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Finite-time boundedness analysis for a new multi-layer switched system with time-delay

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ABSTRACT

In this paper, the finite-time boundedness problem of a multi-layer switched system subject to average dwell time switching signal, Markov jump and time-varying polytopic uncertainties is investigated. The sufficient conditions guaranteed the conclusion are shown in the form of linear matrix inequalities to ease the formulation, and the switched Lyapunov–Krasovskii approach is used in detail proofs. At last, an illustrative example is employed to demonstrate the efficiency of the main result.

1. Introduction

Generally speaking, a switched system is a dynamical system that consists of a finite number of subsystems, which are described by differential/difference equations and are employed to capture the dominant dynamics of the system in different operation modes, and a switching signal, which orchestrates the activation of subsystems at each instant of time. In the past two decades, the switching signal (rule), used by researchers to decide which subsystem is being activated, is deterministic [1–9], or stochastic [10–17]. In this paper, we will study a class of multi-layer switched system consisting of deterministic and stochastic switching signal simultaneously.

The deterministic switching rule is often considered with the average dwell time. Average dwell time switching means that the number of switches in a finite interval is bounded and the average spacing from one switching to the next should not be less than a constant [2]. Many studies have shown that average dwell time switching is more general and flexible in stability analysis and control theories [18]. The systems with this kind of switching are seemed as slow-switched systems and have been extensively researched [3–5]. The Markov jump systems, regarded as a special class of stochastic systems with system matrices changed randomly at discrete time instances governed by a Markov process, can be applied to describe many practical systems such as electric power systems and neural network systems subject to random changes in structure and parameters [10–13,19]. A part of recent extended studies are focused on the Markov jump systems with partly known transition probabilities (TPs) due to the hard and expensive measuring method to get the complete knowledge of the TPs [14–17].

On the other hand, polytopic uncertainties, a kind of parameter uncertainties, are often used in switched systems. This kind of uncertainty means system matrices belong to some convex polytope. When there are some prior structural information on the uncertainties, the polytopic uncertainties can arise. Much work has been studied for polytopic uncertain switched systems [20–23].

Stability is a basic topic for switched systems. Up to now, many existing literatures related to stability of switched systems have paid attention to the finite-time stability [6,7,24]. The problem of finite-time stability has gained its popularity from the 1960s [25,26], because the behavior of system over a certain finite time interval is the major concern in some practical applications, such as the variation of concentration in a chemical process. Finite-time stability means, given a fixed time interval and a bound of initial condition, its states does not exceed the prescribed threshold. In [27], a sufficient condition for the problem of finite-time stabilization via state feedback is presented. The study of finite-time stability is combined with linear matrix inequalities and Lyapunov function theory in [28,29]. A novel finite-time stability criterion using piecewise-like delay method is given in [30]. Moreover, it is noted that system which is finite-time stable may not be Lyapunov

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asymptotic stable [31,32]. Finite-time boundedness is introduced as an extended concept of finite-time stability in [33]. Authors in [8] give the finite-time boundedness conditions for switched linear systems with average dwell time. In [34], the problem of finite-time H_∞ estimation for systems with Markov jumps and average dwell time is studied. The observer design for time-varying polytopic uncertain systems in the framework of finite-time boundedness is studied in [35], and more results for finite-time boundedness can be seen in [9,36–39] and the related references in them. However, to the best of our knowledge, the problem of finite-time boundedness for multi-layer switched systems has not been researched yet.

Motivated by the above discussions, in this paper, we considered the finite-time boundedness for a class of multi-layer switched system. The basic idea of the multi-layer is shown in Fig. 1. Three levels are considered in our model. In layer 1, the deterministic switching signal $\sigma(k)$ is subject to average dwell time. Then in the interval $[k_l, k_{l+1}) = [k_l, k_l+1, \dots, k_{l+1})$, the system switched randomly with known or unknown transition probabilities. In layer 3, polytopic uncertainties are considered in every time instance $k_l+i, i=0, 1, 2, \dots, k_{l+1}-k_l-1$. Moreover, time-delay is also considered in the systems due to its one of the main causes for instability of systems, and all conditions of the problem are established in the form of linear matrix inequalities to ease the formulation.

The contributions of this paper are in two folds. A multi-layer switched system is proposed firstly. Then, the finite-time boundedness problem for the system is considered. The rest of this paper is organized as follows. In the next section, the new system and problem descriptions are stated, and the necessary definitions are recalled. Section 3 presents the result for the new system, and a corollary is given for the system without disturbance. An illustrative example is provided in the fourth section. Finally, conclusions are drawn in Section 5.

Notations: The notations in this paper are fairly standard. The superscript 'T' stands for matrix transposition, the 'I' represents identity matrix, R^n denotes the n-dimensional Euclidean space. In addition, in symmetric block matrices we use '*' as an ellipsis for the terms that are introduced by symmetry. Matrices, if their dimensions are not explicitly stated, are assumed to be compatible for algebraic operations. A matrix $P>0\ (\geq 0)$ means P is a symmetric positive (semi-positive) definite matrix.

2. Model description and preliminaries

Consider the following time-delay multi-layer switched system:

$$x(k+1) = A_{r_k,\sigma_k}(\lambda)x(k) + Ad_{r_k,\sigma_k}(\lambda)x(k-\tau) + D_{r_k,\sigma_k}(\lambda)\omega(k), \quad (1)$$

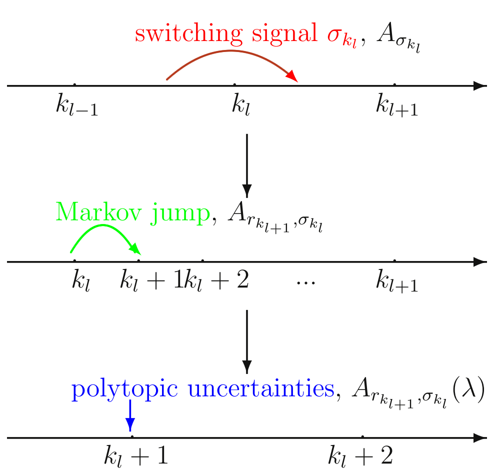

Fig. 1. The basic idea of the multi-layer switched system.

where $x(k) \in R^n$ is the state vector, $\omega(k) \in R^m$ is the disturbance input, τ denotes the delay time. $\{r_k, k \geq 0\}$ is a discrete-time Markov stochastic process, which takes values in a finite set $\chi = \{1, 2, \dots, N\}$ with a transition probabilities matrix $\Lambda = \{\pi_{ij}\}$, $i, j \in \chi$, $N>1$ is the number of subsystems. For $r_k = i, r_{k+1} = j$, the system governs the switching among different subsystems with $\Pr(r_{k+1} = j | r_k = i) = \pi_{ij}^{\sigma_k}$, where $\pi_{ij}^{\sigma_k} \geq 0, \forall i, j \in \chi$, and $\sum_{j=1}^N \pi_{ij}^{\sigma_k} = 1$ for all $i \in \chi$. In addition, σ_k is assumed to be a piecewise high-level average dwell time switching signal ,which takes its values in a finite set $\Theta = \{1, 2, \dots, M\}$. A switching sequence $k_0 < k_1 < \dots < k_l < \cdots$ is continuous from right everywhere. When $k \in [k_l, k_{l+1})$, it means the σ_{k_l} th transition probability mode is active. $\pi_{ij}^{\sigma_k}$ is now a function of σ_k. Moreover, the transition probabilities of the jumping process $\{r_k, k \geq 0\}$ in this paper are assumed to be partly accessed. For example, for system (1) with four subsystems under the switching signal σ_k, the transition probability matrix may be

$$\begin{bmatrix} ? & \pi_{12}^{\sigma_k} & ? & \pi_{14}^{\sigma_k} \\ ? & ? & \pi_{23}^{\sigma_k} & ? \\ \pi_{31}^{\sigma_k} & ? & ? & \pi_{34}^{\sigma_k} \\ ? & \pi_{42}^{\sigma_k} & ? & ? \end{bmatrix},$$

where '?' means the unaccessible probabilities. $\forall i \in \chi$, we denote

$$\chi_K^i = \{j : \pi_{ij}^{\sigma_k} \text{ is known}\}, \quad \chi_{UK}^i = \{j : \pi_{ij}^{\sigma_k} \text{ is unknown}\}. \quad (2)$$

To ease the presentation, we use $(A_{i,m}(\lambda), Ad_{i,m}(\lambda), D_{i,m}(\lambda))$ instead of $(A_{r_k,\sigma_k}(\lambda), Ad_{r_k,\sigma_k}(\lambda), D_{r_k,\sigma_k}(\lambda))$ when $r_k = i, \sigma_k = m$.

In this paper, the matrices of each subsystem have polytopic uncertain parameters. It is assumed that, at each instant of time k, $(A_{i,m}(\lambda), Ad_{i,m}(\lambda), D_{i,m}(\lambda)) \in R_{i,m}$, where $R_{i,m}$ is a given convex bounded polyhedral domain described by

$$R_{i,m} = \left\{ (A_{i,m}(\lambda), Ad_{i,m}(\lambda), D_{i,m}(\lambda)) = \sum_{q=1}^{s} \lambda_q (A_{i,m,q}, Ad_{i,m,q}, D_{i,m,q}), \right.$$
$$\left. \sum_{q=1}^{s} \lambda_q = 1, \lambda_q \geq 0 \right\}, \quad (3)$$

where $(A_{i,m,q}, Ad_{i,m,q}, D_{i,m,q})$ denotes the qth vertex in the ith subsystem of mth mode, s means the total number of vertices, and the uncertain parameter vector $\lambda(k) = [\lambda_1(k), \dots, \lambda_s(k)]^T \in R^s$ is supposed to be time-varying. The process of the system and the relations among the switching can be seen in Fig. 1.

Remark 1. The model (1) includes some very recently existing ones as special cases. When all the transition probability matrices Λ are identity, the model (1) is reduced to the one in [35]. On the other hand, when the number of vertices in the polytope is $s=1$, the model (1) is reduced to the case in [36]. So our model (1) can be used to describe more complicated practical systems.

To describe our main result more precisely, we have the following definitions.

Definition 1. (*Finite-time stability*) The discrete-time linear system

$$x(k+1) = A_{r_k,\sigma_k}(\lambda)x(k) + Ad_{r_k,\sigma_k}(\lambda)x(k-\tau)$$

is said to be finite-time stable with respect to $(\delta_1, \delta_2, R, N_0)$, where R is a positive-definite matrix, $0<\delta_1<\delta_2$, if

$$\sup_{\theta \in [-\tau, 0]} x^T(\theta)Rx(\theta) \leq \delta_1 \Rightarrow x^T(k)Rx(k) < \delta_2, \quad \forall k \in \{1, 2, \dots, N_0\}.$$

Definition 2. (*Finite-time boundedness*) The discrete-time linear system (1) is said to be finite-time bounded with respect to $(\delta_1, \delta_2, R, d, N_0)$, where R is a positive-definite matrix,