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Indirectly regularized variational level set model for image segmentation

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ABSTRACT

In this paper, we propose a variational level set model with indirect regularization term for image segmentation. Instead of using direct regularization on level set function, we introduce an auxiliary function to regularize indirectly the level set function. Our energy functional consists of a data term, a link term of level set function with the auxiliary function and a regularization term of the auxiliary function. We prove that the energy functional is convex in $L^2(\Omega) \times W^{1,2}(\Omega)$ and give the convergence analysis of the alternating minimization algorithm that we utilized. We show that the indirect regularization has some advantages over direct regularization theoretically and experimentally. Experimental results illustrate that the proposed model can better handle images with high noise, angle and weak edges.

1. Introduction

Image segmentation is an important subject in image processing and computer vision which facilitates the subsequent tasks such as image analysis, pattern recognition. Based on some similar characteristics (intensity value, texture, color) of the input image, one would like to partition the image domain into two or more regions, each representing an object. Up to now, a lot of good algorithms and methods have been proposed to address the image segmentation tasks.

Many successful approaches to image segmentation involve partial differential equations (PDE) and variational level set models. The evolution PDE for a PDE-based model is directly constructed or indirectly derived from a minimization problem, while the evolution PDE for a variational level set model is directly derived from the minimization of energy functional over level set functions. So far, there are a lot of researches for PDE-based models [1–7]. In this work, we focus on the variational level set methods for image segmentation.

Among a wealth of variational level set models [8–19], we must state the celebrated "active contours without edges" model proposed by Chan and Vese [8], one of the most widely used models for two-phase image segmentation. Its energy functional consists of two terms: a data term and a regularization term based on zero level set (i.e., length regularization). It works well in processing images with a large amount of noise and can detect objects whose boundaries cannot be defined by gradient. But the Chan–Vese model has also the following limits.

First, it is difficult to handle images with intensity inhomogeneity because of its simple assumption of intensity homogeneity. To overcome this defect, many works [9–12] are proposed in different ways. These methods assume normally that the intensities are homogeneous in local regions of image. By dealing with image in terms of local regions instead of global regions, they perform well on images with intensity inhomogeneity. The popular methods include the well-known region-scalable fitting (RSF) model [9], the local image fitting (LIF) model [10], the local intensity clustering (LIC) model [11] and the local Gaussian intensity clustering (LGIC) model [12], etc.

Second, the minimizer of the energy functional for the Chan–Vese model [8] sometimes becomes a local one due to the non-convexity of functional. This is a serious difficulty because the local minima of functional often offer poor segmentation results. To overcome this difficulty, some researches [13–16] provided convex methods and algorithms to solve the non-convex problem of the Chan–Vese model. Chan et al. [13] provided an efficient global convex energy functional of the Chan–Vese (GCV) model to compute global minimizers by showing that solutions could be obtained from a convex relaxation, in which the total variation (TV) is used as the regularization term. Afterwards, Bresson et al. [14] proposed a global convex segmentation (GCS) model by integrating a weighted TV into the GCV model. Recently, Brown et al. [15] proposed a completely convex formulation of the Chan–Vese model, which is guaranteed to compute a global minimizer of functional under certain conditions. In our previous work [16], we proposed a convex variational level set model based on the coefficient of variation (CoV); we proved that the value of the

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unique global minimizer for the energy functional is within the interval $[-1, 1]$ for any image, and equals to 1 in the object and -1 in the background for an ideal binary image.

Third, when implementing the Chan–Vese model, a re-initialization technique is usually needed periodically to maintain the degraded level set function being a signed distance function in the whole iteration process [8]. However, this regularization method is very time-consuming and introduces some fundamental problems about when and how to apply the re-initialization. To eliminate the re-initialization step, some variational level set formulations [20–23] have been proposed to regularize the level set function during evolution. These methods without re-initialization have many advantages over the traditional methods. The well-known distance regularization level set evolution (DRLSE) methods, including DRLSE1 [20], DRLSE2 [21] and DRLSE3 [22], keep the level set function as a signed distance function during the evolution. Recently, Zhang et al. [5] proposed a reaction–diffusion method for level set evolution by using the H^1 regularization, i.e., $E_R(\phi)=\int_\Omega |\nabla\phi|^2 dx$ where ϕ is the level set function. This method performs well on image with high noise because a standard method minimizing $E_R(\phi)$ is to find the steady-state solution of the gradient flow equation $\partial\phi/\partial t=\Delta\phi$. The role of this diffusion equation is to regularize the level set function ϕ; it controls the smoothness of the zero level set to penalize complicated boundaries of regions and further avoid the occurrence of isolated small regions (e.g., noise points) in final segmentation. However, this processing is essentially equivalent to the Gaussian smoothing that will eventually output a constant function. At the meantime, some other shape-prior based level set methods [24,25] also reduce the re-initialization procedure by integrating a trained shape into the level set energy functional.

The aforementioned models can be categorized into the direct regularization framework, in which the regularization is directly posed on level set functions. In this paper, we propose an indirectly regularized variational level set model in which the regularization is posed indirectly on level set function via an auxiliary function. The energy functional of this model contains three terms: a data term, a link term of level set function with the auxiliary function and a regularization term of the auxiliary function. We prove that the proposed energy functional is convex in $L^2(\Omega)\times W^{1,2}(\Omega)$ and has a unique global minimizer. Since the energy functional is convex, the proposed model can be solved efficiently by means of the alternating minimization algorithm (see for example [26]). We show that the alternating minimization algorithm is convergent for the proposed model under mild conditions. Experiments on synthetic and real images illustrate that our model provides promising segmentation results compared with state-of-the-art works [5,8,13,16].

The organization of the remainder of this paper is as follows. In Section 2, we introduce some related works. In Section 3, we propose an indirectly regularized variational level set model and give a rigorous analysis. Section 4 gives the algorithm and convergence analysis. Section 5 presents experimental results. In Section 6, we discuss initializations of level set function and auxiliary function and the advantages of indirect regularization over direct regularization. This paper is summarized in Section 7.

2. Previous works

Let $\Omega\subset R^2$ be a bounded open connected set with a Lipschitz boundary, and $f:\ \Omega\to R$ be a given image. Let $\phi:\Omega\to R$ denote the level set function. Traditionally, the energy functional for the existing variational level set models can be formulated in the form:

$$E(\phi)=E_D(f,\phi)+E_R(\phi), \tag{1}$$

where $E_D(f,\phi)$ is the data term or external energy which makes the zero level set of ϕ deform so that it fits to the object boundary, and $E_R(\phi)$ is the regularization term or internal energy which penalizes the oscillation of ϕ.

2.1. About the data term

In [8], Chan and Vese constructed a data term which is expressed as the following formulation:

$$E_D(f,\phi)=\lambda_1\int_\Omega (f-c_1)^2 H(\phi)dx+\lambda_2\int_\Omega (f-c_2)^2(1-H(\phi))dx, \tag{2}$$

where λ_1, λ_2 are positive parameters, and H is the Heaviside function. The constants c_1 and c_2 are defined as

$$c_1=\frac{\int_\Omega f\cdot H(\phi)dx}{\int_\Omega H(\phi)dx},\quad c_2=\frac{\int_\Omega f\cdot(1-H(\phi))dx}{\int_\Omega(1-H(\phi))dx}, \tag{3}$$

which represent the mean intensity values of f in $\Omega_1=\{x\in\Omega|\,\phi(x)>0\}$ and $\Omega_2=\{x\in\Omega|\,\phi(x)<0\}$, respectively. The data term (2) is non-convex and so has sometimes local minima that often provide poor results.

To overcome this defect, Chan et al. [13] provided a convex relaxation method, in which the data term is defined as follows:

$$E_D(f,\phi)=\lambda\int_\Omega\left((f-c_1)^2-(f-c_2)^2\right)\phi dx,\quad \phi\in[0,1], \tag{4}$$

where λ is a positive parameter, and the constants c_1 and c_2 are defined as

$$c_1=\frac{\int_{\Omega_1} f(x)dx}{\int_{\Omega_1} dx}, c_2=\frac{\int_{\Omega_2} f(x)dx}{\int_{\Omega_2} dx}, \tag{5}$$

which represent the mean intensity values of f in $\Omega_1=\{x\in\Omega|\,\phi(x)>\alpha\}$ and $\Omega_2=\{x\in\Omega|\,\phi(x)<\alpha\}$ with $\alpha\in(0,\ 1)$, respectively. The energy functional in (4) is homogeneous of degree 1 with respect to ϕ, which does not exist minimizer in general, thus the authors simply restrict the values of ϕ such that $0\le\phi\le 1$.

Recently, Lee and Seo [17] proposed the following data term with two shifted Heaviside functions:

$$E_D(f,\phi)=\lambda_1\int_\Omega (f-c_1)^2\phi H(\alpha+\phi)dx-\ \lambda_2\int_\Omega (f-c_2)^2\phi H(\alpha-\phi)dx, \tag{6}$$

where λ_1,λ_2 are positive parameters, and α is an arbitrary positive value. Here, ϕ is multiplied to prevent from computing a local minimum and $H(\pm\phi)$ is shifted by $\mp\alpha$ to confine the range of ϕ. As is the case for the formulation (2), the two constants c_1 and c_2 in (6) are still defined by (3). The Lee-Seo model has a global minimum, and so works well on two-phase image segmentation problems. Based on the Lee-Seo model, Li and Kim [18] replace the Heaviside function in the Lee-Seo model with the function:

$$H_c(z)=\frac{1+z}{2}, \tag{7}$$

and get the following data term:

$$E_D(f,\phi)=\lambda_1\int_\Omega (f-c_1)^2\phi H_c(1+\phi)dx-\ \lambda_2\int_\Omega (f-c_2)^2\phi H_c(1-\phi)dx, \tag{8}$$

The constants c_1 and c_2 are defined as

$$c_1=\frac{\int_\Omega f\cdot H_c(\phi)dx}{\int_\Omega H_c(\phi)dx},\quad c_2=\frac{\int_\Omega f\cdot(1-H_c(\phi))dx}{\int_\Omega(1-H_c(\phi))dx}. \tag{9}$$

The Li-Kim model can be numerically solved using an unconditionally stable semi-implicit scheme and segments well two-phase images.