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Multiplicity of almost periodic solutions for multidirectional associative memory neural network with distributed delays

Tiejun Zhou [a,*], Yi Wang [b], Min Wang [a]

[a] College of Science, Hunan Agricultural University, Changsha, Hunan 410128, China
[b] College of Information Science and Technology, Hunan Agricultural University, Changsha, Hunan 410128, China

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ABSTRACT

In this paper, the multiplicity of almost periodic solutions is studied for a multidirectional associative memory (MAM) neural network with almost periodic coefficients and continuously distributed delays. Under some assumptions on activation functions, some invariant subsets of the MAM neural network are constructed. The existence of multiple almost periodic solutions are obtained by using the theory of exponential dichotomy and Schauder's fixed point theorem. Furthermore, a sufficient condition is derived for the local exponential stability of some almost periodic solutions and their exponential attracting domains are also given. An example is given to illustrate the effectiveness of the results.
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1. Introduction

Generally, the multistability of neural networks, that is the existence and stability of multiple equilibrium points, periodic solutions or almost periodic solutions, is prerequisite when the neural network is applied to solve some problems of many-to-many association memory. It has attracted much attention in the recent years [1–16]. According to the different characteristics of active function, such as piecewise linear nondecreasing, Mexican-hat-type, discontinuous or concave–convex, the coexistence and local stability of multiple equilibria have been extensively investigated [1–10]. As is known to all, the cyclic behaviors or rhythmic activities are widespread in nature. Therefore, the existence and stability of multiple periodic or almost periodic solutions have also been analyzed [11–17]. From a phenomenological point of view, the almost periodic functions can represent more exactly rhythmic activities [15]. However, to the best of our knowledge, the investigation of multiple almost periodic solutions of neural networks is seldom reported [15,17,16]. In [15], a general methodology that involves geometric configuration of the n-neuron network structure for studying multistability was developed. Through applying the contraction mapping principle, some sufficient conditions guaranteeing the existence of 2^n exponentially stable almost periodic solutions were given, when the connection strengths, time lags, and external bias are almost periodic functions of time. In [16], by considering two classes of activation functions, the authors revealed that under some conditions, there are 2^n locally exponentially stable almost-periodic solutions of a delayed n-neuron neural network. The authors of [17] investigated the dynamics of 2^n almost periodic attractors for cellular neural networks with variable and distributed delays, they obtained exponential attracting domains.

The multidirectional associative memory (MAM) neural network, which was proposed by the Japanese scholar M. Hagiwara in 1990 [18], is an extension of BAM neural network model [19]. In an MAM neural network, its neurons are arranged in three or more fields. The neurons in the same field of an MAM neural network are not connected to each other, but the neurons between every two different fields are fully interconnected. By using of MAM neural networks, one can achieve the many-to-many association memory. Therefore, this class of networks possesses wide application fields such as pattern recognition, image denoising and intelligent information processing [18,20–23]. In order to achieve the many-to-many association memory by MAM neural networks, it is necessary to ensure their multistability. In [24], we studied the multistability issue for a delayed MAM neural network with m fields and n_k neurons in the field m as follows.

$$\frac{dx_{ki}}{dt} = -a_{ki}x_{ki}(t) + \sum_{p=1, p\neq k}^{m} \sum_{j=1}^{n_p} w_{pj}^{ki} f_{pj}\left(x_{pj}(t-\tau_{pj}^{ki})\right) + I_{ki} \tag{1}$$

* Corresponding author. Tel.: +86 0731 84635203; fax: +86 0731 84618071.
E-mail addresses: hntjzhou@126.com (T. Zhou), hunau_wangyi@163.com (Y. Wang), wangmin_407@163.com (M. Wang).

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where $k=1,2,\ldots,m$, $i=1,2,\ldots,n_k$, $x_{ki}(t)$ denotes the membrane voltage of the ith neuron in the field k at time t, $a_{ki}>0$ denotes the decay rate of the ith neuron in the field k, $f_{pj}(\cdot)$ is a neuronal activation function of the jth neuron in the field p, w_{pj}^{ki} is the connection weight from the jth neuron in the field p to the ith neuron in the field k, I_{ki} is the external input of the ith neuron in the field k, τ_{pj}^{ki} is the time delay of the synapse from the j neuron in the field p to the ith neuron in the field k. By using the Brouwer fixed point theorem and Dini upper right derivative, we obtained that some sufficient conditions which ensure the multidirectional associative memory neural network has 3^l equilibria and 2^l equilibria of them are stable, where l is a parameter associated with the number of neurons. System (1) is with discrete delay. It can only reflect the relationship between the state of the neuron x_{ki} at time t and the state of any other neuron x_{pj} at the time ahead of it by a fixed time length τ_{pj}^{ki}.

Since a neural network usually has a spatial extent due to the presence of a multitude of parallel pathways of a variety of axon sizes and lengths, there is a distribution of propagation delays over a duration of time [25]. That is to say, the state of the neural network in the past every time affects its current state. Obviously, the distributed delays can reflect the relationship more really than discrete delays. Hence, we also studied the MAM neural network with the continuously distributed delays [26] as follows.

$$\frac{dx_{ki}}{dt}=-a_{ki}x_{ki}(t)+\sum_{p=1,p\neq k}^{m}\sum_{j=1}^{n_p}w_{pj}^{ki}f_{pj}\left(\int_0^{+\infty}g_{pj}^{ki}(s)x_{pj}(t-s)\,ds\right)+I_{ki}(t), \tag{2}$$

where $g_{pj}^{ki}(s)$ is the delay kernel function of the synapse from the jth neuron in field p to the ith neuron in field k. By constructing a suitable Liapunov function and a Poincaré mapping, we proved the existence and the exponential stability of multiple periodic solutions of the MAM neural network (2).

From the viewpoint of biological nervous system, there exists memory vibration in human's cerebrum. And from the viewpoint of networks implementation, the neural network's external inputs I_{ki}, the neuronal signal decay rates a_{ki}, and the connection weights w_{pj}^{ki} often almost periodically vary with time because there are almost periodic phenomenons in the electrical power system. Therefore, the discussion on existence and stability of multiple almost periodic solutions of MAM neural networks is also very meaningful. Motivated by the above, in this paper, we study the existence and the exponential stability of multiple almost periodic solutions of an MAM neural network with almost periodic coefficients and continuously distributed delays as follows.

$$\frac{dx_{ki}}{dt}=-a_{ki}(t)x_{ki}(t)+\sum_{p=1,p\neq k}^{m}\sum_{j=1}^{n_p}w_{pj}^{ki}(t)f_{pj}\left(\int_0^{+\infty}g_{pj}^{ki}(s)x_{pj}(t-s)\,ds\right)+I_{ki}(t). \tag{3}$$

The initial conditions associated with (3) are of the form

$$x_{ki}(t)=\theta_{ki}(t), \tag{4}$$

where $k=1,2,\ldots,m$, $i=1,2,\ldots,n_k$ and $\theta_{ki}:(-\infty,0]\to\mathbb{R}$ are continuous functions.

In this paper, our main object is to obtain the sufficient conditions ensuring the existence and the exponential stability of multiple almost periodic solutions of the MAM neural network (3) with initial condition (4). After constructing an invariant basin of system (3), we split it to multiple subsets. In every subset, there exists a almost periodic solution. And in the invariant subset, the almost periodic solution is exponentially stable. This paper is organized as follows. In the next section, we introduce some lemmas of exponential dichotomy method and almost periodicity. In Section 3, we construct an invariant basin of MAM neural network (3) and split it into multiple invariant subsets. In Section 4, we investigate the existence of multiple almost periodic solutions of (3) by using exponential dichotomy and Schauder's fixed point theorem. Meanwhile, we construct exponential attracting domain of each almost periodic solution. In Section 5, we investigate the exponential stability of multiple almost periodic solutions of (3) by constructing exponential attracting domain of each almost periodic solution. In Section 6, an example is given to illustrate the effectiveness of our results.

2. Preliminary

We firstly introduce two lemmas on exponential dichotomy [27]. Set the vector

$$x(t)=(x_{11}(t),\ldots,x_{1n_1}(t),\ldots,x_{m1}(t),\ldots,x_{mn_m})^T\triangleq \mathrm{col}\{x_{ki}(t)\}.$$

Lemma 1. *Let* $B(t)=\mathrm{diag}(-a_{11}(t),\ldots,-a_{1n_1}(t),\ldots,-a_{m1}(t),\ldots,-a_{mn_m}(t))$. *If* $a_{ki}(t)(k=1,2,\ldots,m,i=1,2,\ldots,n_k)$ *are almost periodic functions and* $\lim_{T\to+\infty}(1/T)\int_t^{t+T}a_{ki}(s)\,ds>0$, *then system*

$$\frac{dx}{dt}=B(t)x(t) \tag{5}$$

admits an exponential dichotomy.

Lemma 2. *If system (5) admits an exponential dichotomy, the almost periodic system*

$$\frac{dx}{dt}=B(t)x(t)+\eta(t)$$

has a unique almost periodic solution $\varphi(t)$, *and*

$$\varphi(t)=\int_{-\infty}^{t}X(t)PX^{-1}(s)\eta(s)\,ds-\int_t^{+\infty}X(t)(E-P)X^{-1}(s)\eta(s)\,ds.$$

Set $I[n]=\{1,2,\ldots,n\}$. For any $k,p\in I[m]$, $p\neq k$, $i\in I[n_k]$, and $j\in I[n_p]$, we make the following assumptions to MAM neural network (3).

(H1) The signal decay rates $a_{ki}(t)$, connection weights $w_{pj}^{ki}(t)$ and external inputs $I_{ki}(t)$ are all almost periodic functions on $\mathbb{R}$, and

$$\lim_{T\to+\infty}\frac{1}{T}\int_t^{t+T}a_{ki}(s)\,ds>0,\ \underline{a}_{ki}=\inf_{t\in\mathbb{R}}a_{ki}(t)>0.$$

(H2) The delay kernel functions $g_{pj}^{ki}(s):\mathbb{R}^+\to\mathbb{R}^+$ are piecewise continuous, and satisfy

$$\int_0^{+\infty}g_{pj}^{ki}(s)\,ds=1,\ \text{and}\ \int_0^{+\infty}e^{\underline{a}s}g_{pj}^{ki}(s)\,ds<+\infty,$$

where $\underline{a}=\min_{k\in I[m]}\min_{i\in I[n_k]}\underline{a}_{ki}$.

(H3) There exist constants $M_{pj}>0$ such that $|f_{pj}(x)|\le M_{pj}$ for each $x\in\mathbb{R}$.

Remark 1. In (H1), the assumption of almost periodicity and condition $\lim_{T\to+\infty}(1/T)\int_t^{t+T}a_{ki}(s)\,ds>0$ ensure that the linear almost periodic differential system (18) has an almost periodic solution by using Lemma 1. In general, the signal decay rates $a_{ki}(t)>0$. The condition $\underline{a}_{ki}=\inf_{t\in\mathbb{R}}a_{ki}(t)>0$ is a prerequisite to construct the invariant basin Φ and those functions $H_{(k+1)i}^{\pm}$ and $G_{ki}^{\pm}$ in Section 3.

Remark 2. In (H2), the condition $\int_0^{+\infty}g_{pj}^{ki}(s)\,ds=1$ is demanded because the delay kernel functions are equivalent to the weights of the neurons state in different time. In the proof of Theorem 5, we define the functions $H_{ki}(\lambda)$, which need the condition $\int_0^{+\infty}e^{\underline{a}s}g_{pj}^{ki}(s)\,ds<+\infty$.

According to the properties of almost periodic function [27], the coefficients of (3) are all bounded and uniformly continuous. Set $\overline{a}_{ki}=\sup_{t\in\mathbb{R}}a_{ki}(t)$, $\overline{w}_{pj}^{ki}=\sup_{t\in\mathbb{R}}|w_{pj}^{ki}(t)|$, $\overline{I}_{ki}=\sup_{t\in\mathbb{R}}|I_{ki}(t)|$.

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