Neurocomputing 154 (2015) 239–244

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Asymptotic stability of delayed fractional-order neural networks with impulsive effects ☆

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ARTICLE INFO

Article history:
Received 5 June 2014
Received in revised form
8 October 2014
Accepted 29 November 2014
Communicated by T. Heskes
Available online 7 December 2014

Keywords:
Fractional-order
Neural networks
Asymptotic stability
Impulsive
Delay
Riemann–Liouville

ABSTRACT

This paper has investigated the existence, uniqueness and the global asymptotic stability of equilibrium point for delayed fractional-order neural networks with impulsive effects. A lemma has been given based on Riemann–Liouville operator, which plays an important role in the stability analysis. Some sufficient conditions are derived to ensure the existence, uniqueness and the asymptotic stability of the fractional-order neural networks. An illustrative example is given to show the effectiveness of the obtained results by using a new numerical method of fractional-order differential equations.

1. Introduction

Fractional-order calculus has been attracting attention in electrochemistry [1], diffusion [2], viscoelastic materials [3], control [4,5], biological systems [6–8] and so on. As a kind of important biological networks, fractional neural networks have been studied by lots of researchers [9–14]. Due to the finite speed of the signal transmission between neurons, time-delay often exists in almost every neural networks. At the same time, time-delay could also affect the dynamic behavior of neural networks. Thus, time-delay is unavoidable in the analysis of neural networks. In addition, many real-world systems often suddenly receive external disturbance, which makes systems change their trajectory from the original in a very short time. This phenomenon is called impulse. It is clear that such a short time disturbance must have some effects on dynamics of neural networks. Therefore, it is necessary to consider both impulsive effects and delay effects in the dynamic analysis of neural networks.

As an important dynamic behavior, stability is always a prerequisite for the normal work of system. Stability of impulsive delayed integer order neural networks has been widely investigated. In [15–18], stability of impulsive integer order neural networks with delay has been analyzed. In [19–22], the exponential synchronization and the lag synchronization of impulsive delayed integer order neural networks have been discussed. As an extension of integer order neural networks, stability analysis of fractional order neural networks is getting more attention. In [9], the projective synchronization of the fractional order neural networks has been studied. In [10–12], chaotic phenomena of the fractional order neural networks has been investigated. But the results above have considered neither delay nor impulse. In [13,14], the uniform stability and finite time stability of delayed fractional order neural networks have been investigated, respectively, which have not considered impulse. In [23,24], stability and synchronization of impulsive fractional neural networks have been discussed, respectively, but without considering delay. As far as we know, there are few papers on studying the stabilities of the fractional order neural networks with both impulse and delay.

Motivated by the above, this paper will investigate the asymptotic stability of impulsive delayed fractional order neural networks. The well-known Barbalat's lemma [25] has been widely used in dynamic analysis of integer order systems [26–28]. In this paper, a fractional Barbalat's lemma with Riemann–Liouville derivatives will be deriving. Some conditions are presented to guarantee the existence and the asymptotic stability of the equilibrium point of impulsive delayed fractional order neural networks. In addition, for most fractional differential equations, the analytical solutions are difficult to be obtained. Therefore, it is necessary to study some numerical methods of fractional differential equations. In recent

☆This work was jointly supported by the National Natural Science Foundation of China under Grant 11226116, the Fundamental Research Funds for the Central Universities JUSRP51317B.

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http://dx.doi.org/10.1016/j.neucom.2014.11.068
0925-2312/

years, many algorithms about numerical solutions of fractional differential equations have been proposed [29–31]. However, most of them are hard to understand and the form of formula is very complicated. In this paper, we introduced a simple method based on Abel integral equation [32], which has a concise results and a higher accuracy compared with traditional method.

This paper is organized as follows: in Section 2, we introduce some definitions and some lemmas which are needed in later sections. In Section 3, we will give our main results. A new numerical method of fractional differential equations has been derived and an illustrative example will be given in Section 4.

2. Preliminaries and model description

2.1. Fractional calculus

A fractional-order derivative can be considered as a generalization of an integer-order derivative. In this section, some definitions about fractional calculus are given.

Definition 1 (*Li and Deng [33], Petras [34], Diethelm [35]*). The fractional integral of order α for a function $w(t)$ is defined as

$$D_{t_0}^{-\alpha}w(t)=\frac{1}{\Gamma(\alpha)}\int_{t_0}^{t}(t-\tau)^{\alpha-1}w(\tau)\,d\tau \tag{1}$$

where $t\geq t_0$ and $\alpha>0$.

Riemann–Liouville fractional operator often plays an important role in the stability analysis of fractional-order neural networks. The formula of it is defined as follows.

Definition 2 (*Li and Deng [33], Petras [34], Diethelm [35]*). The Riemann–Liouville derivative of fractional order α of function $x(t)$ is defined as

$${}^{RL}D_{t_0,t}^{\alpha}x(t)=\frac{d^m}{dt^m}[D_{t_0,t}^{-(m-\alpha)}x(t)]=\frac{1}{\Gamma(m-\alpha)}\frac{d^m}{dt^m}\int_{t_0}^{t}(t-\tau)^{m-\alpha-1}x(\tau)\,d\tau \tag{2}$$

in which $m-1<\alpha<m$, $m\in Z^+$.

Remark 1. Without loss of generality, the order α of Riemann–Liouville derivative in Definition 2 is given as $0<\alpha<1$. For simply, denote $D^{\alpha}x(t)$ as the Riemann–Liouville derivative ${}^{RL}D_{t_0,t}^{\alpha}x(t)$.

2.2. Some properties of Riemann–Liouville derivative

Some properties of mentioned operators are listed in this section. In addition, a fractional Barbalat's lemma with Riemann–Liouville derivatives will be introduced.

Lemma 1 (*Li and Deng [33]*). *If $w(t),u(t)\in C^1[t_0,b]$, and $\alpha>0$, $\beta>0$, then*

(1) $D^{\alpha}D^{-\beta}w(t)=D^{\alpha-\beta}w(t)$
(2) $D^{-\alpha}D^{\alpha}w(t)=w(t)$
(3) $D^{\alpha}(w(t)\pm u(t))=D^{\alpha}w(t)\pm D^{\alpha}u(t)$.

Lemma 2 (*Barbalat's lemma, Slotine and Li [25]*). *If the differentiable function $w(t)$ has a finite limit as $t\to+\infty$, and if $\dot{w}$ is uniformly continuous, then $\dot{w}\to 0$ as $t\to+\infty$.*

Remark 2. The Barbalat's lemma has been popular due to its usefulness in the stability analysis of time-varying delayed dynamical systems with integer order. It is hard to use the Barbalat's lemma in delayed fractional systems due to some different properties between fractional order derivatives and integer order derivative. Lemma 3 could be an extension of traditional Barbalat's lemma. Maybe it will be an important tool in the analysis of fractional systems.

Lemma 3. *If $\int_{t_0}^{t}w(s)\,ds$ has a finite limit as $t\to+\infty$, $D^{\alpha}w(t)$ is bounded, then $w(t)\to 0$ as $t\to+\infty$, where $0<\alpha<1$.*

Proof. Because $D^{\alpha}w(t)$ is bounded, there exists a constant $M>0$ such that $|D^{\alpha}w(t)|\leq M$. For $t_0\leq T_1<T_2$, we have

$$\begin{aligned}
|w(T_1)-w(T_2)| &= D^{-\alpha}[D^{\alpha}w(T_1)]-D^{-\alpha}[D^{\alpha}w(T_2)]\\
&=\frac{1}{\Gamma(\alpha)}\left|\int_{t_0}^{T_1}(T_1-s)^{\alpha-1}D^{\alpha}w(s)\,ds-\int_{t_0}^{T_2}(T_2-s)^{\alpha-1}D^{\alpha}w(s)\,ds\right|\\
&=\frac{1}{\Gamma(\alpha)}\left|\int_{t_0}^{T_1}[(T_1-s)^{\alpha-1}-(T_2-s)^{\alpha-1}]D^{\alpha}w(s)\,ds-\int_{T_1}^{T_2}(T_2-s)^{\alpha-1}D^{\alpha}w(s)\,ds\right|\\
&\leq\frac{1}{\Gamma(\alpha)}\left[\int_{t_0}^{T_1}|[(T_1-s)^{\alpha-1}-(T_2-s)^{\alpha-1}]D^{\alpha}w(s)|\,ds+\int_{T_1}^{T_2}|(T_2-s)^{\alpha-1}D^{\alpha}w(s)|\,ds\right]\\
&\leq\frac{M}{\Gamma(\alpha)}\left[\int_{t_0}^{T_1}[(T_1-s)^{\alpha-1}-(T_2-s)^{\alpha-1}]\,ds+\int_{T_1}^{T_2}(T_2-s)^{\alpha-1}\,ds\right]\\
&\leq\frac{M}{\Gamma(\alpha+1)}[(T_1-t_0)^{\alpha}-(T_2-t_0)^{\alpha}+2(T_2-T_1)^{\alpha}]\\
&\leq 2\frac{M}{\Gamma(\alpha+1)}(T_2-T_1)^{\alpha}<\varepsilon
\end{aligned}$$

where $|T_2-T_1|<\delta(\varepsilon)=((\varepsilon\Gamma(\alpha+1))/2M)^{1/\alpha}$. Note that $\delta(\varepsilon)$ is independent of T_1 and T_2, which implies that $w(t)$ is uniformly continuous. From Barbalat's lemma, we can get the conclusion. □

Lemma 4 (*Zemyan [32]*). *For $(0<\alpha<1)$, if $\varphi(t)$ satisfies the following Abel integral equation:*

$$\int_{t_0}^{x}\frac{\varphi(t)}{(x-t)^{\alpha}}\,dt=f(x),$$

then

$$\varphi(u)=\frac{\sin\alpha\pi}{\pi}\left[\frac{f(t_0)}{u^{1-\alpha}}+\int_{t_0}^{u}\frac{f'(x)}{(u-x)^{1-\alpha}}\,dx\right].$$

2.3. Problem formulation

Considering the following delayed fractional-order neural network with impulsive effects:

$$\begin{cases}D^{\alpha}x_i(t)=-c_ix_i(t)+\sum_{j=1}^{n}a_{ij}f_j(x_j(t))+\sum_{j=1}^{n}b_{ij}g_j(x_j(t-\tau))+I_i, & t\neq t_k\\ \Delta x_i(t_k)=x_i(t_k^+)-x_i(t_k^-)=-J_{ik}(x_i(t_k)), & t=t_k,\quad k=1,2,3\ldots\end{cases} \tag{3}$$

where $0<\alpha<1$, $c_i>0(i=1,2,\ldots,n)$ are constant, n denotes the number of units in the neural network; $x_i(t)$ denotes the state of the ith unit at time t, $f_j(*),g_j(*)$ denote the activation function of the jth neuron; a_{ij} denotes the constant connection weight of the jth neuron on the ith neuron and I_i denotes an external input of ith neuron. Impulsive moment $\{t_k|k=1,2,3,\ldots\}$ satisfies $0\leq t_0<t_1<t_2<\cdots<t_k<\cdots$, $t_k\to+\infty$ as $k\to+\infty$, and $x(t_k^+)=\lim_{t\to t_k^+}x(t)$ and $x(t_k^-)=x(t_k)$.

Definition 3. A constant vector $x^*=(x_1^*,x_2^*,\ldots,x_n^*)^T$ is an equilibrium point of system (3) if and only if x^* is a solution of the following equations:

$$-c_ix_i+\sum_{j=1}^{n}a_{ij}f_j(x_j)+\sum_{j=1}^{n}b_{ij}g_j(x_j)+I_i=0, \tag{4}$$

and the impulsive jumps $J_{ik}(\cdot)$ are assumed to satisfy $J_{ik}(x_i^*)=0$, $k=1,2,\ldots,i=1,2,\ldots,n$.