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New passivity criteria for uncertain neural networks with time-varying delay ☆

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ABSTRACT

This paper deals with the problem of passivity analysis for neural network with both time-varying delays and norm-bounded parameter uncertainties. A remarkable approach is proposed for constructing a novel Lyapunov–Krasovskii function involving triple integral terms. It does not requiring all the symmetric matrices to be positive definite. Due to the triple-integral terms and relaxation on the positive-definiteness of every Lyapunov-matrix, the conservatism of the results can be successfully reduced. Finally, numerical examples are given to demonstrate the effectiveness of proposed techniques.

1. Introduction

In the past few decades, neural networks (NNS) have been paid considerable attention due to its extensive applications in lots of different areas, such as pattern recognition, signal processing, model identification and optimization problem. As is known that time delay frequently occurs because of the finite switching speed of amplifiers. However, time delay may cause the oscillation and the instability of the system. Stability analysis is thus an important issue in the field of delayed neural networks [1–18]. Recently, many research results have been proposed to deal with the stability problems and the performance of the neural networks with time delay have been improved [19–28].

In real word application, parameter variations, modeling errors and process uncertainties always exist. Particularly, in the case of neural network systems the weight coefficients of neurons often appear uncertainties which are unavoidable and usually time varying. Thus, the stability of uncertain systems has been wide studied. In [34], author researched uncertain neural networks with mixed delays by using an LMI approach and further research results have been developed in [14].

Passivity, as a powerful tool, relates the inputs and outputs to the storage function. Passive properties of systems can keep the systems internally stable. The concept of passivity has been researched in the analysis of stability of dynamical system, nonlinear control and fuzzy control. In a number of practical applications, the problem of passivity analysis for neural network with time-varying delays has been extensively investigated in various fields of science and engineering. In [35], the sufficient conditions for passivity have been established for neural networks. Considering neural networks with time delays, passivity conditions have been presented in [23,24,36–39]. In [23,38], the derived passivity condition is delay-dependent, which are less conservative than the delay-independent results in [35]. In order to further cope with the passivity analysis, passivity condition of neural networks with discrete time has been obtained in [40,41]. In [42–44], passivity of neural networks with discrete and distributed delays has been analyzed. Stability and passivity analyses for different neural networks with Markovian jump parameters and time-varying delays have been investigated in [45,46]. Comparing with [33], an augmented Lyapunov–Krasovskii function with tripe-integral terms is introduced in this research work to improve passivity conditions and relaxed conditions on the activation function. In addition, to bound the integral terms in derivative of Lyapunov–Krasovskii function, Jensen's inequality-based integral inequalities and the free matrix approach are commonly used in [29–33] for stability analysis. In this paper, in order to further enhance the researched results, a new integral inequality based on

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reciprocally convex inequality is applied to present less conservative than other integral inequalities derived in terms of Jensen's inequality.

Referring to the mentioned discussion above, the research work is motivated to deal with the problem of passivity analysis for neural network with both time-varying delay and norm-bounded parameter uncertainties, the relaxed passivity criteria in terms of LMIs are presented. The advantage of this paper lies in that the symmetric matrices involved in the Lyapunov–Krasovskii function are not required to be all positive definite. A new activation functional condition $l_i^- \le \frac{g_i(a)-g_i(b)}{a-b} \le l_i^+$, it is assumed that l_i^- and l_i^+ are constants which are not limited to be positive or negative. Comparing with [33], a novel Lyapunov–Krasovskii function with tripe-integral terms is constructed. In this paper, in addition, a new integral inequality is applied in terms of reciprocally convex inequality for the purpose of conservatism reduction. Less conservative stability criteria are evaluated by using linear matrix inequalities. Finally, three numerical examples are illustrated the effectiveness of the proposed methods.

Notation: Throughout this paper, $\Re^n$ denotes the n-dimensional Euclidean space and $\Re^{n\times n}$ is the set of all $n\times n$ real matrices. For symmetric matrices X, the notation $X>0(X\ge 0)$ means that is a real symmetric positive definite matrix (positive semi-definite). For symmetric matrices X and Y, the notation $X>Y(X\ge Y)$ means that the matrix $X-Y$ is positive definite (nonnegative), $sym(A)$ denotes $A+A^T$, $*$ denotes the elements below the main diagonal of a symmetric block matrix, I denotes the identity matrix with appropriate dimensions. $col(x_1,x_2,\ldots,x_n)$ means $[x_1^T,x_2^T,\ldots,x_n^T]^T$.

2. Preliminaries

Consider the following neural networks with time-varying delay:

$$\dot{x}(t) = -(A+\Delta A(t))x(t)+(W+\Delta W(t))g(x(t))+(W_1 + \Delta W_1(t))g(x(t-\tau(t)))+u(t), \tag{1}$$

$$y(t)=g(x(t)), \quad t\ge 0 \tag{2}$$

$$x(t)=\phi(t), \quad t\in[-\tau_2,0] \tag{3}$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T\in\Re^n$ is the neuron state vector, $A=diag(a_1,a_2,\ldots,a_n)$, and $a_i>0$ $(i=1,2,\ldots,n)$. W, W_1 are interconnection weight matrices. $\tau(t)$ are the time-varying delays, $\phi(t)\in\Re^n$ is a vector-valued initial condition function, respectively. $\Delta A(t)$, $\Delta W(t)$, $\Delta W_1(t)$ represent the time-varying parametric uncertainties. $g(x(\cdot))=[g_1(x_1(\cdot)),g_2(x_2(\cdot)),\ldots,g_n(x_n(\cdot))]^T\in\Re^n$ denotes the neuron activation function, $y(t)=g(x(t))$ are the output of neural networks. $u(t)=[u_1(t),u_2(t),\ldots,u_n(t)]^T\in\Re^n$ stands for the external inputs. Throughout this paper, we shall use the following assumption.

Assumption 2.1. The time delay $\tau(t)$ is a time-varying function in (1) that satisfies $\tau_1\le\tau(t)\le\tau_2$, $\dot{\tau}(t)\le\tau_D$, where $\tau_1\ge 0$, $\tau_2>0$, $\tau_D\ge 0$ are constants and let $\tau_{12}=\tau_1-\tau_2$.

Assumption 2.2. Each activation function $g_i(\cdot)$ in system (1) is continuous and bounded, which satisfies the following inequalities:

$$l_i^- \le \frac{g_i(a)-g_i(b)}{a-b}\le l_i^+ \quad \text{and} \quad g_i(0)=0, \tag{4}$$

where a, $b\in\Re$, l_i^- and l_i^+ are known constants.

Remark 2.1. l_i^- and l_i^+ $(i=1,2,\ldots,n)$ are constants, which can be positive, negative and zero in Assumption 2.2. Consequently, this type of activation function is clearly more general than both the usual sigmoid activation function and the piecewise liner function $g_i(u)=\frac{1}{2}(|u_{i+1}|-|u_i|)$, which is useful to get less conservative result.

Assumption 2.3. The real-valued matrices $\Delta A(t)$, $\Delta W(t)$, $\Delta W_1(t)$ represent the time-varying parameter uncertainties, and are assumed to be of the form

$$[\Delta A(t)\ \Delta W(t)\ \Delta W_1(t)] = HF(t)[E_a\ E_w\ E_{w_1}], \tag{5}$$

where H, E_a, E_w, E_{w_1} are known constant matrices of appropriate dimensions and $F(t)$ are unknown time-varying matrices with Lebesgue measurable elements bounded by $F^T(t)F(t)\le I$. In which I is the identity matrix of appropriate dimensions.

Remark 2.2. The uncertainties $\Delta A(t)$, $\Delta W(t)$, and $\Delta W_1(t)$ satisfy Eq. (5) and reflect the impreciseness of dynamical systems. This could lead to the complexity of systems and increase the difficulty of solving the problem.

We now introduce the following passivity definition.

Definition 2.1 (*Park et al. [20] and Fridman and Shaked [38]*). The system in (1)–(3) is said to be passive if there exists a scalar $\gamma>0$ such that the inequality

$$2\int_0^{t_f} y^T(\alpha)u(\alpha)\,d\alpha \ge \gamma\int_0^{t_f} u^T(\alpha)u(\alpha)\,d\alpha, \tag{6}$$

holds for all $t_f\ge 0$ and under the zero initial condition

The neural networks in (1)–(3) are asymptotically stable and the input and the output of system (1)–(3) satisfy the passivity inequality in (6).

The physical meaning of passive systems is that energy of a nonlinear system can only be increased through the supplement from external sources. In other word, a passive system cannot store more energy than it is supplied.

3. New passivity criteria

Lemma 3.1 (*Jensens inequality, Li and Liao [39]*). *For any constant matrix V, $W\in\Re^{n\times n}$ with $M>0$, scalars $b>a$, vector function $V:[a,b]\to\Re^m$ such that integrations in the following are well-defined, then*

$$(b-a)\int_a^b V^T(s)MV(s)\,ds \ge \left(\int_a^b V(s)\,ds\right)^T M\int_a^b V(s)\,ds, \tag{7}$$

$$\frac{\tau^2}{2}\int_{-\tau}^0\int_{t+\theta}^t W^T(s)MW(s)\,ds\,d\theta \ge \left(\int_{-\tau}^0\int_{t+\theta}^t W(s)\,ds\,d\theta\right)^T M \int_{-\tau}^0\int_{t+\theta}^t W(s)\,ds\,d\theta. \tag{8}$$