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Relaxed H_∞ control design of discrete-time Takagi–Sugeno fuzzy systems: A multi-samples approach

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ABSTRACT

This paper investigates the problem of fuzzy H_∞ control design for a class of discrete-time nonlinear control systems via a multi-samples approach. A new fuzzy H_∞ controller, which is parameter-dependent on not only the current-time but also the one-step-past membership functions, is developed to derive less conservative design conditions for ensuring both the asymptotic stability and the prescribed H_∞ performance index of the obtained closed-loop system. Thanks to the usage of both a new fuzzy Lyapunov function and an extension of the homogenous matrix polynomial methodology, linear matrix inequality (LMI)-based H_∞ control design conditions are obtained for minimizing the normal level in a systematic and relaxed way. In addition, further efforts are made by developing an efficient slack variable approach. Finally, two numerical examples are provided to illustrate the effectiveness of the results given in this paper.

1. Introduction

During the past several decades, the study of robust control has attracted considerable attention since failures of control components often occur in real industrial systems [1–5]. The main objective of the existing literature is to design a fixed controller such that the closed-loop system can maintain stability and acceptable performance, not only when all control components are operational, but also in the case of some admissible control component outages. Among them, it is well known that H_∞ control is an effective control methodology to attenuate the effect of uncertain external disturbances or unmodeled dynamics on control systems, and hence H_∞ control problems for linear systems have been extensively studied, see [6] and the literature therein. In particular, the result of H_∞ control design of linear systems has been extended to nonlinear systems by using the Hamilton–Jacobi inequality (HJI) approach, but it is very difficult to solve an HJI either analytically or numerically until now.

On the other hand, it is well-known that Takagi–Sugeno (T–S) fuzzy model [7] has been proved to be an efficient tool for the representation of complex nonlinear systems and applications. Especially, control synthesis via the so-called T–S fuzzy model has attracted lots of attention [8,9], However, the above-mentioned works use common Lyapunov functions (CLF), and the results based on the CLF method are quite conservative, especially for those used to represent highly nonlinear complex systems. More recently, several approaches have been developed to implement the task of control synthesis of T–S fuzzy control systems with less conservative conditions [10–18]. In [19], robust H_∞ control for discrete-time fuzzy systems has been investigated via basis-dependent Lyapunov functions. By introducing some additional instrumental matrix variables, the stabilization conditions have been relaxed in [20]. Meanwhile, reliable LQ fuzzy control problems for nonlinear systems with actuator faults have been addressed in [21], where different Lyapunov functions for different operating regimes (including the normal and faulty cases) were used to reduce the conservatism of using an CLF. More recently, a type of state feedback controllers, namely, switched parallel distributed compensation (PDC) controllers, has been proposed in [22], which are switched based on the values of membership functions.

It is worth pointing out that all the above fuzzy H_∞ controllers are motivated by the usual PDC theory. One will naturally give a question: whether the conservatism could be further reduced if some different and interesting structures are adopted. In this paper, the problem of fuzzy H_∞ control for a class of discrete-time fuzzy systems is investigated. To the best of our knowledge, this is the first time in the literature that such a novel kind of fuzzy H_∞ controller is designed, which is homogenous polynomially parameter-dependent (HPPD) on both the current and the one-step-past membership functions with some prescribed

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degrees. A new fuzzy Lyapunov function, which is also HPPD on fuzzy membership functions of arbitrary degree, is applied to derive our main results. By using a new fuzzy Lyapunov function and the homogenous matrix polynomial technique, linear matrix inequality (LMI)-based asymptotically necessary and sufficient reliable H_∞ control conditions are proposed for minimizing the normal level while maintaining acceptable levels in an systematic and relaxed way. In particular, the existing results are some special cases of the one given in this paper. Two illustrative examples are finally given to demonstrate the effectiveness of the proposed results.

Notations. The notation $P>0$ means that P is real symmetric and positive definite. $\mathbb{R}$ represents the set of real numbers, $\mathbb{Z}_+$ represents the set of positive integers, $\mathbb{N}$ denotes the natural numbers set $\{0,1,2,\ldots\}$, and $p!$ denotes factorial, i.e., $p!=p(p-1)(p-2)\cdots(2)(1)$ for $p\in\mathbb{N}$ with $0!=1$.

2. Problem formulation and preliminaries

Considering a class of discrete-time nonlinear system that is represented by a set of T–S fuzzy rules as follows [7]:

Rule *i*: If $z_1(t)$ is F_1^i, and $z_2(t)$ is F_2^i, $\cdots$, and $z_p(t)$ is F_p^i, Then

$$\begin{cases} x(t+1)=A_ix(t)+B_{1i}w(t)+B_{2i}u(t) \\ z(t)=C_ix(t)+D_{1i}w(t)+D_{2i}u(t) \end{cases}$$

where $x(t)\in\mathbb{R}^{n_x}$ is the state variable; $u(t)\in\mathbb{R}^{n_u}$ is the control input; $w(t)\in l_1^p[0,\infty)$ is the exogenous disturbance; $z(t)\in\mathbb{R}^{n_z}$ is the controlled output; $A_i, B_{1i}, B_{2i}, C_i, D_{1i}$ and D_{2i} are known constant matrices with appropriate dimensions.

By utilizing the singleton fuzzier and the center-average defuzzier, the overall discrete-time T–S fuzzy model is obtained as [7]

$$\begin{cases} x(t+1)=\sum_{i=1}^{r}h_i(\theta(t))(A_ix(t)+B_{1i}w(t)+B_{2i}u(t)) \\ z(t)=\sum_{i=1}^{r}h_i(\theta(t))(C_ix(t)+D_{1i}w(t)+D_{2i}u(t)) \end{cases} \tag{1}$$

where $\theta(t)=[\theta_1(t),\theta_2(t),\ldots,\theta_g(t)]^T$ is the premise variable, while r is the number of fuzzy rules; $h_i(\theta(t))$ represents the normalized weight for the i-th rule, that is, $h_i(\theta(t))\ge 0$ and $\sum_{i=1}^{r}h_i(\theta(t))=1$.

In this paper, our objective is to propose a H_∞ controller with less conservatism than those existing results in the literature, such that the following two conditions are simultaneously satisfied:

(1) The discrete-time fuzzy system (1) is asymptotically stable when $w(t)=0$.
(2) The discrete-time fuzzy system (1) has prescribed levels γ of H_∞ noise attenuation, i.e., under the zero initial condition $x(0)=0$, $\|e\|_2<\gamma\|w\|_2$ is satisfied for any nonzero $w(t)\in l_2[0,\infty)$.

Throughout this paper, definitions associated with homogeneous polynomials are all consistent with those in [17]. For simplicity, some shortenings are offered as follows:

$$\begin{cases} h_i=h_i(\theta(t)), h=(h_1,\ldots,h_r)^T, h_{-1}=(h_1(\theta(t-1)),\ldots,h_r(\theta(t-1)))^T, \\ h^k=h_1^{k_1}h_2^{k_2}\cdots h_r^{k_r}, h_{-1}^k=h_1^{k_1}(\theta(t-1))h_2^{k_2}(\theta(t-1))\cdots h_r^{k_r}(\theta(t-1)), \\ h_{+1}=(h_1(\theta(t+1)),\ldots,h_r(\theta(t+1)))^T, \\ h_{+1}^k=h_1^{k_1}(\theta(t+1))h_2^{k_2}(\theta(t+1))\cdots h_r^{k_r}(\theta(t+1)). \end{cases} \tag{2}$$

By definition, for r-tuples k and k', one writes $k\ge k'$ if $k_i\ge k'_i, (i=1,\ldots,r)$. The usual operations of summation, $k+k'$, and subtraction, $k-k'$ (whenever $k\ge k'$), are defined componentwise. In particular, two important definitions about the r-tuple $e_i\in\mathcal{K}(1)$ and the coefficient $\pi(k)$ are provided as below

$$e_i=0\cdots0\underbrace{1}_{i-\text{th}}0\cdots0, \pi(k)=(k_1!)(k_2!)\cdots(k_r!). \tag{3}$$

Lemma 1. *For three prescribed positive integers g_1, g_2 and g_3, matrices $R_{k'qk''}^{ij}$ with $k'\in\mathcal{K}(g_1)$, $q\in\mathcal{K}(g_2-2)$, $k''\in\mathcal{K}(g_3)$, and $1\le i\le r, 1\le j\le r$, the following equality (4) always holds:*

$$\sum_{\substack{k'\in\mathcal{K}(g_1),k\in\mathcal{K}(g_2),k''\in\mathcal{K}(g_3),\\ i,j\in\mathcal{A}(k),k-e_i-e_j\ge 0}} h_{-1}^{k'}h^k h_{+1}^{k''}R_{k'(k-e_i-e_j)k''}^{ij} = \sum_{\substack{k'\in\mathcal{K}(g_1),q\in\mathcal{K}(g_2-2),\\ k''\in\mathcal{K}(g_3),1\le i\le r,\ 1\le j\le r}} h_{-1}^{k'}h^q h_{+1}^{k''}h_ih_jR_{k'qk''}^{ij}. \tag{4}$$

Proof. The proof is similar to the proof of Lemma 3 of [17], thus it is omitted here for saving space. □

3. Relaxed fuzzy H_∞ control design

3.1. Relaxed fuzzy H_∞ analysis

In order to reduce the conservatism of existing results, a new fuzzy H_∞ controller, which is parameter-dependent on not only the current-time but also the one-step-past membership functions with any pair of prescribed degrees, is offered as below

$$u(t)=F_{g_1g_2}(h_{-1}h)G_{g_1g_2}^{-1}(h_{-1}h)x(t), \tag{5}$$

where

$$F_{g_1g_2}(h_{-1}h)=\sum_{\substack{k\in\mathcal{K}(g_1),\\ k'\in\mathcal{K}(g_2)}} h_{-1}^k h^{k'}F_{kk'},\ G_{g_1g_2}(h_{-1}h)=\sum_{\substack{k\in\mathcal{K}(g_1),\\ k'\in\mathcal{K}(g_2)}} h_{-1}^k h^{k'}G_{kk'},$$

$F_{kk'}\in\mathbb{R}^{n_u\times n_x}$ and $G_{kk'}\in\mathbb{R}^{n_x\times n_x}$ are control gain matrices, $g_1,g_2\in\mathbb{Z}_+$.

By employing both (1) and (5), the closed-loop control system can be derived as follows:

$$\begin{cases} x(t+1)=\sum_{i=1}^{r}h_i\Big(\Big(A_i+B_{2i}F_{g_1g_2}(h_{-1}h)G_{g_1g_2}^{-1}(h_{-1}h)\Big)x(t)+B_{1i}w(t)\Big) \\ z(t)=\sum_{i=1}^{r}h_i\Big(\Big(C_ix(t)+D_{2i}F_{g_1g_2}(h_{-1}h)G_{g_1g_2}^{-1}(h_{-1}h)\Big)x(t)+D_{1i}w(t)\Big) \end{cases} \tag{6}$$

Theorem 1. *For some prescribed scalars $\gamma>0$, if there exist symmetric matrices $P_{kk'}$, matrices $F_{kk'}$ and $G_{kk'}$, for $k\in\mathcal{K}(g_1), k'\in\mathcal{K}(g_2)$, such that the following matrix inequality (7) is satisfied:*

$$\begin{bmatrix} -P_{g_1g_2}^{-1}(h_{-1}h) & * & * & * \\ 0 & -\gamma^2I & * & * \\ \Pi_{31} & D_1(h) & -I & * \\ \Pi_{41} & B_1(h) & 0 & -P_{g_1g_2}(hh_{+1}) \end{bmatrix}<0, \tag{7}$$

where $\Pi_{31}=C(h)+D_2(h)F_{g_1g_2}(h_{-1}h)(G_{g_1g_2}(h_{-1}h))^{-1}$, $\Pi_{41}=A(h)+B_2(h)F_{g_1g_2}(h_{-1}h)(G_{g_1g_2}(h_{-1}h))^{-1}$, then the closed-loop control system (6) is asymptotically stable when $w(t)=0$, and has prescribed levels γ of H_∞ noise attenuation.

Proof. The proof is twofold: we first certificate that the closed-loop control system (6) is asymptotically stable when $w(t)=0$ and then prove that, under the zero initial condition $x(0)=0$, the H_∞ noise attenuation $\|e\|_2<\gamma\|w\|_2$ is satisfied for any nonzero $w(t)\in l_2[0,\infty)$.

To prove the first part, we choose a fuzzy Lyapunov function candidate which is homogenous polynomially parameter-dependent on both the current and the one-step-past membership