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An improved stability criterion for generalized neural networks with additive time-varying delays

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ABSTRACT

This paper deals with the problem of stability analysis of generalized neural networks with time delays. It should be noted that additive time-varying delays are taken in the state of the neural networks. A novel augmented Lyapunov–Krasovskii (L–K) functional which involves more information on the activation function of the neural networks and upper bound of the additive time-varying delays is constructed. By introducing some zero equations and using the reciprocal convex combination technique and Finsler's lemma, an improved delay-dependent stability criterion is derived in terms of linear matrix inequalities (LMIs), which can be efficiently solved via standard numerical software. Finally, three numerical examples are provided to demonstrate the less conservatism and effectiveness of the proposed results.

1. Introduction

Neural networks have substantial capacity in information processing, due to which, they have been used in many areas such as signal processing, image decryption, pattern recognition, associative memories and fixed-point computations [1–8]. In these applications, one important task is to checkout whether the equilibrium points of the considered networks are stable or not because the applications heavily depend on the dynamic behavior of the equilibrium points. It should be noted that, because of signal transmissions among neurons and the finite switching speed of amplifiers in the implementation of electrical circuits, time delays are an unavoidable factor to be considered in real systems. The existence of time delays may cause oscillation, divergence and even instability. Therefore, stability of neural networks with time-delays has drawn a great deal of attention in recent years [9–24]. Stability results for neural networks may be classified into two categories, i.e., delay-dependent and delay-independent ones. It should be noted that, when the size of time-delay is small, delay-dependent stability criteria are less conservative than delay-independent ones. A main objective of stability problems is to find maximum allowable upper bounds (MAUB) of time-delays, which means our designed delayed neural networks remain asymptotically stable up to MAUB. In order to enhance the upper bound of time delays, how to construct L–K functional and estimate the time-derivative of L–K functional plays a virtual role. In this regard, various methods and approaches have been proposed to drive stability criteria for delayed neural networks such as augmented L–K functional, discretized L–K functional, model transformation, free-weighting matrices, delay-partitioning technique, reciprocally convex combination, quadratic convex combination and so on. For example, Zhang et al. [11] have presented the weighting-delay-based stability criteria for neural networks with time-varying delays by dividing the delay interval into several sub-intervals with weighted parameters, which lead to less conservative result than fixed delay-partitioning methods. By introducing suitable L–K functional and some zero integral inequalities with reciprocally convex optimization approach, stability analysis of neural networks with time-varying delays has been investigated in [16]. Very recently, Kwon et al. [17] have showed the improved stability criteria for neural networks by introducing some new terms in L–K functional that contains more information on activation functions.

In most of the reported results on stability criteria for delayed neural networks, time-delays have been taken in a singular or simple form in the state variable. Meanwhile, the authors in [28] have introduced a new type of neural networks which contains two additive time-varying delay components in the state. Such a system may be encountered in many practical situations such as remote control and networked control. For example, in networked controlled systems, signals transmitted from one point to another may experience a few segments of networks, which can possibly induce successive delays with different properties due to the

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variable network transmission conditions [29,30]. Therefore, the problem of stability analysis of neural networks with two successive time-varying delays in the state has received more and more attention and become more popular in recent years [31–35]. In [32,33], improved stability criteria of neural networks with two additive time-varying delay components have been studied by utilizing reciprocally convex and convex polyhedron approaches, which can lead to less conservative results than in [28,31]. In addition, they have not utilized the information about the lower bound of the activation function in L–K functional. Very recently, the problem of robust stability analysis of neural networks with uncertain parameter has been derived in the work [35].

On the other hand, different types of neural networks such as recurrent back-propagation neural networks, optimization type neural networks, Hopfield neural networks, bidirectional associative memory neural networks and cellular neural networks have been studied in the past few decades. These neural networks have been applied in various fields such as signal processing, parallel computing, optimization problems, secure communication, chemical biology, engineering and so on. For example, the quadruple-tank process can be modeled in the form of neural networks [36], in which four interconnected water tanks and two pumps are composed. In this problem, voltages to the two pumps are considered as inputs and the water level of the lower two tanks is considered as outputs, and the main objective is to control the level of the two lower tanks using the two pumps. In addition, according to states of the neurons (internal/external), neural networks may be classified into two types, namely local field neural networks (LFNNs) and static neural networks (SNNs). From the existing literature, it can be seen that most of the authors have considered the problem of stability analysis of LFNNs [9–17] and SNNs [18–24], separately. It is noted that these two types are not always equivalent, however, it is possible to transfer them into equivalent forms from one to each other under some assumptions. But these assumptions are not always reasonable in many real life applications [10]. In this regard, a unified model has been considered in the works [10,25–27,34] and it is termed as generalized neural networks (GNNs), which is the combination of both LFNNs and SNNs. Therefore, it is enough to study the stability analysis of GNNs instead of studying the stability analysis of LFNNs and SNNs, separately. It should be pointed out that back-propagation neural networks and optimization type neural networks can be modeled in the form of SNNs, whereas Hopfield neural networks, bidirectional associative memory neural networks and cellular neural networks can be modeled in the form of LFNNs. Thus, GNNs have been applied in the areas where different types of neural networks are used.

For example, exponential stability of cellular neural networks (i.e. LFNNs) with both interval time-varying delays and general activation functions has been considered in [15], whereas conditions for the stability analysis of static recurrent neural networks (i.e. SNNs) with interval time-varying delay have been derived in [20,23]. In [23], a new augmented L–K functional has been proposed for the stability analysis of SNNs with interval time-varying delays. Stability analysis for GNNs with time-varying delays has been investigated in [26] and to improve the stability region, an improved integral inequality approach has been used to handle the cross-product terms in [27]. Recently, by using free-weighting matrix and reciprocally convex combination techniques, a delay-dependent stability criterion for generalized continuous neural networks with two delay components has been derived in [34]. To the best of our knowledge, there are a very small number of works that deal with the stability analysis of GNNs with singular time-varying delays and there is a single work on the stability analysis of GNNs with additive time-varying delays. Therefore, there is enough room to improve the stability conditions for GNNs with successive delay components in the state, which is the main motivation of this paper. Also, stability results of these model become more general than the ones in the existing literature.

Motivated by the above discussions, a new delay-dependent stability criterion for generalized neural networks with time-delays is proposed in this paper. It is noted that two successive time-varying delay components are taken in the state. By fully using the available information about time-delays and activation functions, a novel augmented L–K functional is constructed. By utilizing reciprocal convex combination technique and Finsler's lemma, and by proposing some zero equations, the improved delay-dependent stability criterion is derived in terms of LMIs. Finally, three numerical examples are given to show the effectiveness of the proposed method and this shows that we can obtain large maximum delay bounds than ones in recent existing works.

The main contributions and improvements of this paper are summarized as follows:

- Our main aim of this paper is to find a less conservative stability criterion for the network. In order to reduce the conservatism, a novel augmented L–K functional is introduced which includes more information about successive time delays and slope of the activation function. Such type of L–K functional has not yet been considered in the previous studies [31–34] on the stability of neural networks with successive time-varying delay components.
- Inspired by the works [16,17], some zero equations which would include more quadratic and integral terms are introduced. These terms are merged with the time derivative of L–K functional, which in turn can enhance the feasibility region of stability criterion.

Notations: Throughout this paper, the superscripts T and -1 mean the transpose and the inverse of a matrix respectively. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, $\mathbb{R}^{n\times m}$ is the set of all $n\times m$ real matrices. For symmetric matrices P and $Q, P>Q$ (respectively, $P\geq Q$) means that the matrix $P-Q$ is positive definite (respectively, non-negative). I_n, 0_n and $0_{m,n}$ stand for $n\times n$ identity matrix, $n\times n$ and $n\times m$ zero matrices, respectively and symmetric term in a symmetric matrix is denoted by $\star$. $X^{\perp}$ denotes a basis for the null-space of X. If the Matrices are not explicitly stated, it is assumed to compatible dimensions.

2. System description and preliminaries

Consider the following generalized neural networks (GNNs) with additive time-varying delays:

$$\dot{y}(t)=-Ay(t)+W_0h(W_2y(t))+W_1h(W_2y(t-d_1(t)-d_2(t)))+u, \tag{1}$$

where $y(t)=[y_1(t)\ y_2(t)\ \cdots\ y_n(t)]^T$, $y_i(t)$ is the state of the ith neuron at time t; $A=\mathrm{diag}\{a_1,a_2,\ldots,a_n\}$ is a positive diagonal matrix; $W_0\in\mathbb{R}^{n\times n}$, $W_1\in\mathbb{R}^{n\times n}$ and $W_2\in\mathbb{R}^{n\times n}$ are the connection weight matrices; $h(W_2y(\cdot))=[h_1(W_{21}y(\cdot))\ h_2(W_{22}y(\cdot))\ \cdots\ h_n(W_{2n}y(\cdot))]^T$ represents the neuron activation function, where W_{2i} denotes the ith row vector of the matrix W_2; and $u=[u_1\ u_2\ \cdots\ u_n]^T$ is an external constant input vector.

The neuron activation function $h_i(\cdot)$ is continuous and bounded [8,25], and there exist constants k_i^- and k_i^+ such that

$$k_i^-\leq\frac{h_i(\beta_1)-h_i(\beta_2)}{\beta_1-\beta_2}\leq k_i^+,\quad i=1,2,\ldots,n, \tag{2}$$

for any $\beta_1,\beta_2\in\mathbb{R}$, and $\beta_1\neq\beta_2$.

Under the assumption of the activation function in (2), system (1) has an equilibrium point $y^\star=[y_1^\star\ y_2^\star\ \ldots\ y_n^\star]$, i.e., $0=-Ay^\star+W_0h(W_2y^\star)+W_1h(W_2y^\star)+u$. Utilizing the transformation $x(t)=y(t)-y^\star$, one can shift the equilibrium point from $y^\star$ to the