Neurocomputing 171 (2016) 73–81

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Impulsive coordination of nonlinear multi-agent systems with multiple leaders and stochastic disturbance

Wei Li, Hong Zhou, Zhi-Wei Liu*, Yuzhen Qin, Zhenhua Wang

Department of Automation, School of Power and Mechanical Engineering, Wuhan University, Wuhan 430072, PR China

ARTICLE INFO

Article history:
Received 5 March 2015
Received in revised form
15 April 2015
Accepted 11 June 2015
Communicated by Bo Shen
Available online 27 June 2015

Keywords:
Impulsive control
Multi-agent systems
Stochastic disturbance
Multiple leaders
Nonlinear

ABSTRACT

This paper studies the coordination of nonlinear multi-agent systems with multiple leaders and stochastic disturbance. Two impulsive control algorithms are proposed to make all agents to track the convex set determined by the multiple leaders under both fixed and switching topologies. Based on Lyapunov function and comparison principle of impulsive systems, some results on the convergence of the controlled systems are obtained. Finally, some numerical simulations demonstrate the validity of the proposed control algorithms.

1. Introduction

Recent years, there has been an increasing interest in studying the consensus problem of multi-agent systems as its important applications in real world, such as sensor networks [1], unmanned air vehicle formations [2], power systems [3,4], and fault estimation [5,6]. Consensus means the state of agents achieves agreement on certain features through only local interaction. Many results have been derived for consensus of multi-agent systems with different special features, such as time delay [7], switching topologies [8–10], quantization [11–13], and uncertain [14]. For different systems, many kinds of effective control algorithms, including adaptive control [15], impulsive control [7,16–18] and pinning control [19], were reported to make multi-agent systems achieve consensus.

In the above-mentioned works, multi-agent systems are leaderless or only with one leader. However, the consensus problem in multi-agent systems with multiple leaders, which called containment control, has been intensively investigated very recently. In [20], the containment control problem that makes all agents track the convex polygon formed by the leaders was proposed. Then, Li et al. [21] and Liu et al. [22] studied the dynamic/static linear multi-agent system with fixed topology via continuous feedback control. Taking account of the switching communication topology, Cao et al. investigated the containment control of the multi-agent systems with stationary or dynamic leaders in [23], and Li et al. [24] proposed a distributed switching control law for the system with Markovian switching network topologies and measurement noises. While, for the second-order dynamics, Meng et al. [25] discussed the containment control problem of the multiple rigid bodies, and Lou et al. [26] proposed a containment control algorithm for the system with random switching topology. In general, most of the literatures drive followers to the convex space determined by the leaders via continuous control. However, the continuous control strategies are impractical and not cost-effective in some situations.

On the other hand, impulsive control is an effective discontinuous control strategy. Different from the continuous control, the impulsive control only needs to change the state of the system instantaneously at specific instants. Therefore, the impulsive control algorithm has so many advantages, such as flexibility, cost-effectiveness and quick response, that it has been widely applied to the synchronization and consensus problem of multi-agent systems [7,27–29].

Moreover, multi-agent systems often work in complex environment so that external disturbance is unavoidable. Therefore, stochastic disturbance is a vital issue that needs to be taken into consideration when modeling a multi-agent system. In fact, lots of studies on stability and synchronization of multi-agent systems with leaderless or one leader and stochastic disturbance have been done [30–33]. However, how to achieve the coordination of

* Corresponding author.
E-mail address: liuzw@whu.edu.cn (Z.-W. Liu).

http://dx.doi.org/10.1016/j.neucom.2015.06.015
0925-2312/

nonlinear multi-agent systems with multiple leaders and stochastic disturbance is still an unsolved problem.

Motivated by the preceding discussion, this paper studies the coordination of nonlinear multi-agent systems with multiple leaders and stochastic disturbance via impulsive control. Two impulsive controls are proposed to drive all agents of the nonlinear multi-agent system under both fixed and switching topologies to track the convex set determined by leaders, and some results are derived based on the impulsive systems, comparison principle and Lyapunov method.

The rest of this paper is structured as follows. Some basic notions and preliminaries about the graph are given in Section 2. The coordination of nonlinear multi-agent systems with multiple leaders and stochastic disturbance under fixed topology via impulsive control algorithm is discussed in Section 3. While the multi-agent system under switching topology is discussed in the following Section 4. Some numerical simulations are presented in Section 5. The conclusion is shown in the final.

2. Preliminaries

Throughout the paper we will use the following notations. Let $\mathbb{R}$ and $\mathbb{N}$ represent the set of real numbers and positive integers, respectively. $\mathbb{R}^{n\times n}$ denotes an $n\times n$ matrix and $\mathbb{R}^n$ is the n-dimensional vector. $\|x\|$ denotes the Euclidean norm of vector $x\in\mathbb{R}^n$. For $A\in\mathbb{R}^{n\times n}$, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximum and minimum eigenvalues of A, respectively. Let $(\Omega,\mathcal{F},\{\mathcal{F}_t\},\mathcal{P})$ be a complete probability space with filtration $\{\mathcal{F}_t\}_{t\ge 0}$ which satisfies that the filtration contains all $\mathcal{P}$-null sets and is right continuous. Denoted by $\mathfrak{L}^0_{\mathcal{F}_0}([-d,0],\mathbb{R}^n)$, the family of all $\mathcal{F}_0$-measurable $PC([-d,0],\mathbb{R}^n)$-valued random variables $\phi=\{\phi(s): -d\le s\le 0\}$ such that $\sup_{-d\le s\le 0}\mathbb{E}\|\phi(s)\|^2<\infty$, where $\mathbb{E}(\cdot)$ represents the mathematic expectation of corresponding variable.

Next, we will introduce some basic concepts and results about algebraic graph theory. Let $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{A})$ denote a graph, where $\mathcal{V}=\{1,2,\dots,N\}$ is the set of the nodes, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the set of edges and $\mathcal{A}=\{a_{ij}\}$ is the adjacency matrix. If there exists an edge from v_i to v_j, i.e., $(v_i,v_j)\in\mathcal{E}$, $a_{ij}>0$, otherwise $a_{ij}=0$, and the diagonal entries of $\mathcal{A}$ are zeroes, i.e., $a_{ii}=0$. An edge is undirected if $(v_i,v_j)\in\mathcal{E}$ is equal to $(v_j,v_i)\in\mathcal{E}$. A path from v_s to v_e is a sequence of ordered edges $(v_s,v_{p1}),(v_{p1},v_{p2}),\dots,(v_{pk},v_e)$, where $(v_s,v_{p1}),(v_{p1},v_{p2}),\dots,(v_{pk},v_e)\in\mathcal{E}$. $\mathcal{G}_s(\mathcal{V}_s,\mathcal{E}_s,\mathcal{A}_s)$ denotes a subgraph of $\mathcal{G}$, where $\mathcal{V}_s\subseteq\mathcal{V},\mathcal{E}_s\subseteq\mathcal{E}$. The Laplacian matrix $L=(L_{ij})\in{}^{n\times n}$ is defined as

$$L_{ij}=\begin{cases}\sum_{j=1}^{N}a_{ij}, & i=j,\\ -a_{ij}, & j\ne i.\end{cases}$$

In this paper, we consider a nonlinear multi-agent system with stochastic disturbance that consists of N agents. The dynamic model of agent i can be described by the following differential equation:

$$dx_i(t)=h(x_i(t),x_i(t-d_1(t)))\,dt+g(x_i(t),x_i(t-d_2(t)))\,dw(t) \tag{1}$$

where $i=1,\dots,N$, $x_i=[x_{i1},x_{i2},\dots,x_{in}]^T\in\mathbb{R}^n$ is the state vector of the agent i. $d_1(t)$ and $d_2(t)$ are the time delay and they are bounded, i.e., $0\le d_1(t)\le d_1, 0\le d_2(t)\le d_2$. $w(t)=(w_1(t),\dots,w_n(t))\in\mathbb{R}^n$ is an Weiner process defined in $(\Omega,\mathcal{F},\{\mathcal{F}_t\},\mathcal{P})$. $g:\mathbb{R}^+\times\mathbb{R}^n\to\mathbb{R}^{n\times n}$ and $h:\mathbb{R}^+\times\mathbb{R}^n\times\mathbb{R}^n$ are the disturbance intensity and continuous nonlinear function, respectively.

The initial conditions of system (1) are given by

$$x_i(t)=\phi_i(t),\quad -d\le t\le 0,\ i\in\mathbb{F}\cup\mathbb{L}. \tag{2}$$

where $d=\max(d_1,d_2)$.

Assume that there are M followers and $N-M$ leaders in the network. The set of followers and leaders are denoted by $\mathbb{F}\triangleq\{1,2,\dots,M\}$ and $\mathbb{L}\triangleq\{M+1,\dots,N\}$, respectively. An agent is a follower means that it can receive information from others and its state can be controlled. Otherwise, the agent is a leader. A graph $\mathcal{G}(\mathcal{V},\mathcal{E},\mathcal{A})$ is used to model the communication topology among agents. Let $\mathcal{G}_s(\mathcal{V}_s,\mathcal{E}_s,\mathcal{A}_s)$ denote the communication between followers, therefore $\mathcal{G}_s$ is a subgraph of $\mathcal{G}$, $\mathcal{V}_s=\mathbb{F}$ and $\mathcal{E}_s\in\mathcal{E}$.

The objective of this paper is to design a distributed impulsive control to drive the followers to a convex polygon formed by the leaders, which is called the containment control problem. According to [34], the containment control is given as follows.

Definition 1. For any $i\in\mathbb{F}$ and $j\in\mathbb{L}$, there exists $a^i_j\ge 0$ and $\sum_{j=1}^{N}a^i_j=1$, such that

$$\lim_{t\to\infty}\mathbb{E}\left(x_i(t)-\sum_{j=1}^{N}a^i_jx_j(t)\right)=0, \tag{3}$$

then, the nonlinear multi-agent system (1) is said to achieve containment control in mean square, that is the mean square of the states of the followers asymptotically converges to the convex hull formed by the mean square of states of the leaders.

The following propositions are listed for the main results.

Proposition 1. *The communications between the followers are undirected, and each follower has a directed path to at least one leader.*

Remark 1. When the communication between the followers is directed, the eigenvalues of the Laplacian matrix of the followers are complex numbers. This makes the calculation very complex and hard to get the right result, hence, in this paper we only consider the situation of undirected and we will further discussion the situation of directed in the future.

The Laplacian matrix of the graph $\mathcal{G}$ is denoted by L. Based on the property of the leaders and the followers, there are no communication between the leaders and L can be written as the following form:

$$L=\begin{bmatrix}L_1 & L_2\\ 0_{(N-M)\times M} & 0_{(N-M)\times(N-M)}\end{bmatrix}$$

where $L_1\in\mathbb{R}^{M\times M}$ is the Laplacian matrix of the followers (L_1 is the Laplacian matrix of $\mathcal{G}_s$), and $L_2\in\mathbb{R}^{M\times(N-M)}$ denotes the communication topology between the followers and the leaders.

Proposition 2 (*Jie et al.* [35]). *The nonlinear function $h(t,x,y)$ satisfies the following condition*:

$$\left\|h(t,x,y)-\sum_{j=1}^{N-M}l_jh\left(t,x_j,y_j\right)\right\|\le K\left\|x-\sum_{j=1}^{N-M}l_jx_j\right\|+Q\left\|y-\sum_{j=1}^{N-M}l_jy_j\right\|$$

for any $x,y\in\mathbb{R}^n$ and $l_j\ge 0$, $\sum_{j=1}^{N-M}l_j=1$, where K and Q are two nonnegative constants.

Remark 2. This proposition ensures the containment control to be achieved. It can be applied to all linear function and some nonlinear function, such as $x\sin(t), x\cos(t^2)$. If the proposition holds, the Lipschitz condition of the function $f(\cdot)$ holds and it ensures the existence and uniqueness of the solution of the system.