Neurocomputing 149 (2015) 1143–1148

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Classification of committees with vetoes and conditions for the stability of power indices ☆

Jacek Mercik [a,b,*]

[a] Wroclaw University of Technology, Wroclaw, Poland
[b] Wroclaw School of Banking, Poland

ARTICLE INFO

Article history:
Received 2 November 2013
Received in revised form
5 September 2014
Accepted 5 September 2014
Communicated by Dariusz Barbucha
Available online 16 October 2014

Keywords:
Voting
Veto
Power index
Axioms
Classification
Stability

ABSTRACT

Decision making by a committee may be modelled by simple games. Some of the committee's members are equipped with a veto, i.e. they may stop an action temporarily or permanently (by transforming a winning coalition into a losing coalition). A classification of such committees and of power indices is presented in this paper. Special emphasis is given to particular characteristics of winning coalitions and, in consequence, to *a priori* power indices and conditions for their stability from the perspective of both axioms and parameters.

1. Introduction

The analysis of vetoes in committee decision making has a wide spectrum of aspects. From a political science approach (for example: Tsebelis [14] and the state-of-the-art paper of Ganghof [8]), through game theoretical analysis (see for example [10,11] to practical evaluation of the influence of veto power on a decision with special emphasis placed on evaluating the power of a player (power indices). The last approach may be seen, for example, in [6,7,9].

This article has three related goals. First, we want to classify observed cases of veto power during decision making depending on the number of vetoers and on the type of veto, focusing especially on differences between types of vetoes. In doing this, we carry out an analysis of both conditional vetoes and unconditional vetoes. Second, we try to estimate the number of winning coalitions according to the types of veto available and the number of vetoers. Finally, we try to derive conditions for the stability of power indices with respect to changes in the number of winning coalitions and the particular set of axioms used.

Almost all *a priori* power indices are defined as the ratio between the number of particular winning coalitions (with restrictions following from the assumptions made) to the number of all possible coalitions (sometimes majority coalitions only). Therefore, knowing the number of winning coalitions allows us to evaluate a given power index and, what is probably more useful, lets us estimate the likelihood of adopting new acts. Consequently, in practice, the number of winning coalitions including a particular decision-maker will be a measure of the power of that decision-maker acting under various voting procedures. This will help us to choose an appropriate voting system. Such analytical evaluation has not been met in the literature up to now, except for NP-hard permutation algorithms and ad hoc analysis of a committee's regulations.

The article is set up as follows. The next section outlines the concepts of simple games with veto and power indices. Section 3 considers the concepts of conditional veto and unconditional veto and gives three practical examples. Section 4 presents sets of axioms for power indices (and we show that substantial differences may exist between power indices for committees with veto from those for committees without veto) and a classification of cases depending on a veto's type, on the number of vetoers and on whether yes–no or yes–no–abstain voting is used. Section 5 is devoted to the stability of power indices. Finally, there are some conclusions and suggestions for future research.

2. Preliminaries

Let N be a finite set of committee members, q be a quota and w_j be the voting weight of member j, where $j \in N$.

☆This paper was originally intended for the special issue on Computational Collective Intelligence, published in Volume 146.

* Correspondence address: Wroclaw School of Banking, Fabryczna 29-31, 53-609 Wroclaw, Poland.
E-mail address: jacek.mercik@wsb.wroclaw.pl

http://dx.doi.org/10.1016/j.neucom.2014.09.067
0925-2312/

In this paper, we consider a special class of cooperative games called weighted majority games. A weighted majority game G is defined by a quota q and a sequence of nonnegative numbers w_i, $i \in n$, where we may think of w_i as the number of votes, or weight, of player i and q as the threshold or quota needed for a coalition to win. We assume that q and w_j are nonnegative integers. A subset of players is called a coalition.

A game on N is given by a map v: $2^N \to R$ with $v(\varnothing)=0$. The space of all games on N is denoted by G. A coalition $T \in 2^N$ is called a carrier of v if $v(S)=v(S \cap T)$ for any $S \in 2^N$. The domain $SG \subset G$ of simple games on N consists of all $v \in G$ such that:

(i) $v(S) \in \{0,1\}$ for all $S \in 2^N$;
(ii) $v(N)=1$;
(iii) v is monotonic, i.e. if $S \subset T$ then $v(S) \le v(T)$.

A coalition S is said to be winning in $v \in SG$ if $v(S)=1$ and losing otherwise. Therefore, passing a bill, for example, is equivalent to forming a winning coalition consisting of voters. A simple game (N,v) is said to be proper, if and only if the following is satisfied: for all $T \subset N$, if $v(T)=1$ then $v(N \backslash T)=0$.

We analyse only simple and proper games where players may vote either yes–no or yes–no–abstain, respectively.

We shall denote a committee (weighted voting body[1]) with set of members N, quota q and weights w_j, $j \in N$ by $(N,q,w)[=(N,q,w_1,w_2,...,w_n)]$. We shall assume that the w_j are nonnegative integers. Let $t = \sum_{j=1}^{n} w_j$ be the total weight of the committee.

Let V denote the set of all committee members equipped with a veto. We assume that the cardinality of the set V, $V \subset N$, is equal to or greater than 1, i.e. $card(V)=c_v \ge 1$.

A power index is a mapping $\varphi : SG \to R^n$. For each $i \in N$ and $v \in SG$, the ith coordinate of $\varphi(v) \in R^n$, $\varphi(v)(i)$, is interpreted as the voting power of player i in the game v. In the literature, there are two dominant power indices: the Shapley–Shubik power index and the Banzhaf power index. Both are based on the concept of the Shapley value.

The Shapley–Shubik power index [13] for a simple game is the value $\varphi : SG \to R^n$, $v \to (\varphi_1^{SS}(v),\ \varphi_2^{SS}(v),\ \dots, \varphi_n^{SS}(v))$, where for all $i \in N$, $card\ \{N\}=n$; $card\{S\}=s$ and

$$\varphi_i^{SS}(v) = \sum_{S \subset N, i \notin S} \frac{s!(n-s-1)!}{n!} \tag{1}$$

The Banzhaf power index [2] for a simple game is the value: $\varphi : SG \to R^n$, $v \to (\varphi_1^B(v),\ \varphi_2^B(v),\ \dots, \varphi_n^B(v))$, where for all $i \in N$; $card\ \{N\}=n$; $card\{S\}=s$ and

$$\varphi_i^B(v) = \frac{1}{2^{n-1}} \sum_{S \subseteq N \backslash \{i\}} [v(S \cup \{i\}) - v(S)] \tag{2}$$

The above definitions of power indices are directly obtained from characteristic function games, based on the marginal increase in v when a given committee member joins a coalition. In the paper by Turnovec et al. [15], one can find a description of power indices that does not use game theory, but is based on the concept of permutations and their probability. Using either approach, the calculations involved are based on the set of winning coalitions. Hence, in the following chapter we will evaluate the number of such coalitions and its influence on the power index.

If a given committee member can transform any winning coalition into non-winning by using a veto, then that veto is said to be of first degree.

If the veto of a given committee member turns some, but not all, winning coalitions not including that member into non-winning coalitions, then that veto is defined to be of second degree [10]. This type of veto is illustrated by the examples in the following section.

[1] This comes directly from the definition of a weighted game.

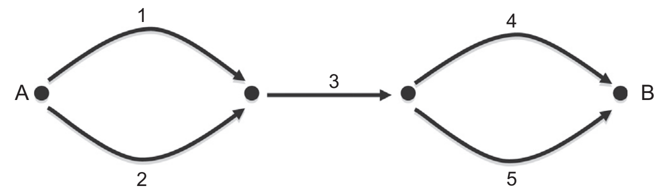

Fig. 1. Transfer of a signal between points A and B.

3. The concept of veto

The two types of veto considered can be illustrated by the following examples:

Example I: Suppose that we wish to send a signal from A to B using the connections numbered from 1 to 5. Each of these connections can be either functioning or non-functioning. In order to transmit the signal, there needs to be a sequence of functioning connections starting at A and finishing at B (Fig. 1). Hence, for a signal to be transmitted

i) connection 3 must be functional,
ii) at least one of connections 1 and 2 must be functional,
iii) at least one of connections 4 and 5 must be functional.

It follows that at least 3 of the five connections must be functional to transmit the signal, but not all sets of connections satisfying this condition lead to the transmission of the signal. The sets of functional connections which lead to the signal being transmitted are as follows:{1,2,3,4,5}, {1,2,3,4}, {1,2,3,5}, {1,3,4,5}, {2,3,4,5}, {1,3,4}, {1,3,5}, {2,3,4}, {2,3,5}.

Consider a voting game based on this scenario in which the connections represent voters, each voter has a single vote and the event “a vote is passed” corresponds to the signal being transferred in the situation described above. Let the quota be 3 and assume that the players have the following veto powers:

a) player 3 can successfully veto any coalition,
b) player 1 (or player 2) can veto the coalition {3,4,5} (but e.g. player 1 cannot veto the coalition {2,3,4}, neither can player 2 veto the coalition {1,3,4}),
c) player 4 (or player 5) can veto the coalition {1,2,3} (but e.g. player 4 cannot veto the coalition {2,3,5}, neither can player 5 veto the coalition {1,3,4}).

Hence, player 3 has a veto of first degree and the remaining players have a veto of second degree. It can be seen that if we assume that the players outside of a coalition use their veto (rather than abstaining), then the set of winning coalitions coincides exactly with the sets of functional connections which lead to the signal being transmitted in the scenario above.

Let P denote a coalition structure $P=\{P_1, P_2, \dots, P_m\}$ over N, i.e. a partition of N, that is $\cup_{k=1}^{m} P_k = N$ and $P_k \cap P_h = \varnothing$ when $k \ne h$. A coalition structure with power of veto $Pv=\{P_1, \dots, \{j\}, \dots, P_m\}$ over N for $j=1,\dots,\ m$ is a coalition structure P where at least one coalition consists of a singleton and at least one of the singletons has power of veto. The veto can be of first or second degree.

Example II: One possible partition of the Security Council of the UN (SC) is $P=\{\{P_1\},\{P_2\},\{P_3\},\{P_4\},\{P_5\},\{NP_6,\dots,NP_{15}\}\}$, where each permanent member P_i has power of veto and the rest of the SC's members create a coalition. Of course, in this example different combinations of partitions are also possible. The classical