Our results establish various relationships between the network parameters only. Therefore, the results of this paper can be easily verified when compared with the previously reported literature results in the LMI forms.

Throughout this paper we will use those notations: For any matrix $P=(p_{ij})_{n\times n}$, $P>0$ will denote that P is symmetric and positive definite, P^T, P^{-1}, $\lambda_m(P)$ and $\lambda_M(P)$ will denote the transpose of P, the inverse of P, the minimum eigenvalue of P and the maximum eigenvalue of P, respectively. We will use the matrix norm

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$\|P\|_2=[\lambda_M(P^TP)]^{1/2}$. For any two positive definite matrices $P=(p_{ij})_{n\times n}$ and $Q=(q_{ij})_{n\times n}$, If $Q>0$, then $P>Q$ will imply that $P>0$. For $\upsilon=(\upsilon_1,\upsilon_2,\ldots,\upsilon_n)^T\in R^n$, we will use the vector norms $\|\upsilon\|_2=\sqrt{\sum_{i=1}^n \upsilon_i^2}$ and $\|\upsilon\|_1=\sum_{i=1}^n|\upsilon_i|$.

2. Problem statement

Consider the following set of nonlinear differential equations that describe the class of neutral-type neural network model with a single delay:

$$\dot{x}_i(t)=-c_ix_i(t)+\sum_{j=1}^n a_{ij}f_j(x_j(t))+\sum_{j=1}^n b_{ij}f_j(x_j(t-\tau))+\sum_{j=1}^n e_{ij}\dot{x}_j(t-\tau)+u_i,\quad i=1,..,n \tag{1}$$

By introducing more delay parameters into the system equations of delayed neutral-type neural networks, we obtain the generalization of neural network model (1), which is in the following form of differential equations:

$$\dot{x}_i(t)=-c_ix_i(t)+\sum_{j=1}^n a_{ij}f_j(x_j(t))+\sum_{j=1}^n b_{ij}f_j(x_j(t-\tau_j))+\sum_{j=1}^n e_{ij}\dot{x}_j(t-\tau_j)+u_i,\quad i=1,..,n \tag{2}$$

where n is the number of the neurons in the network, x_i denotes the state of the ith neuron, the parameters d_i are some constants that keep the solution of system (2) bounded. The constants a_{ij} denote the strengths of the neuron interconnections within the network, the constants b_{ij} denote the strengths of the neuron interconnections with time delay parameters $\tau_j(t)$. e_{ij} are coefficients of the time derivative of the delayed states. Finally, the functions $f_j(\cdot)$ denote the neuron activations, and the constants u_i are some external inputs. In system (2), $\tau_j\geq 0$ represents the delay parameter with $\tau=\max(\tau_j)$, $1\leq j\leq n$. Accompanying the neutral system (2) is an initial condition of the form: $x_i(t)=\phi_i(t)\in C([-\tau,0],R)$, where $C([-\tau,0],R)$ denotes the set of all continuous functions from $[-\tau,0]$ to R.

In this paper, the activation functions $f_i(\cdot)$, $i=1,2,\ldots,n$ are assumed to be Lipschitz continuous, i.e., there exist constants $\ell_i>0$ such that

$$|f_i(x)-f_i(y)|\leq \ell_i|x-y|,\quad i=1,2,\ldots,n\quad \forall x,y\in R, x\neq y \tag{3}$$

Neural network model (2) can be written in the vector–matrix form as follows:

$$\dot{x}(t)=-Cx(t)+Af(x(t))+Bf(x(t-\tau))+E\dot{x}(t-\tau)+u \tag{4}$$

where $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^T\in R^n$, $A=(a_{ij})_{n\times n}$, $B=(b_{ij})_{n\times n}$, $E=(e_{ij})_{n\times n}$, $C=diag(c_i>0)$, $u=(u_1,u_2,\ldots,u_n)^T$, $f(x(t))=(f_1(x_1(t)),f_2(x_2(t)),\ldots,f_n(x_n(t)))^T$ and $f(x(t-\tau))=(f_1(x_1(t-\tau_1)),f_2(x_2(t-\tau_2)),\ldots,f_n(x_n(t-\tau_n)))^T$.

The following lemma will play an important role in the proofs of our main results:

Lemma 1 (*Cheng et al.* [26]). *If a map $H(x)\in C^0$ satisfies the following conditions:*

(i) *$H(x)\neq H(y)$ for all $x\neq y$,*
(ii) *$\|H(x)\|\to\infty$ as $\|x\|\to\infty$,*

then, $H(x)$ is homeomorphism of R^n.

3. Existence and uniqueness analysis

In this section, we present some new delay independent sufficient conditions for the existence and uniqueness of the equilibrium point for neural network model (2) with the activation functions satisfying (3). We first obtain the following result:

Theorem 1. *Under the assumption given* (3), *the neural network model* (2) *has unique equilibrium point for each u if there exist a positive diagonal matrix D and positive definite matrices P, Q and R such that the following conditions hold*:

$$\Omega_1=(C^2-D^2)\mathcal{L}^{-2}-A^TA-A^TP^{-1}A-A^TQ^{-1}A>0$$

$$\Omega_2=D^2\mathcal{L}^{-2}-B^TB-B^TPB-B^TR^{-1}B>0$$

$$\Omega_3=I-E^TE-E^TQE-E^TRE>0$$

$\mathcal{L}=diag(\ell_1,\ell_2,\ldots,\ell_n)$ and I is the identity matrix of dimension of $n\times n$.

Proof. In order to prove the existence and uniqueness of the equilibrium point, we consider the following mapping associated with system (2):

$$H(x)=-Cx+Af(x)+Bf(x)+EH(x)+u \tag{5}$$

If $x^*=(x_1^*,x_2^*,\ldots,x_n^*)^T$ is an equilibrium point of (2), then x^* satisfies the following equation:

$$H(x^*)=-Cx^*+Af(x^*)+Bf(x^*)+EH(x^*)+u=0$$

It is obvious that $H(x)=0$ is an equilibrium point of (2). Therefore, we can directly conclude from Lemma 1 that, for the system defined by (2), there exists a unique equilibrium point for every input vector u if $H(x)$ is homeomorphism of R^n. We will now show that under the conditions of Theorem 1, $H(x)$ is a homeomorphism of R^n. Let us choose two vectors $x,y\in R^n$ such that $x\neq y$. Note that, under the assumptions on the activation functions given by (3), when $x\neq y$, we have either $f(x)\neq f(y)$ or $f(x)=f(y)$. Therefore, for $x\neq y$, the existence and uniqueness analysis must be carried out for the following two cases:

Case 1. $x\neq y$ and $f(x)\neq f(y)$. In this case, $H(x)$ defined by (5) satisfies:

$$H(x)-H(y)=-C(x-y)+A(f(x)-f(y))+B(f(x)-f(y))+E(H(x)-H(y)) \tag{6}$$

First multiplying both sides of (6) by $(2(x-y)C+H(x)-H(y))^T$, and then adding the term $(x-y)^TD^2(x-y)-(x-y)^TD^2(x-y)=0$ to the right hand side of the resulting equation yields

$$\begin{aligned}
&(2(x-y)^TC+(H(x)-H(y))^T)(H(x)-H(y))\\
&\quad=(2(x-y)^TC+(H(x)-H(y))^T)\times(-C(x-y)+A(f(x)-f(y))\\
&\quad+B(f(x)-f(y))+E(H(x)-H(y)))\\
&\quad=(C(x-y)+A(f(x)-f(y))+B(f(x)-f(y))+E(H(x)-H(y)))^T\\
&\quad\times(-C(x-y)+A(f(x)-f(y))+B(f(x)-f(y))+E(H(x)-H(y)))\\
&\quad+(x-y)^TD^2(x-y)-(x-y)^TD^2(x-y)
\end{aligned}$$

where D is a positive diagonal matrix. The above equation is equivalent to:

$$\begin{aligned}
&2(x-y)^TC(H(x)-H(y))\\
&\quad=-(H(x)-H(y))^T(H(x)-H(y))\\
&\quad-(x-y)^TC^2(x-y)+(x-y)^TCA(f(x)-f(y))\\
&\quad+(x-y)^TCB(f(x)-f(y))+(x-y)^TCE(H(x)-H(y))\\
&\quad-(f(x)-f(y))^TA^TC(x-y)+(f(x)-f(y))^TA^TA(f(x)-f(y))\\
&\quad+(f(x)-f(y))^TA^TB(f(x)-f(y))\\
&\quad+(f(x)-f(y))^TA^TE(H(x)-H(y))-(f(x)-f(y))^TB^TC(x-y)\\
&\quad+(f(x)-f(y))^TB^TA(f(x)-f(y))+(f(x)-f(y))^TB^TB(f(x)-f(y))
\end{aligned}$$