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Multiple graph regularized sparse coding and multiple hypergraph regularized sparse coding for image representation

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ABSTRACT

Manifold regularized sparse coding shows promising performance for various applications. The key issue that must be considered in the application is how to adaptively select the suitable graph hyper-parameters in manifold learning for the sparse coding task. Usually, cross validation is applied, but it does not necessarily scale up and easily leads to overfitting. In this article, multiple graph sparse coding (MGrSc) and multiple Hypergraph sparse coding (MHGrSc) for image representation are proposed. Inspired by the Ensemble Manifold Regularizer, we formulate multiple graph and multiple Hypergraph regularizers to guarantee the smoothness of sparse codes along the geodesics of a data manifold, which is characterized by fusing the multiple previously given graph Laplacians or Hypergraph Laplacians. Then, the proposed regularziers, respectively, are incorporated into the traditional sparse coding framework, which results in two unified objective functions of sparse coding. Alternating optimization is used to optimize the objective functions, and two, novel manifold regularized sparse coding algorithms are presented. The proposed two sparse coding methods learn both the composite manifold and the sparse coding jointly, and it is fully automatic for learning the graph hyper-parameters in the manifold learning. Image clustering tests on real world datasets demonstrated that the proposed sparse coding methods are superior to the state-of-the-art methods.

1. Introduction

Image representation plays an essential role in the image processing related field. Researchers have long strived to seek effective sparse and parts-based representation schemes, such as Low Rank Matrix Factorization [1–3], Markov Random Field [4,5] and Sparse Coding [6]. Among them, sparse coding has become more and more popular in real applications due to the following two reasons: (1) In theory, it is consistent with the mechanism humans use to recognize objects and (2) It achieves excellent performance in many applications. Given a set of data feature vectors, organized as an input data matrix, sparse coding aims to find a basis vector pool (dictionary) selecting as few basis vectors as possible (Each basis vector is called the atom of the dictionary.) from the dictionary to linearly reconstruct the data feature vectors while keeping the reconstruction error as small as possible. Different from Low Rank Matrix Factorization, the dictionary in sparse coding is usually overcomplete, that is, the atom number is much larger than feature dimensions; thus, guaranteeing the sparsity of the reconstruction coefficients and leading to sparse and parts-based representation of the data. Various sparse coding methods have been presented [7] and applied to image restoration [8], image super-resolution [9–11], image classification [12,13], compressed sensing [14], face recognition [15,16], texture classification [17,18], action recognition [19,20], object tracking [21,22], etc. Due to the fact that sparse coding can reduce feature dimensions to improve the computation efficiency for sparse feature learning, it has shown promising performances in image classification [12,13], action recognition [19] and other related applications.

Manifold regularized sparse coding methods [23–26] have been proposed where the geometrical structure of the data distribution is exploited and local invariance is considered. These methods are suitable specifically for the data which is drawn from sampling the probability distribution that has support on or near to a manifold of the ambient space. Manifold learning [28–31] aims to discover the underlying geometrical structure of the data distribution. Existing manifold regularized sparse coding methods usually use graph Laplacian [23] or Hypergraph Laplacian [23,24] as the smooth operator to preserve the locality of data space, thus achieving better performance over the original sparse coding. However, the existing

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manifold regularized sparse coding methods have a key issue that must be considered, that is, how to adaptively select the graph hyper-parameter in manifold learning for the sparse coding task. Selecting hyper-parameters is tedious and difficult; no explicit rules can be exploited. We usually apply cross verification to make suitable selections. However, the parameter space is so large that it is impossible to obtain the optimal parameter by hand. In addition, cross verification easily overfits the training and validation sets and does not scale up for the huge amount of possible parameter selections. As a result, automatic manifold estimation is essential to develop the effective manifold regularized sparse coding method.

In this article, multiple graph and multiple Hypergraph regularization are proposed to measure the smoothness of sparse codes along the geodesics of the data manifold. Furthermore, Multiple Graph and Multiple Hypergraph regularizations, respectively, are incorporated into the traditional sparse coding framework, which results in two manifold regularized sparse coding methods where the optimal intrinsic manifold and sparse coding are jointly learned by alternating optimization. The proposed two sparse coding methods enhance the learning performance of the existing manifold regularized sparse coding and avoid the procedure of tuning the graph hyper-parameters.

The contributions of this article are summarized as follows:

(1) Two novel, manifold regularized sparse coding methods (MGrSc and MHGrSc) are presented for image representation. To more effectively explore the intrinsic geometrical structure of the data distribution and allow the sparse codes to move smoothly along the data manifold, we propose the incorporation of multiple graph and multiple Hypergraph regularization, respectively, into the traditional sparse coding framework. The proposed regularizations more effectively preserve the locality of data space by approximating the intrinsic manifold and transforming the graph hyper-parameter selections into the problem of estimating the combination coefficients given composite manifolds.
(2) Our proposed methods are formulated as an alternating optimization problem where the sparse coding and the optimal intrinsic manifold are learned jointly. Two iterative based algorithms are presented to solve this optimization problem. The convergence curve of the algorithm is also provided.
(3) We conduct comprehensive experiments to empirically analyze and compare our methods with the state-of-the-art methods. The experimental results on real world image data sets demonstrate that the proposed algorithms are superior to the following existing methods: Principle Component Analysis, Normal Cut [56], traditional sparse coding [6], Graph regularized sparse coding [23], etc.

The remainder of this article is organized as follows. Section 2 describes the related work. In Section 3, we present the objective function of the proposed methods. Section 4 presents the proposed optimization strategy to optimize the objective function. Section 5 presents the experimental results on the real world datasets and, in particular, comparisons with the state-of-the-art methods. Finally, in Section 5, we present our conclusions.

2. Related work

2.1. Traditional sparse coding

Traditional sparse coding, which consists of two components – the optimization for sparse representation and the learning of overcomplete dictionary, learns the optimal dictionary and finds the corresponding sparse codes. Common optimization methods include matching pursuit [32], orthogonal matching pursuit [33], and basis pursuit [34]. Other less common methods developed to solve the optimization problem are the adaptive Lasso technique [35], reweighted ℓ_1 minimization [36], and multistage convex relaxation [37].

2.2. Learning of overcomplete dictionary

The dictionary is usually directly learned from training rather than by using a predetermined dictionary [38]. One of the more recent contributions is the sparse dictionary [39], which aims to merge the advantages of trained and analytic dictionaries. A category-specific and/or shared dictionary, has been proposed. Perronnin et al. [40] presented a class dictionary adapted from a universal dictionary based on the GMM model. Gao et al. [41] learn (1) for each category, category-specific dictionaries which encode subtle visual differences among the different categories, and (2) a shared dictionary for all the categories which encodes common visual patterns. In addition, another two types of dictionaries are presented: (1) multiscale dictionary [42] where semi-multiscale structure is obtained by arranging several fixed-sized learned dictionaries of different scales and (2) translation-invariance dictionary [43] constructed by collecting all the translations of the trained atoms.

2.3. Variants of traditional sparse coding

Traditional sparse coding is usually suitable to reconstruct data; however, it exhibits poor performance on classification tasks. Consequently, the following five variants of traditional sparse coding are presented:

(1) **Supervised sparse coding.** Label information of a training dataset is exploited to learn the dictionary; the corresponding sparse codes are used for classification tasks. Zhang et al. [44] incorporated labels directly into the sparse coding stage and proposed a discriminative K-SVD method to guarantee class separability.
(2) **Discriminative sparse coding**. The class separability criteria are further incorporated into traditional sparse coding, which leads to discriminative sparse coding methods. Yang et al. [45] introduced Fisher's discriminative criterion to the sparse coding objective function to ensure that sparse representations have large between-class scatter but small within-class scatter. Lian et al. [46] proposed a max-margin sparse coding method, which incorporates the hinge loss function into the sparse coding objective function.
(3) **Structured sparse coding.** Structured sparse coding methods exploit the structure sparsity, such as group sparsity [47]. Gao, Sun et al. [48,49] incorporate additional structured sparsity priors to the sparse coding objective functions, which leads to the promising performances on the applications.
(4) **Kernel sparse coding**. Kernel trick, introduced to a traditional sparse coding framework, leads to Kernel sparse coding [50], which is the method in a high dimensional feature space mapped by some implicit mapping functions.
(5) **Manifold regularized sparse coding.** More recently, manifold regularized sparse coding methods respect the underlying geometry of the data distribution and achieve promising learning performances. Existing sparse coding algorithms usually use graph Laplacian or Hypergraph Laplacian to model the local geometry of the data distribution, for example, Zheng et al. [23] incorporated a graph Laplacian regularizer into a traditional sparse coding objective function and presented graph regularized sparse coding(GrSC). Gao et al. [24] proposed Laplacian sparse coding(LapSc) and Hypergraph Laplacian sparse coding (HLapSc) to preserve the local consistence of data space. In order to exploit the available label to enhance the discriminative ability of sparse coding learning, Wang et al. [25] introduced class label