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Global exponential stability in Lagrange sense for inertial neural networks with time-varying delays

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ABSTRACT

In this paper, the global exponential stability in Lagrange sense related to inertial neural networks with time-varying delay is investigated. Firstly, by constructing a proper variable substitution, the original system is transformed into the first order differential system. Next, some succinct criteria for the ultimate boundedness and global exponential attractive set are derived via the Lyapunov function method, inequality techniques and analytical method. Meanwhile, the detailed estimations for the global exponential attractive set are established. Finally, the effectiveness of theoretical results has been illustrated via two numerical examples.

1. Introduction

The dynamical characteristic analysis of neural networks has attracted extensive attention, due to their successful application in various areas, such as signal processing, automatic control, financial industry and so on. Actually, the time delay ineluctability exists in both biological and artificial neural systems for the finite transfer speed. And the time delay usually leads to instability, oscillations, chaos, etc. However, when coming to complex network synchronization, Dhamala et al. had proved that the synchronous ability of system can be meliorated by controlling the delay [1]. Therefore, there are considerable literatures which investigated the dynamical characteristic of neural networks with time delays. The global asymptotic stability and global exponential stability (GES) with time delays have been intensively studied [2–11]. A weighting-delay-based method was provided to study of the stability of a class of recurrent neural networks [4]. A universal stability analysis method for recurrent neural networks with discrete and distributed delays for neutral-type and high-order recurrent neural networks were given out, and some new succinct criteria for global asymptotic stability were derived [5]. The GES of memristive neural networks with time-varying delays was also studied [9]. Zhang et al. presented a comprehensive review of the research on stability of continuous-time recurrent neural networks, and the future directions of stability analysis of recurrent neural networks were pointed out [11]. Scholars also paid particular attention to the global robust stability of neural networks with time delays [12,13]. By constructing suitable Lyapunov functional and some analysis techniques, Song and Cao have investigated the GES of periodic solutions of BAM neural networks with delays [14].

The inertial item is taken as a critical tool to generate bifurcation and chaos. Comparing to electronic neural networks with the standard resistor–capacitor variety, Babcocka and Westervelt showed that the dynamics could be complex when the neuron couplings included an inertial nature [15]. The Hopf bifurcation and chaos in an inertial neuron system with coupled delay were discussed via the perturbation scheme and the center manifold theorem [16]. In fact, there exist evident biological backgrounds for introducing an inertial term into the standard neural system. Scholars have presented the implementation of the membrane of a hair cell by equivalent circuits which contained an inductance in semicircular canals of some animals, such as pigeon [17,18]. Resorting to matrix measure and Halanay inequality, Cao and Wan investigated GES and synchronization of inertial neural network with time delay [19]. The globally exponentially stable in Lyapunov sense of inertial neural networks was also studied by analysis method and inequality technique [20–22]. On the basis of

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homeomorphism theory and inequality technique, the GES of inertial BAM neural networks was investigated, and the LMI-based sufficient condition was given out [21]. By employing the Lyapunov functional method and the comparison principle, the sufficient conditions guaranteeing GES of inertial BAM neural networks with the impulsive effects was obtained [22].

From the viewpoint of dynamics, the global stable system in Lyapunov sense is a monostable system. Unfortunately, the monostable system has been found some computationally limitations. For example, in the winner-take-all network which depends on the external input (or the initial value), only the neuron with the strongest input (or highest initial value) should remain active. This is possible only if there are multiple equilibrium points with some being unstable [23,24]. The network is required with multistable equilibria when designed for pattern recognition or associative memory storage [25,26]. In these applications, the multistable dynamics system is essential to meet with the requirements, and proper concept of stability is required to investigate multistable systems. It is worth noting that Lagrange stability is concerned with the stability of the total system, different from Lyapunov stability which refers to the stability of equilibrium points. When the Lyapunov stability is discussed, it is necessary to know information of equilibrium, such as existence and quantity. Nevertheless, when come to Lagrange stability, it is not necessary. Once a system is proved to be globally exponentially stable in Lagrange sense, we will only need to concentrate on its dynamics characteristic inside the attractive set, as there is no equilibrium (stable and unstable), periodic state, or chaos attractor outside the attractive set [27–29]. Especially, the global Lyapunov stability can be viewed as a special case of Lagrange stability, by regarding the stable equilibrium point as an attractive set. The Lagrange stability has been studied extensively [27–35]. Resorting to the concept of Lagrange stability, the stability of ecological systems was discussed [30]. By constructing appropriate Lyapunov function and inequality techniques, several succinct criteria were presented to ascertain the Lagrange stability of neutral type BAM neural networks with multiple time-varying delays [32]. With the help of nonsmooth analysis and control theory, the globally exponentially stability in Lagrange sense of memristive neural networks was studied [33,34].

To the best of our knowledge, the problem of the Lagrange stability of inertial neural networks is still open. Accordingly, techniques and methods for it should be explored and developed. In this paper, we are devoted to investigating the Lagrange GES of inertial neural networks with time-varying delays, and give out the specific estimation of the global exponential attractive (GEA) sets.

The reminder of this paper is organized as follows. Section 2 presents model description, definitions and some useful lemmas. The globally exponentially stability in Lagrange sense of inertial neural networks is studied in Section 3. Section 4 gives two numerical examples to show the validity of our results. Finally, conclusions are drawn in Section 5. To streamline the presentation of the results, the following notations are employed. R^n denotes the n-dimensional Euclidean space, $\Gamma=\{1,2,\dots,n\}$. Let $\lambda_{\max}(P)$ and $\lambda_{\min}(P)$ be maximum and minimum eigenvalues of matrix P, respectively. A real symmetric matrix $P<0$ represents that P is negative definite. For a symmetric matrix, $*$ denotes the symmetric elements in it.

2. Preliminaries

In this paper, the following inertial neural networks are considered:

$$\frac{d^2x_i(t)}{dt^2}=-a_i\frac{dx_i(t)}{dt}-b_ix_i(t)+\sum_{j=1}^{n}c_{ij}f_j(x_j(t))+\sum_{j=1}^{n}d_{ij}f_j(x_j(t-\tau_j(t)))+I_i(t),\quad i\in\Gamma,\tag{1}$$

where $x_i(t)$ represents the state of the ith neuron at time t, and the second derivative is called an inertial term of (1). $a_i>0$ and $b_i>0$ are constants. c_{ij} and d_{ij} are connection weights related to the neurons without delays and with delays, respectively. $f_j(\cdot)$ stands for neuron activation function of jth neuron at time t and $f_j(0)=0$, $j\in\Gamma$. $\tau_j(t)$ is the time-varying delay of jth neuron at time t which satisfies $0\le\tau_j(t)\le\tau$ and $\dot{\tau}_j(t)\le\mu$. $I_i(t)$ is the external input on the ith neuron at time t and $|I_i(t)|\le I_i$.

The initial conditions of inertial neural networks (1) are

$$x_i(s)=\varphi_i(s),\quad \frac{dx_i(s)}{dt}=\psi_i(s),\quad s\in[-\tau,0],\quad i\in\Gamma,\tag{2}$$

where $\varphi_i(s)$ and $\psi_i(s)$ are real-valued continuous functions on $[-\tau,0]$.

Assumption 1 (H). There exist two diagonal matrixes $\check{L}=\mathrm{diag}\{l_1^-,l_2^-,\dots,l_n^-\}$ and $\hat{L}=\mathrm{diag}\{l_1^+,l_2^+,\dots,l_n^+\}$ such that for any distinct $x,y\in R$, the following inequalities hold

$$l_i^-\le\frac{f_i(x)-f_i(y)}{x-y}\le l_i^+,\quad i\in\Gamma.$$

Resorting to the following variable transformation:

$$y_i(t)=\frac{dx_i(t)}{dt}+\xi_ix_i(t),\quad i\in\Gamma,\tag{3}$$

the inertial neural network (1) can be written as

$$\begin{cases}\dfrac{dx_i(t)}{dt}=-\xi_ix_i(t)+y_i(t),\\ \dfrac{dy_i(t)}{dt}=-\beta_iy_i(t)+\alpha_ix_i(t)+\displaystyle\sum_{j=1}^{n}c_{ij}f_j(x_j(t))+\sum_{j=1}^{n}d_{ij}f_j(x_j(t-\tau_j(t)))+I_i(t),\quad i\in\Gamma,\end{cases}\tag{4}$$

and the initial conditions can be written as

$$\begin{cases}x_i(s)=\varphi_i(s),\\ y_i(s)=\xi_i\varphi_i(s)+\psi_i(s)\triangleq\phi_i(s),\quad s\in[-\tau,0],\quad i\in\Gamma,\end{cases}\tag{5}$$

where $\alpha_i=\beta_i\xi_i-b_i$, $\beta_i=a_i-\xi_i$.

Remark 1. In contrast to a certain variable transformation, based on the idea of [22], we also introduce free-weighted coefficient ξ_i into variable transformation (3), and different variable transformations can be obtained by selecting ξ_i with different values, which makes our results possess better application.

Denote $x(t)=(x_1(t),x_2(t),\dots,x_n(t))^T$, $y(t)=(y_1(t),y_2(t),\dots,y_n(t))^T$, $A=\mathrm{diag}\{\alpha_1,\alpha_2,\dots,\alpha_n\}$, $B=\mathrm{diag}\{\beta_1,\beta_2,\dots,\beta_n\}$, $\Lambda=\mathrm{diag}\{\xi_1,\xi_2,\dots,\xi_n\}$, $\tau(t)=(\tau_1(t),\tau_2(t),\dots,\tau_n(t))^T$, $C=(c_{ij})_{n\times n}$, $D=(d_{ij})_{n\times n}$, $I(t)=(I_1(t),I_2(t),\dots,I_n(t))^T$, $I=(I_1,I_2,\dots,I_n)^T$, $W=\mathrm{diag}\{w_1,w_2,\dots,w_n\}$, $w_i=\max\{|l_i^-|,|l_i^+|\}$, $i\in\Gamma$. Correspondingly, the network (4) can be written as

$$\begin{cases}\dfrac{dx(t)}{dt}=-\Lambda x(t)+y(t),\\ \dfrac{dy(t)}{dt}=-By(t)+Ax(t)+Cf(x(t))+Df(x(t-\tau(t)))+I(t).\end{cases}\tag{6}$$

Next, we will introduce some definitions and lemmas such that our main results could be clearly presented. According to [28], the following definitions are presented.

Definition 1. A set $\Omega\subseteq R^n$ is said to be a attractive set of (6), if for $\forall s\in[-\tau,0]$, $x(s)\in R^n\backslash\Omega$, $\overline{\lim}_{t\to+\infty}\rho(x(t),\Omega)=0$ holds, where $R^n\backslash\Omega$ is the complement set of Ω, and $\rho(x,\Omega)=\inf_{y\in\Omega}\|x-y\|$ is the distance between x and Ω.

Definition 2. If there exists a radially unbounded and positive definite function $V(\cdot)$, a nonnegative continuous function $K(\cdot)$, and two positive constants ℓ and α such that for any solution $x(t)$ of (6),