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Dissipativity-based filtering of nonlinear periodic Markovian jump systems: The discrete-time case ☆

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ABSTRACT

This paper is concerned with the problem of dissipative filtering for discrete-time periodic Markovian jump systems with bounded nonlinearity. It is assumed that the data measurements from the plant to the filter are subject to randomly occurred packet dropouts satisfying Bernoulli distribution. The purpose of this paper is to design a filter such that the filtering error system is stochastically stable and strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative. To eliminate the cross coupling between the Lyapunov matrix and system matrices, some slack matrix variables are introduced. Based on a mode-dependent and basis-dependent Lyapunov function, a sufficient condition of the desired dissipative filter in terms of linear matrix inequalities (LMIs) for discrete-time Periodic Markovian Jump Systems with bounded nonlinearity is derived. Finally, a numerical example is exploited to demonstrate the effectiveness of the proposed filter design method.

1. Introduction

Over the past few decades, Markovian jump systems (MJSs) have drawn much attention since such types of systems are very common in physical dynamical systems whose structure is subject to random abrupt changes, such as aerospace systems, power systems and networked control systems [1,2]. So far, many researches on stability analysis and robust controller synthesis for MJSs have been reported, and many important results have been obtained. For example, the filtering problems of MJSs have been studied in [3,4]. The control problems have been investigated in [5,6]. The stability and stabilization problems have been reported in [7–11]. The state estimation problems have been reported in [12], and the synchronization problems have been studied in [13].

Periodic systems, as a kind of time-varying systems, are very common in fields like economics, physics and computer science. For example, the pendulum and the satellite run periodically. In event-triggered control systems, systems operate periodically due to its fixed sampling period. Computer services and applications run according to clock cycles of CPU. Therefore, periodic systems have received much attention and many problems have been solved including stability analysis and stabilization [14,15], event-triggered control [16,17]. However, few results exist in the MJSs framework for periodic systems.

Due to the temporary failure (data packet dropout) which results in incomplete signal transmission between the plant and the filter, conventional filter may lead to poor performance in practical systems. Nonlinearity is another phenomenon that is frequently encountered in actual implementation. These two facts just aforesaid are two sources causing instability and unsatisfactory performance. Therefore, it is necessary and significant to attach more importance to MJSs with data packet dropout and nonlinearities.

Besides, since the notion of dissipative systems was introduced in [18], it has aroused great concern due to its general applications in electromechanical system, power system, complex chemical process, etc. In recent years, some research efforts about dissipative control and dissipative filtering have been obtained (see [19,20] and references therein).

It should be pointed out that the considered transition probabilities in the Markov process are time-invariant in the majority of the references in the field of MJSs, i.e., the matrix coefficients are time-independent. However, the assumption cannot always be satisfied in real applications [21–23]. Although time-invariant transition probabilities are expected to simplify the study of MJSs, the ideal transition probabilities limit the practical applications to some extent inevitably. Therefore it is important to attach great importance to the study of nonhomogeneous MJSs. However, up to now, MJSs with time-varying transition probabilities have not yet

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been fully investigated, and this constitutes the primary motivation of our present research.

In this paper, the problem of dissipative filtering for discrete-time periodic MJSs with nonhomogeneous finite state Markov chain and nonlinearity has been studied. Besides, the measurements transmission from the plant to the filter is subject to randomly occurred packet dropouts satisfying Bernoulli distribution. Resorting to a basic-dependent Lyapunov function, a sufficient condition in term of LMIs has been proposed for ensuring the considered system stochastically stable and strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipative. By introducing some slack matrix variables to eliminate the coupling between the system matrices and Lyapunov matrix among different modes, the filter design is obtained by solving a set of LMIs. An numerical example is given to show the effectiveness of the proposed approach.

The rest of this paper is organized as follows. Section 2 presents the definitions and the preliminary results. Stochastically stability and strictly $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipation of the filter error system are given in Section 3. The filter design problem is solved in Section 4. A numerical example is given in Section 5, and we conclude the paper in Section 6.

Notation: The notations used throughout this paper are fairly standard. For notation $(\Omega,\mathcal{F},\mathcal{P})$, Omega is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathcal{P}$ is the probability measure on $\mathcal{F}$. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space. The notation $P>0(\geq 0)$ means that P is real symmetric and positive definite (positive semidefinite). I and 0 represent the identity matrix and the zero matrix, respectively. The superscript "T" represents the transpose. $l_2[0,\infty)$ is the space of square-integrable vector functions over $[0,\infty)$. $\|\cdot\|$ denotes the Euclidean norm of a vector. For an arbitrary matrix B and two symmetric matrices A and C,

$$\begin{bmatrix} A & B \\ * & C \end{bmatrix}$$

denotes a symmetric matrix, where "$*$" denotes the term that is induced by symmetry, and diag$\{\cdots\}$ stands for a block-diagonal matrix. Besides, $E\{x\}$ and $E\{x|y\}$ will, respectively, mean expectation of x and expectation of x condition on y. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Definitions and preliminary results

Fix an underlying probability space $(\Omega,\mathcal{F},\mathcal{P})$ and consider the following discrete-time MJSs:

$$\begin{cases} x(k+1)=A(k,\theta_k)x(k)+B(k,\theta_k)\omega(k) \\ \qquad\qquad +E(k,\theta_k)f(x(k)) \\ y(k)=e(k)C(k,\theta_k)x(k)+D(k,\theta_k)\omega(k) \\ z(k)=H(k,\theta_k)x(k)+L(k,\theta_k)\omega(k) \end{cases} \tag{1}$$

where $x(k)=[x_1^T(k),x_2^T(k),\ldots,x_n^T(k)]^T\in\mathbb{R}^n$ is the state vector, $\omega(k)\in\mathbb{R}^m$ is the external disturbance signal which belongs to $l_2[0,\infty)$, $f(x(k))=[f_1^T(x(k)),f_2^T(x(k)),\ldots,f_n^T(x(k))]^T\in\mathbb{R}^n$ is the nonlinear input, $y(k)=[y_1^T(k),y_2^T(k),\ldots,y_p^T(k)]^T\in\mathbb{R}^p$ is the measured output, $z(k)=[z_1^T(k),z_2^T(k),\ldots,z_q^T(k)]^T\in\mathbb{R}^q$ is the signal to be estimated.

The stochastic variable $e(k)$ is Bernoulli distributed white sequence taking values on either 0 or 1 with

$$\text{Prob}\{e(k)=1\}=E\{e(k)\}=\overline{e},\ \text{Prob}\{e(k)=0\}=1-\overline{e}$$

where Prob$\{\cdot\}$ stands for the probability.

The process $\{\theta_k,k\geq 0\}$ is defined on a finite statespace $\Xi=\{1,\ldots,\sigma\}$. It is assumed that the mode transition probabilities are given as

$$\pi_{ij}(k)=\text{Prob}\{\theta_{k+1}=j|\theta_k=i\},\quad i,j\in\Xi \tag{2}$$

where $\Pi_k=[\pi_{ij}(k)]$, $0\leq\pi_{ij}(k)\leq 1$ and $\sum_{j=1}^{\sigma}\pi_{ij}(k)=1$, $\forall k\geq 0$. $\pi_{ij}(k)$ means the transition probabilities from ith mode at time k to jth mode at time $k+1$. For convenience of later analysis, we denote the matrices associated with the ith mode by $U_i(k)=U(k,\theta_k=i)$, $U_{Fi}(k)=U_F(k,\theta_k=i)$. The system matrices of the ith mode are denoted by $([A_i(k)]_{k\geq 0}$, $[B_i(k)]_{k\geq 0}$, $[C_i(k)]_{k\geq 0}$, $[D_i(k)]_{k\geq 0}$, $[E_i(k)]_{k\geq 0}$, $[H_i(k)]_{k\geq 0}$, $[L_i(k)]_{k\geq 0})$, $i\in\Xi$, which are p-periodic matrix sequences. Obviously these matrices are real known with appropriate dimensions. A set of matrices $[U_i(k)]_{k\geq 0}$ is said to be p-periodic if $U_i(k)=U_i(k+p), i\in\Xi, k\geq 0$ [24]. Based on this, the system is turned into

$$\begin{cases} x(k+1)=A_i(k)x(k)+B_i(k)\omega(k) \\ \qquad\qquad +E_i(k)f(x(k)) \\ y(k)=e(k)C_i(k)x(k)+D_i(k)\omega(k) \\ z(k)=H_i(k)x(k)+L_i(k)\omega(k). \end{cases} \tag{3}$$

Remark 1. In this paper, we know that Markov chain in the system is nonhomogeneous because transition probabilities are time-dependent. However, when $\Pi_k=\Pi$ for all $k\geq 0$, the Markov chain is known as a homogeneous Markov chain. In this paper, transition probability matrices $[\Pi_k(k)]_{k\geq 0}$ are also p-periodic.

Here, we are interested in designing a full-order p-periodic Markovian jump linear filter of the form

$$\begin{cases} x_F(k+1)=A_{Fi}(k)x_F(k)+B_{Fi}(k)y(k) \\ z_F(k)=H_{Fi}(k)x_F(k)+L_{Fi}(k)y(k) \\ x_F(0)=0 \end{cases} \tag{4}$$

where $A_{Fi}(k)$, $B_{Fi}(k)$, $H_{Fi}(k)$, $L_{Fi}(k)$, $\forall i\in\Xi$ are filter gains to be determined. The filter with the above structure is assumed to jump synchronously with the modes in system (3), which is hereby mode-dependent.

Applying this filter to the system (3), we obtain the following estimation error system:

$$\begin{cases} \overline{x}(k+1)=\overline{A}_{i1}(k)\overline{x}(k)+\tilde{e}(k)\overline{A}_{i2}(k)\overline{x}(k) \\ \qquad\qquad +\overline{B}_i(k)\omega(k)+\overline{E}_i(k)\overline{f}(\overline{x}(k)) \\ \overline{z}(k)=\overline{C}_{i1}(k)\overline{x}(k)+\tilde{e}(k)\overline{C}_{i2}(k)\overline{x}(k) \\ \qquad +\overline{D}_i(k)\omega(k) \end{cases} \tag{5}$$

where $\overline{x}(k)=\left[x^T(k)\ x_F^T(k)\right]^T\in\mathbb{R}^{2n}$, $\overline{f}(\overline{x}(k))=\left[f^T(x(k))\ 0\right]^T\in\mathbb{R}^{2n}$, $\overline{z}(k)=z(k)-z_F(k)$, $\tilde{e}(k)=e(k)-\overline{e}$ and

$$\overline{A}_{i1}(k)=\begin{bmatrix} A_i(k) & 0 \\ \overline{e}B_{Fi}(k)C_i(k) & A_{Fi}(k) \end{bmatrix}$$
$$\overline{A}_{i2}(k)=\begin{bmatrix} 0 & 0 \\ B_{Fi}(k)C_i(k) & 0 \end{bmatrix}$$
$$\overline{B}_i(k)=\begin{bmatrix} B_i(k) \\ B_{Fi}(k)D_i(k) \end{bmatrix}$$
$$\overline{C}_{i1}(k)=\left[H_i(k)-\overline{e}L_{Fi}(k)C_i(k)-H_{Fi}(k)\right],$$
$$\overline{C}_{i2}(k)=\left[-L_{Fi}(k)C_i(k)0\right]$$
$$\overline{D}_i(k)=\left[L_i(k)-L_{Fi}(k)D_i(k)\right]$$
$$\overline{E}_i(k)=\begin{bmatrix} E_i(k) & 0 \\ 0 & 0 \end{bmatrix}. \tag{6}$$

Note that (5) is a p-periodic discrete-time MJLS. In other words, $\overline{A}_{i1}(k)$, $\overline{A}_{i2}(k)$, $\overline{B}_i(k)$, $\overline{C}_{i1}(k)$, $\overline{C}_{i2}(k)$, $\overline{D}_i(k)$, $\overline{E}_i(k)$ are p-periodic matrix sequences. It is clear that $E\{\tilde{e}(k)\}=0$ and $E\{\tilde{e}(k)\tilde{e}(k)\}=\overline{e}(1-\overline{e})$.

Assumption 1 (*Liu et al. [25]*). Each function $f_i(x(k))$ in (1) is continuous and bounded, and there exist constants δ_i and ρ_i such