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Exponential lag synchronization for delayed memristive recurrent neural networks

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ABSTRACT

In this paper, by using the parallel-memristors connection corresponding to the capacitors and memristors synaptic connection in usual recurrent neural networks, general delayed memristive recurrent neural networks are proposed. Basing on nonsmooth analysis and control theory, several sufficient conditions concerning global exponential lag synchronization are obtained for the proposed system. In addition, the obtained results complement and extend earlier publications on memristive or conventional neural network dynamical systems with continuous or discontinuous right-hand side. Finally, numerical simulations illustrate the effectiveness of our results.

1. Introduction

In these days, dynamics analysis of memristive neural networks has attracted many researchers' attention and numerous studies with respect to memristor have been carried out, e.g., in 2010, Hu and Wang [1] study asymptotic stability of memristive neural networks by constructing proper Lyapunov functionals and using differential inclusion theory, and then, stability and synchronization control of memristive neural networks were widely investigated in [2–17].

As it is well known, stability and control in nonlinear science has been known for a rather long time, and its applications to diverse areas such as secure communications, energy system, and biological reactions, e.g., see [18–31]. Transmission delay is ubiquitous when signals are communicated among neurons, e.g., in the signal transmission system, the signal one gets on the receiver side at time $t+\overline{\tau}$ is the signal from the transmitter side at time t. So, it is reasonable to require one neural network to synchronize the other neural network at a constant time lag. Compared with complete synchronization, lag synchronization may be a more appropriate technique to clearly indicate the fragile nature of neurons systems. Therefore, lag synchronize between two chaotic neural networks may be an important problem of both theoretical and potentially practical application in secure communications. In the past few years, great efforts have been devoted to the study for lag synchronization of chaotic systems and neural networks with or without delays [32–37].

The memristive neural network is a class of state-dependent nonlinear systems from a systems-theoretic point of view [2–17]. Such system can reveal coexisting solutions, jumped, transient chaos of rich and complex nonlinear behaviors [8,10]. However, on the lag synchronization of memristive neural networks, few results are found in literatures. On the other hand, the memristive neural networks with discontinuous right-hand side, this problem brings challenges to investigate the exponential lag synchronization of such system.

Motivated by the above discussions, in this paper, exponential lag synchronization for delayed memristive recurrent neural networks is studied. The study of lag synchronization for delayed memristive recurrent neural networks is very tough and up to now, few results focus on the lag synchronization problem for delayed memristive recurrent neural networks, since an over-abundance of complex nonlinear behavior appears even in a simple memristive system. The main purpose of this paper, therefore, is to shorten up such a gap by making the attempt to deal with the lag synchronization of delayed memristive recurrent neural networks.

The organization of this paper is as follows. Section 2, some preliminaries are introduced. Some algebraic conditions concerning lag synchronization are derived in Section 3. In Section 4, numerical simulations are given to demonstrate the obtained results. Finally, this paper ends by a conclusion.

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2. Preliminaries

2.1. Model

In this paper, we discuss a class of delayed memristive neural networks described by the following equations:

$$\frac{dx_i(t)}{dt} = -d_i(x_i(t))x_i(t) + \sum_{j=1}^{n} a_{ij}(x_j(t))f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_j(t-\tau_j)) \times g_j(x_j(t-\tau_j)) + I_i, \quad t \ge 0, \quad i \in N = \{1, 2, \ldots, n\}. \tag{1}$$

where

$$d_i(x_i(t)) = \frac{1}{\mathbf{C}_i}\left[\sum_{j=1}^{n}(\mathbf{M}_{ij}+\mathbf{W}_{ij}) \times \delta_{ij} + \mathbf{Q}_i\right],$$

$$a_{ij}(x_j(t)) = \frac{\mathbf{M}_{ij}}{\mathbf{C}_i} \times \delta_{ij}, \quad b_{ij}(x_j(t-\tau_j)) = \frac{\mathbf{W}_{ij}}{\mathbf{C}_i} \times \delta_{ij}, \tag{2}$$

where $\delta_{ij} = 1$, if $i \ne j$ holds, otherwise, -1. $x_i(t)$ is the state of the i-th neuron at time t, $d_i(x_i(t))$ is the i-th neuron self-inhibitions at time t. $\mathbf{M}_{ij}$ and $\mathbf{W}_{ij}$ denote the memductances of memristors $\mathbf{R}_{ij}^*$ and $\mathbf{R}_{ij}^{**}$, respectively. $\mathbf{R}_{ij}^*$ represents the memristor between the neuron activation function $f_j(x_j(t))$ and $x_i(t)$, $\mathbf{R}_{ij}^{**}$ represents the memristor between the neuron activation function $g_j(x_j(t-\tau_j))$ and $x_i(t)$. And $a_{ij}(x_j(t))$, $b_{ij}(x_j(t-\tau_j))$ are memristors synaptic connection weights, denote the strengths of the jth unit on the ith unit at time t and time $t-\tau_j$, respectively. $\mathbf{R}_i^*$ represents the parallel-memristor corresponding to the capacitor $\mathbf{C}_i$, $\mathbf{Q}_i$ denote the memductance of memristor $\mathbf{R}_i^*$, and $f_j(\cdot)$, $g_j(\cdot)$ are the activation functions, I_i is the external input. $\tau_j$corresponds to the transmission delays and satisfies $0 \le \tau_j \le \tau$ ($\tau = \max_{j \in N}\{\tau_j\} > 0$), $i, j \in N$, $N = \{1, 2, \ldots, n\}$.

Basing on the analysis in [38], we know that memristor needs to exhibit only two sufficient distinct equilibrium states since digital computer applications requiring only two memory states, and a basic form of a memristor consists of a junction can be switched from a low- to a high-resistive state and vice versa. The switch opens at a threshold voltage. In this paper, as the special case, we let the threshold voltage is zero, then

$$d_i(x_i(t)) = \begin{cases} d_i^*, & x_i(t) \le 0, \\ d_i^{**}, & x_i(t) > 0, \end{cases}$$

$$a_{ij}(x_j(t)) = \begin{cases} a_{ij}^*, & x_j(t) \le 0, \\ a_{ij}^{**}, & x_j(t) > 0, \end{cases}$$

$$b_{ij}(x_j(t-\tau_j)) = \begin{cases} b_{ij}^*, & x_j(t-\tau_j) \le 0, \\ b_{ij}^{**}, & x_j(t-\tau_j) > 0, \end{cases} \tag{3}$$

in which $d_i^* > 0$, $d_i^{**} > 0$, a_{ij}^*, a_{ij}^{**}, $i, j \in N$, are all constant numbers. Obviously, the memristive recurrent neural networks (1) are state-dependent switched systems, which are the generalization of those for conventional neural networks.

Throughout this paper, we consider system (1) as the drive system and corresponding response system is as follows:

$$\frac{dy_i(t)}{dt} = -d_i(y_i(t))y_i(t) + \sum_{j=1}^{n} a_{ij}(y_j(t))f_j(y_j(t)) + \sum_{j=1}^{n} b_{ij}(y_j(t-\tau_j)) \times g_j(y_j(t-\tau_j)) + I_i + u_i(t), \quad t \ge 0, i \in N, \tag{4}$$

where

$$d_i(y_i(t)) = \begin{cases} d_i^*, & y_i(t) \le 0, \\ d_i^{**}, & y_i(t) > 0, \end{cases}$$

$$a_{ij}(y_j(t)) = \begin{cases} a_{ij}^*, & y_j(t) \le 0, \\ a_{ij}^{**}, & y_j(t) > 0, \end{cases}$$

$$b_{ij}(y_j(t-\tau_j)) = \begin{cases} b_{ij}^*, & y_j(t-\tau_j) \le 0, \\ b_{ij}^{**}, & y_j(t-\tau_j) > 0, \end{cases} \tag{5}$$

and $u_i(t)$ is a feedback controller which is defined by

$$u_i(t) = \sum_{j=1}^{n} \pi_{ij}(y_j(t) - x_j(t-\tau_0)), \tag{6}$$

where π_{ij} is a constant for all $i, j \in N$, which denote the control gain, and $\tau_0 \ge 0$ is the lag delay.

Remark 1. This paper considers a class of neural networks are different from the neural networks with discontinuous neuron activations (see [39]), and also are different from the neural networks with discontinuous right side which switching depend on time t. Therefore, the obtained results here complement and extend earlier publications on conventional neural network dynamical systems with continuous or discontinuous right-hand side.

Remark 2. Because of the memductances $\mathbf{M}_{ij}$, $\mathbf{W}_{ij}$, $\mathbf{Q}_i$ switch at the threshold voltage zero, therefore, from (2), we know that $d_i(x_i(t))$ will also change corresponding to memductances $\mathbf{M}_{ij}$, $\mathbf{W}_{ij}$, $\mathbf{Q}_i$, however, in the previous works [2,10–12,17], the authors considered $d_i(x_i(t)) = 1$ or $d_i(x_i(t)) = d_i > 0$, $i, j \in N$, in their models. So, the delayed memristive recurrent neural networks (1) in this paper are more reasonable and general than the models discussed in previous works [2,10–12,17].

2.2. Notations

In this paper, solutions of all systems considered in the following are intended in Filippov's sense [40]. For any $h = (h_1, h_2, \ldots, h_n)^T \in \mathbb{R}^n$, the norms are defined by $\|h\|_p = (\sum_{i=1}^n |h_i|^p)^{1/p}$, where $p \ge 1$ is a positive integer. We define $\|\phi\| = \sup_{-\tau \le t \le 0}[\sum_{i=1}^n |\phi_i(t)|^p]^{1/p}$, for $\forall \phi = (\phi_1(t), \phi_2(t), \ldots, \phi_n(t))^T \in \mathbb{C}([-\tau, 0], \mathbb{R}^n)$. Let $\underline{\xi}_i = \min\{\hat{\xi}_i, \check{\xi}_i\}$, $\overline{\xi}_i = \max\{\hat{\xi}_i, \check{\xi}_i\}$, $\mathrm{co}\{\hat{\xi}_i, \check{\xi}_i\} = \theta\underline{\xi}_i + (1-\theta)\overline{\xi}_i = [\underline{\xi}_i, \overline{\xi}_i]$, $0 \le \theta \le 1$. For a continuous function $k(t): \mathbb{R} \to \mathbb{R}$, $D^+k(t)$ is called the upper right dini derivative and defined as $D^+k(t) = \overline{\lim}_{h \to 0^+}(1/h)(k(t+h) - k(t))$. The initial conditions of systems (1) and (2) are assumed to be $x_i(s) = \varphi_i(s)$, $y_i(s) = \psi_i(s)$, respectively, $s \in [-\tau, 0]$, $\varphi_i(s), \psi_i(s) \in \mathbb{C}([-\tau, 0], \mathbb{R})$, $i \in N$.

Now, we do the following assumption for system (1):

(Δ_1) For $j \in N$, $\forall s_1, s_2 \in \mathbb{R}$, the neuron activation functions f_j, g_j are bounded, $f_j(0) = g_j(0) = 0$ and satisfy

$$\sigma_j^- \le \frac{f_j(s_1) - f_j(s_2)}{s_1 - s_2} \le \sigma_j^+, \quad \rho_j^- \le \frac{g_j(s_1) - g_j(s_2)}{s_1 - s_2} \le \rho_j^+, \tag{7}$$

where $s_1 \ne s_2$, and σ_j^-, σ_j^+, ρ_j^-, ρ_j^+ are constants.

2.3. Definitions, properties and lemmas

Through the theories of differential inclusions and set-valued maps [40–42], from (1) and (4), it follows that

$$\frac{dx_i(t)}{dt} \in -\mathrm{co}[d_i(x_i(t))]x_i(t) + \sum_{j=1}^{n} \mathrm{co}[a_{ij}(x_j(t))]f_j(x_j(t)) + \sum_{j=1}^{n} \mathrm{co}[b_{ij}(x_j(t-\tau_j))]g_j(x_j(t-\tau_j)) + I_i, \text{ for a.a. } t \ge 0, \quad i \in N, \tag{8}$$

and

$$\frac{dy_i(t)}{dt} \in -\mathrm{co}[d_i(y_i(t))]y_i(t) + \sum_{j=1}^{n} \mathrm{co}[a_{ij}(y_j(t))]f_j(y_j(t)) + \sum_{j=1}^{n} \mathrm{co}[b_{ij}(y_j(t-\tau_j))]g_j(y_j(t-\tau_j)) + I_i + u_i(t), \quad \text{for a.a. } t \ge 0, \quad i \in N, \tag{9}$$