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Diagonal principal component analysis with non-greedy ℓ_1-norm maximization for face recognition

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ABSTRACT

Diagonal principal component analysis (DiaPCA) is an important method for dimensionality reduction and feature extraction. It usually makes use of the ℓ_2-norm criterion for optimization, and is thus sensitive to outliers. In this paper, we present a DiaPCA with non-greedy ℓ_1-norm maximization (DiaPCA-L1 non-greedy), which is more robust to outliers. Experimental results on two benchmark datasets show the effectiveness and advantages of our proposed method.

1. Introduction

Principal component analysis (PCA) is a classical tool for feature extraction and face recognition [1]. In the domain of image analysis, two-dimensional PCA (2DPCA) [2] and diagonal PCA (DiaPCA) [3] were developed to capture spatial information. The rationale behind 2DPCA is to select computational "samples" from the row vectors of an image, instead of the vector stacked by the whole image pixels in PCA [4,5]. In contrast to 2DPCA, DiaPCA seeks the optimal projective vectors from the row vectors of *diagonal images*, which makes DiaPCA better than 2DPCA in many aspects [3,6–8]. It is noteworthy that the algorithms including PCA, 2DPCA and DiaPCA make use of the ℓ_2-norm for optimization [9]. Although the ℓ_2-norm is optimal for the case of independent and identically distributed (i.i.d.) Gaussian noise but not robust to outliers [10,11]. Owing to this intrinsic drawback, the methods mentioned above are all sensitive to outliers.

Some recent methods including L1-PCA [12], R1-PCA [13] and PCA-L1 [14] attempt to attenuate this sensitivity by adopting the ℓ_1-norm, which is known more robust to outliers than the ℓ_2-norm. Among them, PCA-L1 is attractive for being robust to outliers and having a relatively low computational complexity. By referring to the techniques of PCA-L1, 2DPCA-L1 was proposed and validated with competitive performance in many computer vision problems [15]. Since it is difficult to directly solve the ℓ_1-norm maximization problem, PCA-L1 and 2DPCA-L1 resort to greedy strategies in order to optimize all projection vectors sequentially. However, the projection vectors are prone to being struck in local solutions.

Recently, a PCA with non-greedy ℓ_1-norm maximization (PCA-L1 non-greedy) was developed by Nie et al. [16]. Compared with PCA-L1, it optimized all projection vectors simultaneously, and thus effectively avoids the projection vectors being struck in local solutions. In this paper we propose a DiaPCA with non-greedy ℓ_1-norm maximization, termed as DiaPCA-L1 non-greedy, for face recognition. This method has three major advantages: (1) it is more robust to outliers than the ℓ_2-norm based methods; (2) it shares the advantage of DiaPCA in preserving image characteristics; and (3) it directly deals with the ℓ_1-norm maximization problem and optimizes all projection vectors simultaneously.

The rest of this paper is organized as follows: Section 2 gives an introduction to DiaPCA and the representation of diagonal image preprocessing technique. The DiaPCA-L1 non-greedy is elaborated in Section 3. Section 4 reports all experimental results, and conclusions are finally drawn in Section 5.

2. Brief review of DiaPCA

In DiaPCA, the original images are firstly transformed into the corresponding *diagonal images*. Similar to 2DPCA, the *diagonal covariance matrix* is defined based on the *diagonal images*. Then the eigen-decomposition is utilized to obtain the optimal projection vectors.

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Suppose that there are n training images, denoted by $\boldsymbol{A}_i, i=1,2,\ldots,n$. The size of the matrix $\boldsymbol{A}_i$ is $h\times w$, where h and w are the image height and width, respectively. For each training image, the corresponding *diagonal image* was defined as follows:

1. If the width w is equal to or bigger than the height h, the technique illustrated in Fig. 1(a) is utilized to generate the *diagonal image* $\boldsymbol{B}$ for the original image $\boldsymbol{A}$.
2. If the width w is smaller than the height h, the technique illustrated in Fig. 1(b) is utilized to generate the *diagonal image* $\boldsymbol{B}$ for the original image $\boldsymbol{A}$.

A further representation of the diagonal image preprocessing technique mentioned above in DiaPCA has been introduced by Lu and Tan [8]. It is shown that transforming an image to its diagonal one is equivalent to assigning an appropriate weight to each pixel to emphasize its different importance. Thus, the *diagonal image* can also be obtained by multiplying the corresponding pixel in the original image with a positive weight, namely

$$\boldsymbol{B}=\boldsymbol{A}\otimes\boldsymbol{T}, \tag{1}$$

where $\boldsymbol{T}$ is the weighting matrix, "$\otimes$" denotes element-wise multiplication and $\boldsymbol{B}$ is the *diagonal image* matrix.

Furthermore, if $w \ge h$,

$$\boldsymbol{T}=\begin{bmatrix} 1 & 1 & \cdots & 1 & 1 \\ \frac{a_{22}}{a_{21}} & \frac{a_{23}}{a_{22}} & \cdots & \frac{a_{2w}}{a_{2(w-1)}} & \frac{a_{21}}{a_{2w}} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \frac{a_{hh}}{a_{h1}} & \frac{a_{h(h+1)}}{a_{h2}} & \cdots & \frac{a_{h(w-2)}}{a_{h(w-1)}} & \frac{a_{h(w-1)}}{a_{hw}} \end{bmatrix}. \tag{2}$$

if $w<h$,

$$\boldsymbol{T}=\begin{bmatrix} 1 & \frac{a_{22}}{a_{12}} & \cdots & \frac{a_{ww}}{a_{1w}} \\ 1 & \frac{a_{32}}{a_{22}} & \cdots & \frac{a_{(w+1)w}}{a_{2w}} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \frac{a_{h2}}{a_{(h-1)2}} & \cdots & \vdots \\ 1 & \frac{a_{12}}{a_{h2}} & \cdots & \frac{a_{(w-1)w}}{a_{hw}} \end{bmatrix}. \tag{3}$$

Fig. 2 shows five original images, the corresponding *diagonal images* and weighting matrices. It is shown that the weighting matrices $\boldsymbol{T}$ applies different weights for pixels around texture or edge regions on the original images. For example, some important parts such as eyes and eyebrows are assigned with bigger weights, while the other regions are assigned with smaller ones. According to the weighting representation, DiaPCA can be regarded as weighted 2DPCA and the weighting matrix emphasizes the importance of different face parts for recognition [8].

Without loss of generality, assume that the image width w is equal to or bigger than the image height h. Then, for each image $\boldsymbol{A}_i$, the corresponding *diagonal image* $\boldsymbol{B}_i$ is obtained with the technique illustrated in Fig. 1(a) or (1).

Let $\boldsymbol{X}_i \in \mathbb{R}^{h\times w}, i=1,\ldots,n$, denote all *diagonal images*. The *diagonal covariance matrix* is defined as

$$\boldsymbol{G}=\frac{1}{n}\sum_{i=1}^{n}(\boldsymbol{X}_i-\overline{\boldsymbol{X}})^T(\boldsymbol{X}_i-\overline{\boldsymbol{X}}), \tag{4}$$

where $\overline{\boldsymbol{X}}=\frac{1}{n}\sum\boldsymbol{X}_i$. The eigenvectors $\boldsymbol{\omega}_1,\boldsymbol{\omega}_2,\ldots,\boldsymbol{\omega}_l$ corresponding to the l largest eigenvalues are obtained by using the eigen-decomposition on $\boldsymbol{G}$.

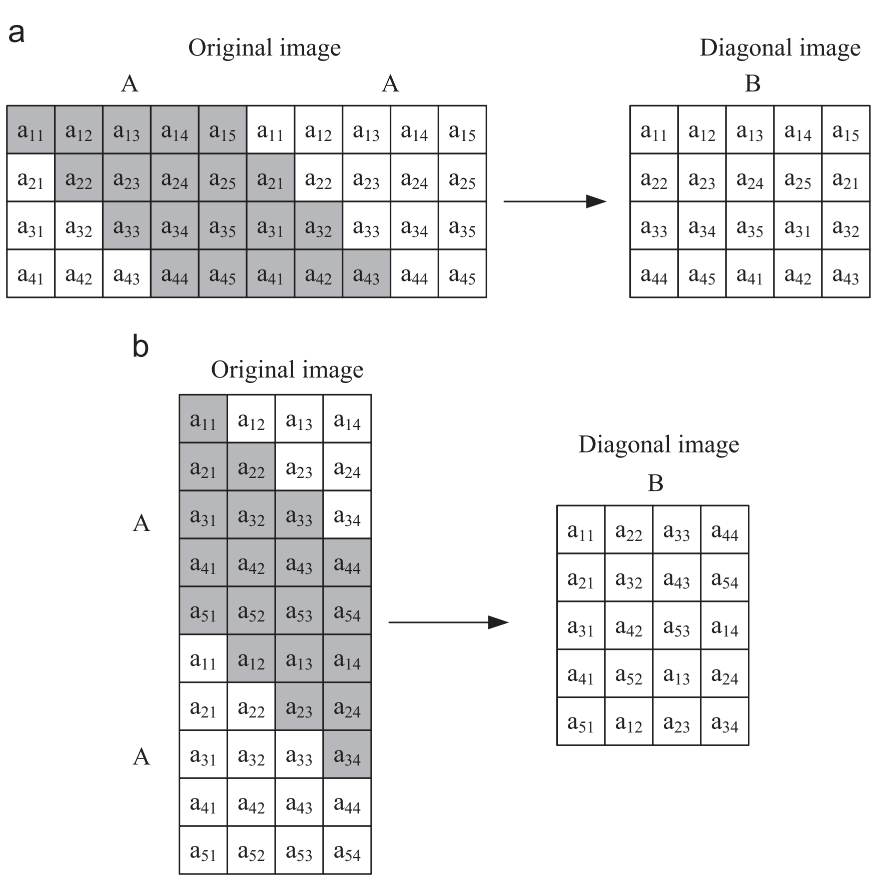

Fig. 1. Two techniques of deriving the *diagonal images*. (a) Image width is equal to or bigger than image height. (b) Image width is smaller than image height.