Neurocomputing 171 (2016) 434–443

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Non-uniform sampled-data control for stochastic passivity and passification of Markov jump genetic regulatory networks with time-varying delays

Li Lu [a,*], Zhen Xing [b], Bing He [c]

[a] College of Electrical Engineering, Shanghai Dianji University, Shanghai 200240, China
[b] Cooper (Ningbo) Electric Co., Ltd., Ningbo 315336, China
[c] Maintenance company, State Grid Shanghai Municipal Electric Power Company, Shanghai 200063, China

ARTICLE INFO

Article history:
Received 27 March 2015
Received in revised form
18 May 2015
Accepted 25 June 2015
Communicated by Lixian Zhang
Available online 4 July 2015

Keywords:
Markov jump genetic regulatory networks
Passivity and passification
Non-uniform sampled-data
Time-varying delays

ABSTRACT

This paper investigates the passivity and passification problems for Markov jump genetic regulatory networks with time-varying delays. A sampled-data control approach with non-uniform sampling period is taken into account. By invoking the input-delay approach and employing the Lyapunov–Krasovskii functional method, delay-dependent sufficient conditions on passivity are first given, upon which a mode-dependent passification controller design procedure is then presented. Finally, two numerical examples are provided to illustrate the applicability and effectiveness of the theoretical results.

1. Introduction

The past decades have witnessed the rapid development of genetic regulatory networks (GRNs) in mathematics, biology and engineering applications (see [1–6] and the cited references therein). It is a crucial issue to understand how mRNAs and proteins work collectively and interact with each other to perform the complicated biological functions, see, e.g., the process of transcriptions and translations. Moreover, time delays subject to the slow process of transcription, translation, and translocation processes therein are commonly regarded as main obstacles to reach desired dynamic behaviors, which may lead to instability of GRNs. A large number of results on this issue have been reported, see [7–14].

On another research frontier, with the wide utilization of computers, the digital controllers have been adopted for controlling dynamical systems, which gives rise to the sampled-data control approach. Compared with traditional feedback control systems in continuous-time context, sampled-data control systems involve both continuous-time and discrete-time signals, which can effectively reduce the information occupation and improve the dynamic behavior [15,16]. However, some potential issues accompanied by the employment of sampled-data control approach under networked environment are challengeable, for instance, non-uniform sampling periods induced by network loads or sporadic faults. It is worth mentioning that numerous studies about sampled-data control schemes have been conducted for non-GRNs [17–20]. To the best of the author's knowledge, the non-uniform sampled-data control problems for GRNs have not been fully investigated, especially for Markov jump GRNs, which motivates us for this study.

In addition, passivity and passification problems have been paid increasing attention recently. Passivity theory was first proposed in the circuit analysis [21] and then extended to many other systems, including high-order nonlinear systems and electrical network [22]. It can be widely applied to perform the stability analysis, observer design and signal processing [23,24], etc. Recently, the passivity of linear systems with delays and the passivity of neural networks with time-varying delays have been studied by employing appropriate storage Lyapunov–Krasovskii functionals [25,26]. Although the importance of sampled-data control and passivity property have been widely recognized, up to now, the sampled-data problems of passivity and passification for Markov jump GRNs with time-varying delays have not yet been reported and remain open.

* Corresponding author.
E-mail addresses: lul@sdju.edu.cn (L. Lu), jason.xing@cooperindustries.com (Z. Xing), hebing.qh@gmail.com (B. He).

http://dx.doi.org/10.1016/j.neucom.2015.06.057
0925-2312/

Motivated by the aforementioned analysis, this paper investigates the passivity and passification problems for Markov jump GRNs by a non-uniform sampled-data approach. Compared with the existing results, a new Markov jump GRNs model is established to describe the non-uniform sampled-data control scheme by utilizing the input delay approach [27,28]. Some new passivity conditions are presented in the forms of linear matrix inequalities (LMIs) by applying Lyapunov–Krasovskii functional method. Based on the obtained conditions, a further mode-dependent passification controller design procedure is provided to guarantee the required performance of the resulting closed-loop Markov jump GRNs.

The remainder of this paper is organized as follows. Section 2 gives some preliminaries and formulates the passivity and passification problems. In Section 3, sufficient conditions are derived to ensure the passivity of Markov jump GRNs by the sampled-data controllers. Section 4 gives two numerical examples to show the effectiveness of the proposed passification results. Finally, the paper is concluded in Section 5.

Notation: The notation used in this paper is fairly standard. $\mathbb{R}^n$ denotes the n dimensional Euclidean space, $\mathbb{R}^{m\times n}$ represents the set of all $m\times n$ real matrices. I and 0 represent identity matrix and zero matrix with appropriate dimensions, respectively. $\mathbb{L}_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$. The notation $P>0$ means P is real symmetric and positive definite. The superscript "T" denotes matrix transposition. The notation $X>Y$ where X and Y are symmetric matrices means that $X-Y$ is positive definite. In addition, in symmetric block matrices, $*$ is used as an ellipsis for the terms that are introduced by symmetry and diag$\{\cdots\}$ denotes a block-diagonal matrix. For the notation $(\Omega,\mathcal{F},\mathcal{P})$, Ω represents the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $E\{\cdot\}$ stands for the mathematical expectation. Finally, all matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. Problem formulation and preliminaries

Consider the Markov jump GRNs defined in a complete probability $(\Omega,\mathcal{F},\mathcal{P})$, which can be described by the following equations [13]:

$$\begin{cases}\dot{x}(t)=-A(r_t)x(t)+B(r_t)g(y(t-\sigma(t)))+U(r_t)u(t)+H_x(r_t)\varpi_x(t)\\ \dot{y}(t)=-C(r_t)y(t)+D(r_t)x(t-\tau(t))+V(r_t)v(t)+H_y(r_t)\varpi_y(t).\end{cases}\tag{1}$$

$x(t)=[x_1(t),\ldots,x_n(t)]^T\in\mathbb{R}^n$, $y(t)=[y_1(t),\ldots,y_n(t)]^T\in\mathbb{R}^n$, $x_k(t)$ and $y_k(t)$ $(k=1,2,\ldots n)$ denote the concentrations of mRNA and protein of the kth node at time t, respectively. The nonlinear function $g(y(t))=[g_1(y_1(t)),\ldots,g_n(y_n(t))]^T$ represents the feedback regulation of the protein on the transcription. The control inputs $u(t)=[u_1(t),\ldots,u_n(t)]^T\in\mathbb{R}^n$ and $v(t)=[v_1(t),\ldots,v_n(t)]^T\in\mathbb{R}^n$ satisfy $u_k(t)$ and $v_k(t)$ $(k=1,2,\ldots n)\in\mathbb{L}_2[0,\infty)$. $\varpi_x(t)=[\varpi_{x1}(t),\ldots,\varpi_{xn}(t)]^T\in\mathbb{R}^n$ and $\varpi_y(t)=[\varpi_{y1}(t),\ldots,\varpi_{yn}(t)]^T\in\mathbb{R}^n$ denote the external disturbances satisfying $\varpi_{xk}(t)$ and $\varpi_{yk}(t)(k=1,2,\ldots n)\in\mathbb{L}_2[0,\infty)$. $A(r_t)=\mathrm{diag}\{a_1,\ldots,a_n\}$, $C(r_t)=\mathrm{diag}\{c_1,\ldots,c_n\}$ and $D(r_t)=\mathrm{diag}\{d_1,\ldots,d_n\}$ denote the decay rates of mRNA, protein and the translation rate of the kth node, respectively. $B(r_t)=(B_{ij}(r_t))\in\mathbb{R}^{n\times n}$ is the coupling matrix of the GRNs. $\tau(t)$ and $\sigma(t)$ denote time-varying translation delay and feedback regulation delay, respectively. All $A(r_t)$, $B(r_t)$, $C(r_t)$, $D(r_t)$, $U(r_t)$, $V(r_t)$, $H_x(r_t)$, $H_y(r_t)$ are constant matrices with appropriate dimensions for a fixed mode r_t and all modes can be detected.

The parameter r_t $(t\ge 0)$ denotes a right-continuous Markov process on the given probability space $(\Omega,\mathcal{F},\mathcal{P})$, which takes values in a finite set $\mathcal{I}\triangleq\{1,\ldots,N\}$ with generator $\Pi=\{\pi_{ij}\},\forall i,j\in\mathcal{I}$ described as

$$\Pr(r_{t+\Delta t}=j:r_t=i)=\begin{cases}\pi_{ij}\Delta t+o(\Delta t) & \text{if } i\ne j\\ 1+\pi_{ii}\Delta t+o(\Delta t) & \text{if } i=j,\end{cases}\tag{2}$$

with $\Delta t>0$, $\lim(o(\Delta t)/\Delta t)=0$ and $\pi_{ij}\ge 0$ $(i,j\in\mathcal{I},\ j\ne i)$ is the transition rate from mode i at time t to mode j at time $t+\Delta t$, while $\sum_{j=1}^N\pi_{ij}=0,\ \forall i\in\mathcal{I}$.

For the sake of convenience, denote the Markov process r_t $(t\ge 0)$ by i indices [29–31]. Consequently, system (1) can be rewritten as

$$\begin{cases}\dot{x}(t)=-A_ix(t)+B_ig(y(t-\sigma(t)))+U_iu(t)+H_{ix}\varpi_x(t)\\ \dot{y}(t)=-C_iy(t)+D_ix(t-\tau(t))+V_iv(t)+H_{iy}\varpi_y(t).\end{cases}\tag{3}$$

In this paper, the control input signals are generated by a sampled-data controller. More precisely, the control signals are represented with a sequence of sampling times: $0=t_0<t_1<\cdots<t_k<\cdots$, and $\lim_{k\to\infty}t_k=\infty$, such that only $u(t_k)$ and $v(t_k)$ are available for interval $t_k\le t<t_{k+1}$. Then, the mode-dependent state feedback controller can be designed as

$$\begin{cases}u(t_k)=K_{ix}x(t_k)\\ v(t_k)=K_{iy}y(t_k)\end{cases}\quad t_k\le t<t_{k+1},\tag{4}$$

where K_{ix} and K_{iy} are the state feedback gain matrices. Therefore, it can be obtained that

$$\begin{cases}\dot{x}(t)=-A_ix(t)+B_ig(y(t-\sigma(t)))+U_iK_{ix}x(t_k)+H_{ix}\varpi_x(t)\\ \dot{y}(t)=-C_iy(t)+D_ix(t-\tau(t))+V_iK_{iy}y(t_k)+H_{iy}\varpi_y(t).\end{cases}\tag{5}$$

Moreover, the sampling period T defined as $T:t_{k+1}-t_k$ is not constant. Noting that $t_k=t-(t-t_k):t-d(t)$ holds for $t_k\le t<t_{k+1}$, then system (5) is rewritten as

$$\begin{cases}\dot{x}(t)=-A_ix(t)+B_ig(y(t-\sigma(t)))+U_iK_{ix}x(t-d(t))+H_{ix}\varpi_x(t)\\ \dot{y}(t)=-C_iy(t)+D_ix(t-\tau(t))+V_iK_{iy}y(t-d(t))+H_{iy}\varpi_y(t).\end{cases}\tag{6}$$

Remark 1. The study of Markov jump GRNs has received much attention due to their theoretical importance and potential applications. It is worth mentioning that most of the researches mainly focus on the stability problem or the state estimation problem of GRNs and have achieved remarkable results [6,12,32,33]. In this paper, we investigate the non-uniform sampled-data control scheme to solve the passivity and passification problems of Markov jump GRNs.

Remark 2. The input-delay approach has been an effective method to deal with the non-uniform sampling case, which is more complicated but more practical in the applications. By introducing the concept of virtual delay, the sampled-data control system can be transformed to the continuous-time system with time-varying delays, which is the key idea of this paper.

Throughout this paper, the following assumptions are given.

Assumption 1. The time-varying delays satisfy

$$0<\tau(t)\le\tau_M,\quad 0<\sigma(t)\le\sigma_M,\tag{7}$$

where τ_M, σ_M are known positive constants.

Assumption 2. The sampling period is bounded by h $(h>0)$, then it follows that

$$0<t_{k+1}-t_k\le h,\quad 0<d(t)\le h.$$

Assumption 3 (*Wei et al. [11]*). The nonlinear regulatory function $g_i(t)$ $(i=1,2,\ldots,n)$ is a monotonically increasing function with saturation, which satisfies $0\le g_i(a)/a\le k$ for any $a>0$, or