Neurocomputing 154 (2015) 230–238

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Short Communication

# Observer-based consensus for nonlinear multi-agent systems with intermittent communication

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ARTICLE INFO

*Article history:*
Received 3 July 2014
Received in revised form
9 October 2014
Accepted 28 November 2014
Available online 11 December 2014

*Keywords:*
Multi-agent system
Nonlinear dynamical system
Consensus
Observer
Intermittent communication

ABSTRACT

The article studies a consensus control problem for second-order nonlinear multi-agent systems with a leader where the velocity of the active leader cannot be obtained in real time. Under the common assumption that the multiple agents can communicate with their neighbors only during a sequence of discontinuous time intervals, a new neighbour-based consensus control protocol based on observers is developed for each agent. With the help of multiple Lyapunov function technology, some sufficient conditions are derived such that the follower agents can track the leader and keep pace with the leader at the same time under fixed topology. Moreover, the similar results are given under the switching topology. Finally, numerical examples are presented for illustration.



## 1. Introduction

Over the past few years, cooperative control of multi-agent systems has attracted the attention of researchers from different disciplines. A multi-agent system usually consists of many interconnected agents cooperating to complete a certain task. Extensive application for multi-agent systems include mobile autonomous robots, unmanned aerial vehicle formation, surveillance, hazardous material handling and distributed sensor networks.

As an important fundamental problem in the distributed cooperative control of multi-agent systems, consensus problem for multi-agent systems has received considerable attention recently. The basic idea of a distributed consensus problem means that a local control law is designed for each agent based on the information of the agent itself and that of its neighbors such that all the agents asymptotically reach a certain common agreement. Consensus has applications in formation control [1–4], networks [5,6], flocking [7–11] and so on. Many interesting results of consensus issue for multi-agent systems have been investigated from many views and a great deal of results has been obtained, for example, see [12–19] and reference therein. Among the studies of consensus problem for multi-agent systems, leader-follower consensus problem is one of the important research topics [20–22]. Even in leaderless situation, leaders which specify an objective for all agents are usually introduced in multi-agent systems [23,24].

Generally, the agents in multi-agent systems usually have second-order dynamics. Second-order consensus problems for multi-agent systems aim to guide all the agents to move with leaders' velocity and converge to the same position. Many works, on consensus problem for multi-agent systems, focus on agents described by linear second-order integrator dynamics. However, in reality, the agents may be governed by inherent nonlinear dynamics and hence the second-order consensus problem with nonlinear dynamics is more challenging.

In practical applications, the active leader may be a moving reference signal or a target that is not completely known, thus distributed control protocols based on distributed observers are usually designed for leader-following multi-agent systems. For the agent dynamics are either first-order or second order, [25,26] studied the distributed observer-based consensus for leader-following systems with an unknown velocity of a variable leader. In [27], based on the gain vector associated with agent's estimated state and the relative estimated states between agents, a distributed protocol is proposed such that the consensus of high-order continuous and discrete-time integral multi-agent systems is obtained. In [28], due to that the state of the agent is not measurable, a linear observer is designed to estimate the agent's state using its own output information. Based on the observer, a modified consensus protocol with the group decision value

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http://dx.doi.org/10.1016/j.neucom.2014.11.069
0925-2312/

associated with initial states and initial estimated states of all agents is designed and the modified consensus problem for linear multi-agent systems can be solved. Moreover, taking into account the constraints on controller is needed in physical and engineering systems. In [29], the semi-global leader-following consensus of a linear multi-agent system is investigated. Considering the input saturation, low gain feedback-based distributed consensus protocol is designed such that the consensus for multi-agent systems can be achieved. Compared with the low gain state feedback consensus of linear multi-agent systems in [29], based on the designed observer, a novel output feedback consensus algorithm [30] is proposed which is sure that the semi-global consensus of the multi-agent system can be reached by a new Lyapunov function. In addition, in consideration of that it is hard to apply control actions to all agents in a massive multi-agent system, the pinning control [31–33] has been introduced which refers to that only a small percentage of follower agents have access to the leader node.

It should be noticed that most of the works mentioned above studied the multi-agent systems with the presuppositions that the information shared among agents is transmitted continuously, whereas, this may not be realistic due to the limitations of sensing abilities, external disturbances or the failure of devices. In many cases, the agents can obtain the information from their neighbors intermittently. To deal with this situation, motivated by the work [35], consensus control for nonlinear multi-agent systems with intermittent protocols based on observers is studied in this article. Different from the work [35], we design distributed intermittent protocols based on observers for consensus problems for second-order nonlinear multi-agent systems.

In this article, we provide some new approaches to investigate the consensus control for nonlinear multi-agent systems. The main contributions can be listed as follows:

(1) The consensus problem for second-order nonlinear multi-agent systems is investigated with an active leader. In some practical leader-follower multi-agent systems, the velocity of the leader may not be obtained by the follower agents, and hence an observer is designed to estimate the leader's velocity by the follower in this article.
(2) Due to that the communication of the information between the agents and their neighbors may not be continuous, a distributed intermittent consensus control protocol is designed for nonlinear multi-agent systems under fixed and switching topologies. Compared with the work [35], this work extends the existing results [35] on state-based feedback intermittent control protocol to the case of distributed intermittent protocol through state observer.

The remainder of the article is organized as follows. In Section 1, some preliminaries are briefly outlined. In Section 2, by designing a distributed intermittent control law based on observers, some sufficient conditions are given such that the agents achieved the consensus with the leader. Some simulation examples are presented to illustrate the theoretical results in Section 3. Finally, some conclusions are given.

## 2. Preliminaries and problem formulation

### 2.1. Notation

In this article, $I_n \in R^{n\times n}$ is the $n$-dimensional identity matrix. $\|\cdot\|$ denotes the Euclidean norm. For a symmetrical matrix $A \in R^{n\times n}$, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ represent the maximal and minimum real eigenvalues of matrix $A$, respectively. The symbol $\otimes$ denotes the Kronecker product.

### 2.2. Preliminary knowledge

In this section, some important knowledge about algebraic graph theory and some lemmas are reviewed.

Suppose that a multi-agent system consists of a leader and $n$ agents with the interconnection topology graph $\overline{G}$. Graph $\overline{G}$ includes $n$ followers related to graph $G$ and a leader. The undirected graph $G(V,\varepsilon,A)$ consists of a set of vertices $V=\{v_1,v_2,\cdots,v_n\}$, a set of edges $\varepsilon \subseteq V\times V$ and an adjacency matrix $A=[a_{ij}]\in R^{n\times n}$. An edge of the graph $G$ is denoted by an unordered pair of vertices $e_{ij}=(v_i,v_j)\in\varepsilon$, which indicates that agent $i$ can received information from agent $j$ and vice versa.

The set of neighbours of agent $i$ is denoted by $N_i=\{v_j\in V: e_{ij}\in\varepsilon\}$. We assume that the adjacency element $a_{ij}>0$ if $e_{ij}\in\varepsilon$, otherwise $a_{ij}=0$ while $e_{ij}\notin\varepsilon$, and $a_{ii}=0$ for all $i=1,2,\cdots n$.

The Laplacian matrix $L=[l_{ij}]\in R^{n\times n}$ of $G$ is defined as $l_{ii}=\sum_{j\neq i}a_{ij}, l_{ij}=-a_{ij}$ for $i\neq j$. Denote a diagonal matrix $B=\operatorname{diag}\{b_1,\cdots b_n\}$ be a leader adjacency matrix related to $\overline{G}$, where $b_i>0$ if the follower $i$ connected to the leader, otherwise $b_i=0$.

A maximal induced subgraph $G_1$ of undirected graph $G$ that is connected, is named as a component of $G$. Graph $\overline{G}$ is connected if at least one agent in each component of $G$ is linked to the leader by a directed edge. Moreover, when the adjacent graph is variable, we define a switching signal $\sigma(t):[0,\infty)\to P=\{1,2,\cdots,N\}$, $p=\sigma(t),\quad p\in P$ which is a piecewise constant right continuous function. Assume that the switching moments $0<t_1<t_2<\cdots<t_k<\cdots$ satisfy $\inf\{t_{k+1}-t_k\}=h, h>0, k=1,2,\cdots$, In view of the adjacent topology is time-varying, the neighbour set of agent $i, N_i(t)$, the adjacent elements $a_{ij}, i,j=1,2,\cdots,n$, Laplacian $L_p(p\in P)$ and $B_p(p\in P)$ associated with the switching topology graph $\overline{G}_p(p\in P)$ are time-varying.

Based on the above presentation, the following lemmas borrowed from [25,26] will be used in next part.

**Lemma 2.2.1.** [26] *Consider a symmetric matrix*

$$D=\begin{bmatrix} A & E \\ E^T & C \end{bmatrix},$$

where $A$ and $C$ are square. Then $D$ is positive definite if and only if both $A$ and $C-E^TA^{-1}E$ are positive definite.

**Lemma 2.2.2.** [25] *If the entire graph $\overline{G}_p, p\in P$ is connected, then the symmetric matrix $H_p=L_p+B_p, p\in P$ associated with $\overline{G}_p$ is positive definite.*

### 2.3. Model formulation

Consider a second-order nonlinear multi-agent system consisting of $n$ followers and one leader.

Assume that the state of leader is changing throughout the entire process with dynamics expressed as

$$\begin{cases} \dot{x}_0 = v_0, \\ \dot{v}_0 = f(t,x_0), \end{cases} \tag{1}$$

where $x_0, v_0\in R^m$ are the position and velocity states of the leader, respectively. $f(t,x_0)\in R^m$ is a continuously differentiable function, which describes the time-varying nonlinear intrinsic dynamics of the leader, and $f(t,0)=0$. The leader system (1) can be seen as a virtual leader or a real agent leader. The leader is usually independent of their followers, but has influence on the followers' behaviors. In this article, we always assume that the position information $x_0$ of the leader can be obtained by all the follower agents.