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Impulsive consensus of multi-agent nonlinear systems with control gain error

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ABSTRACT

In this paper, the consensus problem of multi-agent nonlinear systems with control gain error is studied. Based on the theory of impulsive differential equations, Lyapunov stability theory and algebraic graph theory, some impulsive consensus conditions are given to realize the consensus of a class of multi-agent nonlinear systems. Compared with the existing investigations of impulsive consensus of multi-agent systems, the proposed impulsive control protocol with control gain error is more rigorous and effective in practical systems. Two numerical simulations are verified to confirm the effectiveness of the proposed methods.

1. Introduction

In recent years, there has been an increasing interest in the research on multi-agent systems due to its widespread applications in many fields, such as physics, science and mathematics [1–10]. Generally speaking, according to the number of leaders existing in the multi-agent systems, the cooperative control of multi-agent systems can be classified into two classes, namely, leaderless consensus problem and cooperative tracking problem. For the leaderless consensus problem, the distributed controllers are designed for each node (agent) such that all nodes eventually converge to an unprescribed common value [11–13]. For the cooperative tracking one, a leader node is considered as a command generator that generates the desired reference trajectory and ignores information from the follower nodes [14,15]. It is worth mentioning that, in practical system, the introduction of a leader can broaden the scope of applications by guaranteeing the states of the followers to converge to some desired trajectories, and thus it is very significant to design cooperative tracking protocol for leader-following multi-agent systems which is also regarded as generalization of well-known master-slave synchronization [16–20].

Recently, many control techniques and significant results have been developed for consensus of linear and nonlinear multi-agent systems, such as observer-based protocol, adaptive control, pinning control, etc. Compared with these continuous control methods, impulsive control is an efficient method to deal with the dynamical systems which cannot be controlled by continuous control methods [21–25]. In addition, in consensus processes, one node receives the information from its neighbor nodes only at the discrete time instants, which dramatically reduces the amount of synchronization information transmitted between the nodes of multi-agent systems and makes the method more efficient in a large number of real-life applications [26–31]. In the literatures dealing with the impulsive consensus problem, several important topics have been addressed, including impulsive consensus with communication delay [32–34], some investigations about average consensus [35,36], networks with switching topology [37], etc. Furthermore, in [38], the authors proposed an impulsive hybrid cooperative control scheme which is useful while the operating time of the controller is smaller for the consensus of a network with nonlinear identical nodes. All these mentioned literatures are commonly devoted to investigate impulsive dynamical control systems without disturbances and uncertainties. However, in many real-world networks, nonlinear dynamics with disturbances occur periodically rather than desired models. Therefore, it is more practical to consider a multi-agent system with control gain error. In [39,40], the impulsive synchronization schemes with uncertain perturbations are investigated. In fact, the disturbance often happens when the impulsive control input is given, which has been investigated in our previous work in [41]. The extension to multi-agent systems is also important and significant, which motivates our research in this paper.

Motivated by the aforementioned discussion, this paper addresses the scheme of consensus of multi-agent nonlinear systems with control gain error based on the impulsive control protocol. Compared with the error acting on continuous control

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moment, it can be found that the error in the discrete time called control gain error has bigger influence on stable regions. Thus, it's more important to consider the impulsive control gain error with an acceptable boundary in multi-agent nonlinear system. Moreover, the impulsive control system with gain error considered in this paper is more coincident with the real world.

The rest of this paper is organized as follows. Some basic preliminaries are described in Section 2. In Section 3, we also formulate the synchronization problem for nonlinear dynamical systems. In Section 4, we further investigate the new impulsive control criteria and with the control gain error. Numerical examples are given to show the effectiveness of the theoretical results in Section 5. Finally, some conclusions are drawn in Section 6.

Throughout the letter, the superscript 'T' stands for the transpose of a matrix. $\mathbb{R}$, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the real number, n-dimensional Euclidean space, the set of all $n\times n$ real matrices. Let $\mathbb{N}_+=\{1,2,\ldots\}$. I_n is the n dimensional identity matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions. $\mathrm{diag}\{d_1,\ldots,d_N\}$ indicates the diagonal matrix with diagonal elements d_1 to d_N. $\otimes$ denotes the Kronecker product. $\lambda_{\max}(L)(\lambda_{\min}(L))$denotes the maximal (minimal) eigenvalue of L.

2. Graph theory

Throughout this paper, the communication graph among the agents is represented by a directed graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ with a nonempty finite set of N nodes $\mathcal{V}=\{v_1,\ldots,v_N\}$, a set of edges or arcs $\mathcal{E}\in(\mathcal{V}\times\mathcal{V})$, and the associate adjacency matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$. An edge rooted at node j and ended at node i is denoted by (v_j,v_i) and $a_{ij}>0$ if $(v_j,v_i)\in\mathcal{E}$, otherwise $a_{ij}=0$. We assume that there are no repeated edges and no self-loops, i.e., $a_{ii}=0$, $\forall i\in\mathbb{N}_+$. Node j is called a neighbor of node i if $(v_j,v_i)\in\mathcal{E}$. The set of neighbors of node i is denoted as $N_i=\{j|(v_j,v_i)\in\mathcal{E}\}$. Define the in-degree of node i is $d_i=\sum_{j=1}^N a_{ij}$ and in-degree matrix as $D=\mathrm{diag}\{d_i\}\in\mathbb{R}^{N\times N}$. The Laplacian matrix is $L=D-\mathcal{A}$. In a directed graph, a sequence of successive edges in the form $\{(v_i,v_k),(v_k,v_l),\ldots,(v_m,v_j)\}$ is a direct path from node i to node j. An undirected graph $\mathcal{G}$ is connected if and only if there exists an undirected path between any two vertices in $\mathcal{G}$.

Lemma 1. *(Horn and Johnson [42]). For matrices A, B, C and D with appropriate dimensions, for α is a constant, with the Kronecker product $\otimes$ the following properties*

1) $(\alpha A)\otimes B=A\otimes(\alpha B)$
2) $(A+B)\otimes C=A\otimes C+B\otimes C$
3) $(A\otimes B)(C\otimes D)=(AC)\otimes(BD)$
4) $(A\otimes B)^T=A^T\otimes B^T$

3. Problem description

Suppose a system consists of N identical agents with nonlinear dynamics is described by

$$\dot{x}_i(t)=Ax_i(t)+\psi(x_i(t))+u_i(t), \tag{1}$$

where $x_i\in\mathbb{R}^n$ is the state of node i $(i=1,\ 2,\ \cdots,\ N)$, $\psi:\mathbb{R}^n\to\mathbb{R}^n$ is a continuous nonlinear function of agent i at time t, $A\in\mathbb{R}^{n\times n}$ is a known constant matrix, all the considered agents share a common state space $\mathbb{R}^n$, $x_i=(x_{i1},x_{i2},\ldots,x_{in})^T$.

The impulsive control input of agent i is designed as

$$u_i(t)=B_k\left(\sum_{j\in N_i}a_{ij}(x_i(t)-x_j(t))+c_i(x_i(t)-x_0(t))\right)\sum_{k=1}^{\infty}\delta^*(t-t_k),\quad k\in\mathbb{N}_+,\ i=1,2,\cdots,N, \tag{2}$$

where $B_k\in\mathbb{R}^{N\times n}$ is the impulsive control gain to the nonlinear dynamics system, $\delta^*(t)$ is impulsive control function satisfy $\delta^*(t)=0$ for $t\neq 0$, the time sequence $\{t_k\}$ satisfy $0\le t_0<t_1<t_2<\cdots<t_{k-1}<t_k<\cdots$, $\lim_{k\to\infty}t_k=\infty$.

Consider the dynamics of the leader as

$$\dot{x}_0(t)=Ax_0(t)+\psi(x_0(t)), \tag{3}$$

where $x_0\in\mathbb{R}^n$ is the state of the leader. It can be regarded as an exosystem or a command generator, which generates the desired target trajectory. We refer to $c_i\ge 0$ as the weight of edge from the leader node to node i $(i\in\{1,\ldots,N\})$. $c_i>0$ if and only if there is an edge from the leader node to node i, and $C=\mathrm{diag}\{c_i\}\in\mathbb{R}^{N\times N}$.

Assumption 1. The parametric uncertainty Δb_k is a constant assumed as follows

$$\Delta b_k=m\varphi(t_k)b_k,\ k\in\mathbb{N}_+, \tag{4}$$

where $m>0$ is a known constant, $|\varphi(t_k)|<1$, and we choose $B_k=b_kI_n$ for simplicity.

This paper aims to design appropriate control inputs of follower agents with gain error to track the leader. To achieve the tracking synchronization for $x_i(t)$, it is assumed that nodes can communicate with their neighbors at discrete instants. The distributed cooperative impulsive controlled system is denoted as

$$\begin{cases}\dot{x}_i=Ax_i+\psi(x_i), & t\neq t_k,\\ \Delta x_i(t_k)=x_i(t_k^+)-x_i(t_k^-)=(B_k+\Delta B_k)e_i(t_k) & \\ =(b_k+\Delta b_k)\left(\sum_{j\in N_i}a_{ij}(x_i(t_k)-x_j(t_k))+c_i(x_i(t_k)-x_0(t_k))\right), & t=t_k,\\ x_i(t_0^+)=x_i(t_0),t_0\ge 0, & \\ i=1,2,\cdots N,\ k\in\mathbb{N}_+, & \end{cases} \tag{5}$$

where $\Delta x_i(t_k)$ is the jump of the state of follower agent i at the time instant t_k, B_k is the impulsive control gain at time t_k and ΔB_k is the error gain of the controller. $x_i(t_k^+)=\lim_{t\to t_k^+}x_i(t)$ and $x_i(t_k^-)=\lim_{t\to t_k^-}x_i(t)$, assume that $x_i(t)$ is left consensus at t_k, that is, $x_i(t_k)=x_i(t_k^-)$. The local neighborhood synchronization error $e_i(t)$ is defined in subsequent Definition 2.

Assumption 2. Continuous nonlinear function $\psi:\mathbb{R}^n\to\mathbb{R}^n$ satisfies the following condition

$$\psi(x_1)-\psi(x_2)=\Psi(x_1,x_2)(x_1-x_2), \tag{6}$$

where $\Psi(x_1,x_2)\in\mathbb{R}^{n\times n}$ is the function of vector x_1 and x_2.

Remark 1. A lot of nonlinear systems can be considered as (1), especially some well-known chaotic systems, such as Lorenz system, Chen system, Lü system, unified system, Chua's circuit, and so on.

Definition 1. The global synchronization error is defined as

$$\delta_i(t)=x_i(t)-x_0(t),\delta_i(t)\in\mathbb{R}^n, \tag{7}$$

where$x=[x_1^T(t),\ \ x_2^T(t),\cdots,x_N^T(t)]\in\mathbb{R}^{nN}$, $\underline{x}_0(t)=1_N\otimes x_0(t)\in\mathbb{R}^{nN}$.

Definition 2. [43] The local neighborhood synchronization error for node i is defined as

$$e_i(t)=\sum_{j\in N_i}a_{ij}(x_i(t)-x_j(t))+c_i(x_i(t)-x_0(t)), \tag{8}$$

where $e_i(t)=[e_{i1}(t),e_{i2}(t),\cdots,e_{in}(t)]^T\in\mathbb{R}^n$.

Subtracting (3) from (1), we can get the impulsive error system described as

$$\begin{cases}\dot{\delta}_i=A\delta_i+\Psi(x_i,x_0)\delta_i,\\ \Delta\delta_i(t_k)=(b_k+\Delta b_k)e_i(t_k).\end{cases} \tag{9}$$