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A new RAIM algorithm based on multivariate cumulative sum and its improvement

Jianhua Ran [a,b], Maozhu Jin [c,*], Xingli Gan [d]

[a] School of Resource and Environmental Science, Wuhan University, Wuhan, China
[b] Jiuzhaigou International Park Administration, Sichuan, China
[c] Business School of Sichuan University, Chengdu, China
[d] The 54th Institute of China Electronic Technology Corporation, Shijiazhuang, Hebei, China

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ABSTRACT

"Snapshot" RAIM algorithm based on parity space can detect a larger bias fault in one step by using redundant BeiDou satellites, but it does not have the ability to detect a medium cumulative fault. First, a method that uses multivariate cumulative sum is designed to detect receiver positioning equations parity residual, and it derives multivariate cumulative RAIM (MCRAIM) alarm threshold, and then we compared the performances of the two kinds of algorithm with RAIM and MCRAIM. Secondly, the RAIM and MCRAIM are combined to be improved to a new algorithm that can detect not only a big mutational mean shift in one step, but also moderate continuous mean shift, and the Calculation formula of detecting Function and threshold will be given. Finally, the average run length indicator and fault detection performance of RAIM–MCRAIM combination algorithm are analyzed, and it proves that a combination of the two algorithms have more advanced features, it can make up for the defects of using either RAIM or MCRAIM individually.

1. Introduction

Receiver Autonomous Integrity Monitoring(RAIM) is able to provide an autonomous fault detection capability as well as an alarm capability when the BeiDou navigation satellite has something wrong and causes positioning accuracy drops. At present, "snapshot" RAIM detection algorithm [1–4] based on the parity space is mainly used. For example, an American company called Honeywell has used the parity space RAIM algorithm to manufacture a BeiDou/IRS combination sensor, and it has achieved very good results. But the largest defect of "snapshot" RAIM algorithm that bases on the parity space is that it does not have the ability to detect the fault with a cumulative type.

First, we use multivariate cumulative sum to detect the parity residuals of the redundant measurement equation. It can deduce the multivariate cumulative RAIM (MCRAIM), and solve the problem that the traditional RAIM algorithm could not detect slow varying cumulative failure, and then we compare the performance of the two kinds of algorithms with RAIM and MCRAIM. Secondly, the RAIM and MCRAIM are combined and improved to a new algorithm that can detect not only a big mutational mean shift in one step, but also moderate continuous mean shift, and Calculation formula of detecting function and threshold can be got. Finally, average run length indicator and the fault detection performance of RAIM–MCRAIM combinative algorithm are evaluated by a simulation experiment, and it proves that the combination of the two kinds of algorithms have more advanced features, it could make up for the defects of using either RAIM or MCRAIM individually.

2. "Snapshot" RAIM algorithm

2.1. BeiDou positioning equation

Linear Measurement Equation [5,6] can be expressed as

$$z = Hx + b + \varepsilon \tag{1}$$

In the above Eq. (1), z denotes the Measurement vector of $M \times 1$; x denotes the State vector of $N \times 1$; b denotes the Failure vector of $M \times 1$; ε is the Gaussian measurement noise of $M \times 1$, with $E(\varepsilon) = 0$, $\text{cov}(\varepsilon) = \sigma^2 I_M$, and I_M is a unit matrix of order M; H is a transition matrix from the state space to the measurement space of $M \times N$, rank$[H] = N$; $M \geq N + 1$.

* Corresponding author.
E-mail address: jinmaozhu@scu.edu.cn (M. Jin).

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2.2. Generation of parity residuals

Relying on the Eq. (1), use the ordinary least square, we can deduce the Parity Residual Vector q [7]of $(M-N)\times 1$ which is

$$q=Vz=V(b+\varepsilon) \tag{2}$$

We can find that the Parity Residual Vector q and the State Vector x are independent, and $E(q)=Vb_i$, $\mathrm{cov}(q)=\sigma^2 I_{M-N}$, thus the Parity Residual Vector q follow the Gaussian Distribution of $N(Vb_i,\sigma)$. Then, we can judge whether bugs will be developed by relying on the Distribution Characteristics of the Parity Residual Vector q: If there is no fault, then we have$E(q)=0$; else $E(q)=Vb_i\neq 0$.

2.3. Criterion of the fault detection

In order to detect faults, We present two opposing hypotheses [8,9]. The original hypothesis is H_0, $E(q)=0$, $\mathrm{cov}(q)=\sigma^2 I_{M-N}$; and the alternative hypothesis is H_1, $E(q)=Vb_i=\mu$, $\mathrm{cov}(q)=\sigma^2 I_{M-N}$. Since q follows the Normal Distribution of $M-N$, We can construct a log-likelihood ratio function under the case of "no fault" and "fault". Then a Maximum Likelihood Function:$L_{max}=q^T q/2\sigma^2$ can be got. So, we can get the Fault Detection Function constructed by f

$$D=\left|q^T q\right|^{1/2},\ D^2\sim\chi^2(M-N) \tag{3}$$

The criterion of the fault detection is: if $D\geq T$, then there is fault; otherwise no fault, and T is the alarming threshold.

3. Multidimensional cumulative RAIM algorithm

3.1. Criterion of the fault detection

By judging whether the mean of the Parity Residuals q offsets, we can detect if any faults exist in the data of BeiDou redundant measurement. However, RAIM algorithm of the previous study is the "snapshot" algorithm, we could only use the current measurement data to detect a very big abnormal mean shift in one step [10], a continuous moderate abnormal mean shift could not be detected. To solve the problems, a new algorithm called multidimensional cumulative RAIM algorithm is deduced, abbreviated as MCRAIM.

The Parity Residuals of $M-N$ dimension is $q_t=[q_{1t},q_{2t},\cdots,q_{(M-N)t}]$, which follows the Normal Distribution of $M-N$ dimension. Its density function [11–14]

$$f(q_t)=\frac{1}{(2\pi)^{(M-N)/2}\left|\Sigma\right|^{1/2}}e^{-(X^2/2)},\ X^2=(q_t-\mu_0)^T\Sigma^{-1}(q_t-\mu_0) \tag{4}$$

Where q_{it} is a element of the parity residual vector, $-\infty<q_{it}<+\infty$, $i=1,2,\cdots,M-N$; μ_0 is the mean of the parity residuals q_t, $\mu_0=[0,0,\cdots,0]$; Σ is variance of the parity residuals, $\Sigma=\sigma^2 I_{M-N}$.

Let $q_1,q_2,\cdots,q_\theta$ denote variables of the independent and identically distribution $N(\mu_0,\Sigma)$, and $q_{\theta+1},q_{\theta+2},\cdots$ denote variables of the independent and identically distribution $N(\mu_1,\Sigma)$, with the variable point θ unknown. Set that there are a series of random variables $q_1,q_2,\cdots,q_t$.

The original hypothesis is H_0:$q_i\sim N(\mu_0,\Sigma)$, $i=1,2,\cdots,t$; and the alternative hypothesis is H_1: we have $\theta<\infty$, make $q_i\sim N(\mu_0,\Sigma)$, $i=1,2,\cdots,\theta$, while $q_i\sim N(\mu_1,\Sigma)$, $i=\theta+1,\theta+2,\cdots$, and $\mu_1>\mu_0$. The likelihood ratio of the alternative hypothesis to the original hypothesis is

$$L_{t,\beta}=\exp\left\{\frac{(u_1-u_0)}{\sigma}\sum_{i=\nu+1}^{n}\left(\frac{(q_i-u_0)}{\sigma}-\frac{(u_1-u_0)}{2\sigma}\right)\right\} \tag{5}$$

Table 1
The available navigation satellite and its parameters.

PRN	Longitude	Latitude	Height (km)	Azm (°)	Elv (°)
05	173°11′62″	6°29′86″	20128.48	119.8	20.9
08	145°93′89″	55°92′48″	20117.20	45.0	25.0
09	124°32′40″	18°24′38″	20005.40	184.7	54.4
18	40°37′84″	44°60′74″	20337.40	302.7	19.4
21	107°76′87″	10°09′60″	19947.27	210.3	40.3
22	36°76′37″	42°05′82″	20239.32	302.1	15.4

Table 2
The average run length ARL of MCRAIM algorithm.

Threshold	Mean offset ($ARL_0=10^4$, $k=1$)						
	0	1	1.5	2	3	4	5
Dim=2, h=5. 21	10^4	22	8. 6	5. 2	3. 0	2. 2	1.88
Dim=2, h=4. 3	10^4	938	219	60	8. 5	2. 32	1.26

Let $k_u=(u_1-u_0)/2\sigma$, $y_i=(q_i-u_0)/\sigma$, then its logarithmic likelihood ratio is

$$\Lambda_{n,\nu}=\ln L_{n,\nu}=2k\sum_{i=\nu+1}^{n}(y_i-k_\mu) \tag{6}$$

Thus, as for the maximum log-likelihood ratio of the alternative hypothesis "drift" to the original hypothesis "no drift", the statistic value is

$$\Lambda_n=\max_{1<\nu<n}\Lambda_{n,\nu}=\max\left\{2k\sum_{i=\nu+1}^{n}(y_i-k_\mu)\right\} \tag{7}$$

We can obtain one-dimensional cumulative recurrence formula

$$Z_t=\max\{0,Z_{t-1}+y_t-k\},\ t=1,2,\cdots \tag{8}$$

In the above equation: $Z_0=0$, $k=(u_1-u_0)/2\sigma$, $y_i=(q_i-u_0)/\sigma$.

In order to get the multidimensional cumulative recursive formula, the recurrence formula (8) with a scalar form is transformed into a vector form $\vec{Z}_t=\max\left\{0,\vec{Z}_{t-1}+\vec{y}_t-\vec{k}\right\}$. Therefore, MCRAIM algorithm can be expressed as

Let the length C_t of $\vec{Z}_{t-1}+\vec{y}_t$ be

$$C_t=(Z_{t-1}+y_t)^T\Sigma^{-1}(Z_{t-1}+y_t) \tag{9}$$

If $C_t\leq k$

$$Z_t=0 \tag{10}$$

Else

$$Z_t=(Z_{t-1}+y_t)(1-k/C_t) \tag{11}$$

where: $Z_0=0$; $k>0$.

Then, the detection function is

$$D_t=\sqrt{Z_t^T\Sigma^{-1}Z_t} \tag{12}$$

The judgment criterion: if a certain $D_t\geq h$, then set an alarm flag as we think there is drift in the process, h is the alarm threshold.

3.2. MCRAIM alarm threshold settings

MCRAIM treats the Average Run Length as the evaluation criteria of the algorithm performance, And it can be divided into the Average Run Length ARL_0 under a controlled state and the Average Run Length ARL_1 under a uncontrolled state. Generally, ARL_0 is a fixed part, so the performance of the algorithm is only