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Synchronization of switched complex dynamical networks with non-synchronized subnetworks and stochastic disturbances

Guang He [a,b], Jian-an Fang [a,*], Wenbing Zhang [c], Zhen Li [a]

[a] College of Information Science and Technology, Donghua University, Shanghai 201620, PR China
[b] Department of Mathematics, Anhui Polytechnic University, Anhui 241000, PR China
[c] Department of Mathematics, Yangzhou University, Jiangsu 225002, PR China

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ABSTRACT

In this paper, the global synchronization problem of switched complex dynamical networks (SCDNs) with stochastic disturbances is investigated. Different from the existing results concerning synchronization of switched complex networks, all the subnetworks of the SCDNs here are not self-synchronized. Based on the dwell time approach and the discretized Lyapunov function technique, a sufficient synchronization criterion is obtained in term of linear matrix inequalities (LMIs) and the corresponding switching signal is obtained. The obtained switching signal depends on time rather than the system states, which makes it easier to be implemented. Finally, two examples are provided to illustrate the effectiveness of the theoretical results.

1. Introduction

In the past few years, a strong upsurge of the study of complex networks has been witnessed in many fields of science, engineering and society [1–6]. Complex networks can exhibit many interesting phenomena, such as spatio-temporal chaos, synchronization, spiral waves, self-organization [7–9]. As the major collective behavior of complex networks, synchronization is one of the key issues that has been extensively investigated [10–14].

In practice, complex dynamical networks may be affected more or less by uncertainties such as unmodeled dynamics, link failure and new link creation that may happen at times [15]. And then the jumps between different topologies happen occasionally, for example, biological neural networks [16], flying object motions [16], power grid [17,18], and so on. As is well known, a large power grid consisting of a large number of local power generators can work properly only if the generators are kept in synchronism, and need to retain their stability to provide normal power supplies. If some local power generators cannot be synchronized, it may lead to instability of the power grid or collapse. Therefore, when a local power system happens to have a severe fault, it will be automatically cut off from the network by a relay protection device in order to avoid further the damage to the global power grid. It means that the topology structures switch from one to another [17,18]. Therefore, it is important to consider the switching networks when modeling the complex networks. Recently, synchronization of SCDNs has been a hot research issue, and various SCDNs have been proposed [7,17–24]. For example, in [19], synchronization of discrete time neural networks with node switching was addressed. In [17], exponential synchronization problem was investigated for a class of complex delayed dynamical networks with switching topology. In [7,20–24], synchronization of complex networks with both node and topology switching was studied.

With respect to the synchronization of SCDNs, two main problems have been investigated in the literature: (1) Developing synchronization conditions for the SCDNs under arbitrary switching signals; (2) Identifying the controlled switching signals under which the SCDNs can be synchronized. In regard to the first issue, recently, many significant results have been reported [25–27,18]. For example, in [18], synchronization problem for complex dynamical networks with switching topology was transformed into the stability problem for time-varying switched system and a common Lyapunov function was constructed. It is implied that the complex networks in [18] are synchronized under arbitrary switching signals. On the other hand, as is well known, even when all the subnetworks are self-synchronized, the whole network may fail to preserve synchronization under arbitrary

* Corresponding author.
E-mail addresses: hg-1211ok@163.com (G. He), jafang@dhu.edu.cn (J.-a. Fang), zwb850506@126.com (W. Zhang), lzqz12@hotmail.com (Z. Li).

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switching [17]. Therefore, it is of great significance to investigate the second problem, i.e., how to design the switching signal such that the complex networks with such switching signal can be synchronized. In the past decade, some useful methods have been proposed to design switching signals, for example, in [28], by using the maximal dwell time length of subnetworks and the average dwell time approach, stability of switched stochastic neural networks was investigated; In [17], by using the ratio of the total activation time of self-synchronized subnetworks and non-synchronized subnetworks, exponential synchronization of complex delayed dynamical networks with switching topology was addressed. However, in the above results, it is implicitly assumed that there exists at least one subsystem that is self-synchronized.

Due to disturbances, unmodeled dynamics or possible faults, some modes may be unstable in a given switched system [29]. As is well known, even if all subsystems are unstable, one may carefully orchestrate switching between unstable modes to make the switched system asymptotically stable [30,31]. Therefore, how to design appropriate switching signals to stabilize the switched system composed fully of unstable modes is very interesting. In the past decade, how to design the appropriate switching laws to stabilize the switched system composed of unstable subsystem has been discussed in [32–34]. This implies that the switching law plays a good role in synchronization under some circumstances. Although, the stability of switched systems with unstable subsystems has been investigated in [32–34]. However, synchronization problem of switched complex networks with non-synchronized subnetworks has received relative little attention primarily due to the coexistence of the coupling terms and the switching signals [18]. In [18], when all subnetworks are not self-synchronized, synchronization of the SCDNs was studied by designing restricted switching law depending on the state of nodes. But, in [18], the node switching was neglected and the coupled matrix is assumed to be simultaneous triangularization. Moreover, in [18], the switching law depends on the states, which is hard to be implemented. In this paper, we consider more general switching, e.g., both node and topology switching. Moreover, the considered switching signal only depends on time rather than system states.

On the other hand, when signals transmit between dynamical nodes, the state of each individual node is often subjected to various types of noise, uncertainty from external random fluctuations in the process of transmission and other probabilistic causes, which may lead information to be lost. These undesired phenomena may have a great influence on the behavior of dynamical networks [35]. Therefore, when investigating and simulating more realistic networks, it is important to take stochastic perturbations into account. In addition, the stochastic disturbances could better describe the dynamical behavior of a coupled complex network presented within a noisy environment [36]. Recently, synchronization of complex dynamical networks with stochastic perturbations has been extensively investigated in the literatures [35,37–39]. In [35], synchronization was investigated for a class of delayed complex dynamical networks with impulsive and stochastic effects. In [39], distributed robust synchronization was investigated for a class of dynamical networks with stochastic coupling. However, to the best of the authors' knowledge, the synchronization problem for complex networks with both switching and stochastic perturbations, in which all subnetworks are not self-synchronized, has not been fully investigated.

Motivated by the above discussions, in this paper, the synchronization problem is investigated for a class of SCDNs with non-synchronized subnetworks. Based on the dwell time approach and the discretized Lyapunov function technique, a synchronization criterion for such complex dynamical networks is obtained in terms of LMIs. The main contributions of this paper can be listed as follows: Eq. (1) all the subnetworks of the SCDNs are not self-synchronized; Eq. (2) the switching signals only depend on time rather than the state of nodes, which are easy to be physically implemented; Eq. (3) the effects of both switching and stochastic perturbations are simultaneously considered.

Notations: Throughout this paper, $\mathbb{N}$ and $\mathbb{R}^n$ denote, respectively, the set of nonnegative integers and the n-dimensional space. $\mathbb{R}^{m\times n}$ denote $m\times n$ real matrix. For vector $x\in\mathbb{R}^n$, $|x|$ and x^T denote, respectively, the Euclidean norm and its transpose. We use $\lambda_{\max}(\cdot)$ (respectively $\lambda_{\min}(\cdot)$) to denote the maximum (respectively the minimum) eigenvalue of a real matrix. The notation $A\le B$ (respectively $A<B$) means that the matrix $A-B$ is negative semidefinite (respectively negative definite). I_n is the identity matrix of order n.

2. Model and preliminaries

In this section, we first present the network model, then some basic lemmas, definitions and assumptions are given.

Consider the following SCDNs with stochastic disturbances:

$$d[x_i(t)] = [C_{\sigma(t)}x_i(t) + B_{\sigma(t)}f_{\sigma(t)}(x_i(t)) + \vartheta\sum_{j=1}^{N} a_{ij}^{\sigma(t)}\Gamma_{\sigma(t)}x_j(t)]\,dt + g_{\sigma(t)}(t,x_i(t))d[w(t)],\quad i=1,2,\dots,N \tag{1}$$

where ϑ is the coupling strength; $w(t)=[w_1(t),w_2(t),\dots,w_n(t)]^T$ is an n-dimensional Weiner process, and $w_i(t)$ is independent of $w_j(t)$ for $i\ne j$. $\sigma(t):[0,\infty)\to\mathfrak{M}=\{1,2,\dots,m\}$ is the switching signal, which is a piecewise constant function continuous from the right. For each fixed $\sigma(t)=r\in\mathfrak{M}$, $C_r\in\mathbb{R}^{n\times n}$ is a real matrix; $B_r\in\mathbb{R}^{n\times n}$ represents the connection weight matrix; $f_r(x_i(t))$ is the activation function, and $g_r:\mathbb{R}^+\times\mathbb{R}^n\to\mathbb{R}^{n\times n}$ is the noise intensity function matrix. $\Gamma_r\in\mathbb{R}^{n\times n}>0$ is the diagonal inner coupling matrix which represents the way of linking the components in each pair of connected two nodes; $A_r\in\mathbb{R}^{N\times N}$ is the outer coupling configuration matrix which represents the structure of the network in which a_{ij}^r is defined as follows: if there is a connection from node j to node i $(j\ne i)$, then $a_{ij}^r\ne 0$; otherwise, $a_{ij}^r=0$. The diagonal entries of matrix a_{ii}^r are determined by the following coupling condition:

$$a_{ii}^r = -\sum_{j=1,j\ne i}^{N} a_{ij}^r,\quad i=1,2,\dots,N,\quad r\in\mathfrak{M}. \tag{2}$$

Let $x(t)=[x_1^T(t),x_2^T(t),\dots,x_N^T(t)]^T$. $\mathbf{F}_r(x(t))=[f_r^T(x_1(t)),\ f_r^T(x_2(t)),\dots,f_r^T(x_N(t))]^T$, $\mathbf{G}_r(t,x(t))=[g_r^T(x_1(t)),g_r^T(x_2(t)),\dots,g_r^T(x_N(t))]^T$, $\Omega(t)=[w^T(t),\ w^T(t),\dots,w^T(t)]^T$. For a clear presentation, here we let $\mathbf{C}_r^N=I_N\otimes C_r$, $\mathbf{B}_r^N=I_N\otimes B_r$, $\mathbf{A}_r=A_r\otimes\Gamma_r$, Then, the SCDN in Eq. (1) can be rewritten in the following Kronecker product form:

$$d[x(t)] = [\mathbf{C}_r^N x(t) + \mathbf{B}_r^N\mathbf{F}_r(x(t)) + \vartheta\mathbf{A}_r x(t)]dt + \mathbf{G}_r(t,x(t))d[\Omega(t)]. \tag{3}$$

Remark 1. Due to disturbances, unmodeled dynamics or possible faults, all subnetworks may not be synchronized in a given switched complex network [18,29,40]. Therefore, in this situation, it is interesting to investigate how to tolerate the existence of non-synchronized subnetworks without destroying synchronization of the overall networks. Recently, the synchronization problems for SCDNs with non-synchronized subnetworks have been investigated in [7,17,18]. However, in the existing results concerning synchronization of SCDNs, it is implicitly assumed that there exists at least a subnetwork that is self-synchronized [7,17,20,25–27]. In this paper, we consider the SCDNs in Eq. (1) where all the subnetworks are not self-synchronized.

In order to derive the main results of this paper, we need the following assumptions, definitions and lemmas.