Neurocomputing 171 (2016) 179–184

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Stability criterion of complex-valued neural networks with both leakage delay and time-varying delays on time scales

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ARTICLE INFO

Article history:
Received 5 February 2015
Received in revised form
16 May 2015
Accepted 18 June 2015
Communicated by Haijun Jiang
Available online 30 June 2015

Keywords:
Complex-valued neural networks
Exponential stability
Leakage delay
Time-varying delay
Complex-valued linear matrix inequality

ABSTRACT

In this paper, the problem on the global exponential stability of complex-valued neural networks with both leakage delay and time-varying delays on time scales is discussed. By constructing appropriate Lyapunov–Krasovskii functionals and using matrix inequality technique, a delay-dependent condition to assure the global exponential stability for the considered neural networks is established. The condition is expressed in complex-valued linear matrix inequality, which can be checked numerically using the effective YALMIP toolbox in MATLAB. An example with simulations is given to show the effectiveness of the obtained result.

1. Introduction

Over the past few decades, neural networks have been widely investigated due to their extensive applications in pattern recognition, associative memory, signal processing, image processing, combinatorial optimization, and other areas [1]. In implementation of neural networks, however, time delays are unavoidably encountered [2]. It has been found that, the existence of time delays may lead to instability and oscillation in a neural network. Therefore, stability analysis of neural networks with time delays has received much attention, for example, see [1–8] and references therein.

As an extension of real-valued neural networks, complex-valued neural networks with complex-valued state, output, connection weight, and activation function become strongly desired because of their practical applications in physical systems dealing with electromagnetic, light, ultrasonic, and quantum waves [9]. In fact, complex-valued neural networks (CVNNs) make it possible to solve some problems which cannot be solved with their real-valued counterparts. For example, the XOR problem and the detection of symmetry problem cannot be solved with a single real-valued neuron, but they can be solved with a single complex-valued neuron with the orthogonal decision boundaries, which reveals the potent computational power of complex-valued neurons [10]. Besides, CVNNs has more different and more complicated properties than the real-valued ones [11]. Therefore it is necessary to study the dynamic behaviors of CVNNs deeply [12].

In recent years, there have been some researches on the stability of various CVNNs, for example, see [10–21]. In [10], authors proposed CVNNs, and its weight matrix was supposed to be Hermitian with nonnegative diagonal entries in order to preserve the stability of the network. A computational energy function was introduced and evaluated in order to prove network stability for asynchronous dynamics. In [11], author weakened the assumption on weight matrix in [10], and derived a new stability condition. In [12], authors considered the complex-valued Hopfield neural networks which possess the energy function and analyzed the phase dynamics of the network with certain forms of an activation function. In [13], higher order CVNNs were proposed, and several criteria for checking stability were given by considering a new energy function. In [14], a class of CVNNs with constant delays was considered, and some sufficient conditions were obtained for assuring the stability of the equilibrium point of CVNNs with two classes of activation functions. In [15], the boundedness and complete stability of CVNNs with constant delay were investigated. Several criteria to guarantee the boundedness and complete stability were derived. In [16], the asymptotical stability of CVNNs with constant delay was investigated, where the activation functions can be expressed by separating their real and imaginary parts. In [17], authors considered CVNNs

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http://dx.doi.org/10.1016/j.neucom.2015.06.032
0925-2312/

with time-varying delays and unbounded distributed delays whose activation functions can be expressed by separating their real and imaginary parts. In [18], when the activation functions satisfy the Lipschitz continuity condition in the complex domain, the asymptotical stability of CVNNs with constant delay was studied. In [19], authors considered discrete-time CVNNs, and obtained several sufficient conditions for checking global exponential stability of a unique equilibrium. In [20], authors discussed a class of discrete-time recurrent neural networks with complex-valued linear threshold neurons, and derived some conditions for the boundedness, global attractivity, and complete stability of such networks. In [21], authors introduced the delay into discrete-time CVNNs in [20], and investigated the boundedness and complete stability of the considered discrete-time CVNNs with constant delay. By constructing appropriate Lyapunov–Krasovskii functionals and employing linear matrix inequality technique and analysis method, several new delay-dependent criteria for checking the boundedness and global exponential stability were established.

The above-mentioned CVNNs are either continuous-time CVNNs or discrete-time neural networks, it is troublesome to study stability in two kinds of models. Therefore, it is necessary to unify the study of continuous-time and discrete-time neural networks under the same framework. In [22], based on the theory of time scales, authors considered CVNNs on time scales and established a main criterion guaranteeing the existence, uniqueness and global exponential stability of equilibrium point. In [23], authors considered CVNNs with both leakage time delay and discrete constant delay on time scales. By using the fixed point theory, a criterion for checking the existence, uniqueness of the equilibrium point for the considered CVNNs was presented. By constructing appropriate Lyapunov–Krasovskii functionals, and employing the free weighting matrix method, several delay-dependent criteria for checking the global stability of the addressed CVNNs were established. In [24], the global exponential stability of CVNNs with time-varying delays is investigated. By constructing appropriate Lyapunov–Krasovskii functionals and using matrix inequality technique, a new delay-dependent criterion for checking the global exponential stability of the addressed CVNN was established in terms of real linear matrix inequalities (LMIs). However, the previous criteria in [22–24] for checking the stability of the addressed CVNNs are somewhat conservative due to the construction of constructed Lyapunov functionals and technicality of used mathematical method. Hence, it is our intention in this paper to reduce the possible conservatism.

Motivated by the above discussions, the objective of this paper is to study the exponential stability for CVNNs with both leakage delay and time-varying delays. By constructing new Lyapunov–Krasovskii functionals and using matrix inequality technique, we obtain a new sufficient condition for checking the global exponential stability of CVNNs with both leakage delay and time-varying delays.

Notations: The notations are quite standard. Throughout this paper, I represents the unitary matrix with appropriate dimensions; C^n and $C^{n\times m}$ denote, respectively, the set of all n-dimensional complex-valued vectors and the set of all $n\times m$ complex-valued matrices. A^* shows the complex conjugate transpose of complex-valued matrix A. $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ are defined as the largest and the smallest eigenvalue of Hermitian matrix P, respectively. The subscript T denotes the matrix transposition. The notation $X>Y$ means that X and Y are Hermitian matrices, and that $X-Y$ is positive definite. i shows the imaginary unit, i.e., $i=\sqrt{-1}$. $|a|$ denotes the module of complex number $a\in C$, and $\|z\|$ denotes the norm of $z\in C^n$, i.e., $\|z\|=\sqrt{z^*z}$. If $A\in C^{n\times n}$, denotes by $\|A\|$ its operator norm, i.e., $\|A\|=\sup\{\|Ax\|:\|x\|=1\}=\sqrt{\lambda_{\max}(A^*A)}$. Set $[a,b]_T$ is defined as $[a,b]_T:\{t\in T, a\le t\le b\}$. Sometimes, the arguments of a function or a matrix will be omitted in the analysis when no confusion can arise.

2. Model description and preliminaries

In this section, we will recall some definitions and lemmas which will be used in the proofs of our main results.

Let T be an arbitrary nonempty closed subset (time scale) of R. The forward and backward jump operators $\sigma,\rho:T\to R$ and the graininess $\mu:T\to R^+$, respectively, by $\sigma(t){:=}\inf\{s\in T: s>t\}$, $\rho(t){:=}\sup\{s\in T: s<t\}$ and $\mu(t)=\sigma(t)-t$. Throughout this paper, we always assume that time scale T has a bounded graininess $\mu(t)\le\mu<\infty$.

For a point $t\in T$, t is called right-dense if $\sigma(t)=t$, right-scattered if $\sigma(t)>t$, left-dense if $\rho(t)=t$, left-scattered if $\rho(t)<t$.

For $f:T\to C^n$ and $t\in T$, we note that the real and imaginary parts of f are real valued and one can use the time scales results below for the real-valued entries of Ref and Imf. We say that $f:T\to R$ is delta differentiable at $t\in T$ provided there exists an α such that for all $\epsilon>0$ there is a neighborhood $\mathcal{N}$ of t with

$$|f(\sigma(t))-f(s)-\alpha(\sigma(t)-s)|<\epsilon|\sigma(t)-s|,$$

for all $s\in\mathcal{N}$. In this case we denote α by $f^{\Delta}(t)$, and call $f^{\Delta}(t)$ the delta derivative of f at t. It is easy to see that

$$f^{\Delta}(t)=\begin{cases}\lim\limits_{s\to t, s\in T}\dfrac{f(t)-f(s)}{t-s} & \text{if} \quad \mu(t)=0\\ \dfrac{f(\sigma(t))-f(t)}{\sigma(t)-t} & \text{if} \quad \mu(t)>0\end{cases}$$

and

$$(fg)^{\Delta}=f^{\Delta}g+(f+\mu f^{\Delta})g^{\Delta}.$$

Let f be right-dense continuous, if $F^{\Delta}(t)=f(t)$, we define the delta integral by

$$\int_a^t f(s)\Delta s=F(t)-F(a).$$

It is easy to check that the following formula holds

$$\int_t^{\sigma(t)} f(s)\Delta s=\mu(t)f(t).$$

A function $f:T\to R$ is called rd-continuous provided it is continuous at right-dense points on T and its left sided limits exist at left-dense points on T. The set of rd-continuous functions $f:T\to R$ is denoted by $C_{rd}=C_{rd}(T,R)$. A function $f:T\to R$ is called regressive if $1+\mu(t)f(t)\ne 0$ for all $t\in T$. The set of all regressive and rd-continuous functions is denoted by $\mathcal{R}$. The set $\mathcal{R}^+$ of all positively regressive function consists of those $p\in\mathcal{R}$ that satisfy $1+\mu(t)f(t)>0$ for all $t\in T$. It is known that if $p\in\mathcal{R}$ and $t_0\in T$, then the initial value problem $y^{\Delta}(t)=py(t)$, $y(t_0)=1$ possesses a unique solution. This solution is called the exponential function on the time scale and is denoted by $e_p(\cdot,t_0)$.

In this paper, we consider the following CVNNs with both leakage delay and time-varying delays on time scale

$$z^{\Delta}(t)=-Dz(t-\sigma)+Af(z(t))+Bf(z(t-\tau(t)))+J \tag{1}$$

for $t\in T$, where $z(t)=(z_1(t),z_2(t),\dots,z_n(t))^T\in C^n$ is the state vector of the neural network with n neurons at time t; $D=\text{diag}\{d_1,d_2,\dots,d_n\}\in\mathbb{R}^{n\times n}$ is the self-feedback connection weight matrix, where $d_i>0$; A and B are $n\times n$ matrix with complex entries; $f(z(t))=(f_1(z_1(t)),f_2(z_2(t)),\dots,f_n(z_n(t)))^T\in C^n$ denotes the neuron activation at time t; $J=(J_1,J_2,\dots,J_n)^T\in C^n$ is the external input vector; σ denotes leakage delay, $\tau(t)$ is the transmission delay which satisfy that $0\le\tau(t)\le\tau$ and $t-\tau(t)\in T$ for all $t\in T$,