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Global exponential stability in a Lagrange sense for memristive recurrent neural networks with time-varying delays

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ABSTRACT

In this paper, we consider the global exponential stability in a Lagrange sense for memristive recurrent neural networks with time-varying delays. Here, we adopt nonsmooth analysis and control theory to handle memristive neural networks with discontinuous right-hand side, and by constructing proper Lyapunov functionals and using inequality technique, several new sufficient conditions in linear matrix inequality form are given to ensure the ultimate boundedness and global exponential attractivity of the memristor-based neural networks in the sense of Filippov solutions. In addition, these conditions do not require the connection weight matrices to be symmetric and the delay functions to be differentiable. Finally, numerical simulations illustrate the effectiveness of our results.

1. Introduction

With the development of application as well as many technologies, memristive neural networks have proven as a promising architecture in neuromorphic systems for the non-volatility, high-density and unique memristive characteristic [1–4]. As we know, the neural networks are very important nonlinear circuit networks because of their wide applications in associative memory, pattern recognition, signal processing, systems control, optimization problem, and the integration and communication delays are unavoidably encountered both in biological and artificial neural systems, which may lead to poor performance such as oscillation, instability, and chaos. Hence, there are a large number of results on the stability in a Lyapunov sense for neural networks [5–11]. On the other hand, due to the finite speed of transmission and spreading in practical, time delays are unavoidable exist [10–14]. Therefore, dynamics analysis of memristive recurrent neural networks with time delays has been attracted increasing attention, and have appeared many results, e.g., by constructing proper Lyapunov functionals and using the differential inclusion theory, some sufficient conditions were obtained for global uniform asymptotic stability [15], exponential stability [16–18], exponential synchronization [19,20].

It is worth mentioning that Lyapunov stability in [5–11] refers to the stability of equilibrium points which requires the existence of equilibrium points, while Lagrange stability refers to the stability of the total system which does not require the information of equilibrium points. Moreover, the global stability in a Lyapunov sense can be viewed as a special case of stability in a Lagrange sense by regarding an equilibrium point as an attractive set [6,11]. So it is necessary and rewarding to study Lagrange stability. Basically, the goal of the study on global stability in a Lagrange sense is to determine global attractive sets. Once a global attractive set is found, a rough bound of periodic states and chaotic attractors can be estimated.

Lagrange stability has long been studied in theory and applications of engineering systems, e.g., Rekasius [21] considered asymptotic stability in a Lagrange sense for nonlinear feedback control systems, Passino and Burgess [22] adopted the concept of Lagrange stability to investigate discrete event systems, and Hassibi et al. [23] studied the Lagrange stability of hybrid engineering systems. Recently, a considerable number of works also have appeared to study the Lagrange stability for neural networks with time-delays these years (see [24–32]).

However, to our best knowledge, few authors have discussed the stability in a Lagrange sense of memristive recurrent neural networks. Motivated by the above analysis, in this paper, we will study the global exponential stability in a Lagrange sense and the existence of globally exponentially attractive sets for memristive recurrent neural networks with time-varying delays as follows:

$$\frac{dx_i(t)}{dt} = -d_i(x_i(t))x_i(t) + \sum_{j=1}^{n} a_{ij}(x_i(t))f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(x_i(t)) \times f_j(x_j(t-\tau_j(t))) + I_i(t), \quad t \ge 0, \ \ i = 1, 2, \ldots, n, \tag{1}$$

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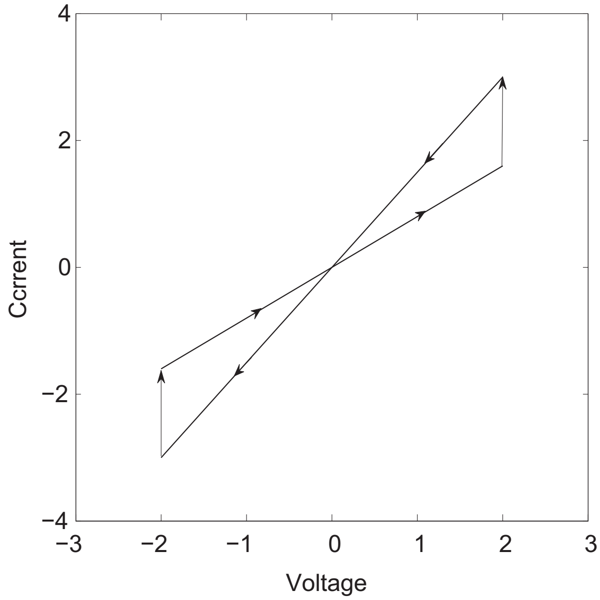

Fig. 1. The typical current–voltage characteristics of memristor.

according to the feature of the memristor and the current–voltage characteristics in Fig. 1, as the previous works [15–19], we can get

$$d_i(x_i(t)) = \begin{cases} d_i^*, & |x_i(t)| \le T_i, \\ d_i^{**}, & |x_i(t)| > T_i, \end{cases} \quad a_{ij}(x_i(t)) = \begin{cases} a_{ij}^*, & |x_i(t)| \le T_i, \\ a_{ij}^{**}, & |x_i(t)| > T_i, \end{cases}$$

$$b_{ij}(x_i(t)) = \begin{cases} b_{ij}^*, & |x_i(t)| \le T_i, \\ b_{ij}^{**}, & |x_i(t)| > T_i, \end{cases}$$

in which switching jumps $T_i > 0, d_i^* > 0, d_i^{**} > 0, a_{ij}^*, a_{ij}^{**}, b_{ij}^*, b_{ij}^{**}$ are all constant numbers. In system (1), $x_i(t)$ is the state of the i-th neuron at time t; $d_i(x_i(t))$ is the i-th neuron self-inhibitions at time t; $a_{ij}(x_i(t))$ is the connection weight; $b_{ij}(x_i(t))$ is the delayed connection weight; $f_j(x_j(t))$ denotes the neuron activation function; $\tau_j(t)$ corresponds to the transmission delays and satisfies $0 \le \tau_j(t) \le \tau$ ($\tau = \max_{1 \le j \le n, t \ge 0}\{\tau_j(t)\}$ is a positive constant); $I_i(t)$ is a continuous bounded external input function, $i,j = 1,2,\ldots,n$. Obviously, the memristive recurrent neural network (1) is a state-dependent switched system, which is the generalization of those for conventional recurrent neural networks.

The organization of this paper is as follows. Some preliminaries are introduced in Section 2. Some new sufficient conditions in linear matrix inequality concerning global exponential stability in a Lagrange sense and the existence of globally exponentially attractive sets of the memristive recurrent neural networks (1) are derived in Section 3. Numerical simulations are given to demonstrate the effectiveness of the proposed approach in Section 4. Finally, this paper ends by a conclusion.

2. Preliminaries

For convenience, we first make the following preparations.

Throughout this paper, solutions of all the systems considered in the following are intended in Filippov's sense (see [33]). And $[\cdot,\cdot]$ represents the interval. Let $\mathcal{C}([-\tau,0],\mathbb{R}^n)$ be the Banach space of continuous functions $\varphi : [-\tau,0] \to \mathbb{R}^n$ with the norm $\|\varphi\| = \sup_{s \in [-\tau,0]}|\varphi(s)|$. For vector $v = (v_1, v_2, \ldots, v_n)^T \in \Re^n$, $\|v\|$ is said to be the Euclidean norm, i.e., $\|v\| = \sqrt{[\sum_{i=1}^n (v_i)^2]}$. Let $\overline{d}_i = \max\{d_i^*, d_i^{**}\}$, $\underline{d}_i = \min\{d_i^*, d_i^{**}\}$, $\overline{a}_{ij} = \max\{a_{ij}^*, a_{ij}^{**}\}$, $\underline{a}_{ij} = \min\{a_{ij}^*, a_{ij}^{**}\}$, $\overline{b}_{ij} = \max\{b_{ij}^*, b_{ij}^{**}\}$, $\underline{b}_{ij} = \min\{b_{ij}^*, b_{ij}^{**}\}$, $A_{ij} = \max\{|\overline{a}_{ij}|, |\underline{a}_{ij}|\}$, $B_{ij} = \max\{|\overline{b}_{ij}|, |\underline{b}_{ij}|\}$, for $i,j = 1,2,\ldots,n$. Let $M = (m_{ij})_{n\times n} \in \mathbb{R}^{n\times n}$ denote real square matrix, M^T, M^{-1} and $\lambda(M)$ denote the transpose, inverse and the eigenvalues of the square matrix M, respectively. $M < 0$ means that M is a negative definite matrix, and $M_1 < M_2$ indicates $M_1 - M_2 < 0$. co$[\underline{\xi}_i, \overline{\xi}_i]$ denotes the convex hull of $[\underline{\xi}_i, \overline{\xi}_i]$, clearly, in this paper, we have co$[\underline{\xi}_i, \overline{\xi}_i] = [\underline{\xi}_i, \overline{\xi}_i]$. And $[\Theta, \Delta]$ by the interval matrix, where $\Theta = (\theta_{ij})_{n\times n}$, $\Delta = (\vartheta_{ij})_{n\times n}$, it follows that $\Theta \ll \Delta$, which means $\Theta \ll \Sigma = (\sigma_{ij})_{n\times n} \ll \Delta$, we have $\theta_{ij} < \sigma_{ij} < \vartheta_{ij}, i,j = 1,2,\ldots,n$, where $\Sigma \in [\Theta, \Delta]$. For a continuous function $k(t): R \to R$, $D^+k(t)$ is called the upper right dini derivative and defined as $D^+k(t) = \overline{\lim}_{h\to 0^+}(1/h)(k(t+h) - k(t))$. For any initial function $\phi(s) \in \mathcal{C}$, $s \in [-\tau, 0]$, the solution of system (1) that starts from the initial condition $\phi(s)$ will be denoted by $x(t, \phi(s))$. In this paper, we first make the following assumption for system (1).

Assumption 1. The neuron activation functions $f_i(x_i)$ in system (1) are bounded, that is, there exist positive constant $h_i > 0$ such that $|f_i(x_i)| \le h_i$. And there exists a diagonal matrix $L = \text{diag}(L_1, L_2, \ldots, L_n)$, for $\forall s_1, s_2 \in \mathbb{R}, s_1 \ne s_2$, the neuron activation of system (1) satisfies

$$0 \le \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \le L_i, \quad f_i(0) = 0, \tag{2}$$

where $L_i, i = 1, 2, ., n$, are positive constants.

Now, as the literature [15–20], by applying the theories of set-valued maps and differential inclusions [33–35], from system (1), we have

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} \in -[\underline{d}_i, \overline{d}_i]x_i(t) + \sum_{j=1}^n [\underline{a}_{ij}, \overline{a}_{ij}]f_j(x_j(t)) + \sum_{j=1}^n [\underline{b}_{ij}, \overline{b}_{ij}]f_j(x_j(t - \tau_j(t))) + I_i(t),$$
$$\text{for a.a. } t \ge 0, \tag{3}$$

or equivalently, there exist $\hat{d}_i(t) \in [\underline{d}_i, \overline{d}_i], \hat{a}_{ij}(t) \in [\underline{a}_{ij}, \overline{a}_{ij}]$, $\hat{b}_{ij}(t) \in [\underline{b}_{ij}, \overline{b}_{ij}]$ such that

$$\frac{\mathrm{d}x_i(t)}{\mathrm{d}t} = -\hat{d}_i(t)x_i(t) + \sum_{j=1}^n \hat{a}_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^n \hat{b}_{ij}(t)f_j(x_j(t - \tau_j(t))) + I_i(t),$$
$$t \ge 0, \quad i = 1, 2, \ldots, n. \tag{4}$$

Now, let $\underline{D} = \text{diag}(\underline{d}_1, \underline{d}_2, \ldots, \underline{d}_n)$, $\overline{D} = \text{diag}(\overline{d}_1, \overline{d}_2, \ldots, \overline{d}_n)$, $\underline{A} = (\underline{a}_{ij})_{n\times n}$, $\overline{A} = (\overline{a}_{ij})_{n\times n}$, $\underline{B} = (\underline{b}_{ij})_{n\times n}$, $\overline{B} = (\overline{b}_{ij})_{n\times n}$, then, (3)and (4) can be rewritten as follows:

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} \in -[\underline{D}, \overline{D}]x(t) + [\underline{A}, \overline{A}]f(x(t)) + [\underline{B}, \overline{B}]f(x(t - \tau(t))) + I(t)$$
$$\text{for a.a. } t \ge 0, \tag{5}$$

or equivalently, there exist $\hat{D}(t) \in [\underline{D}, \overline{D}], \hat{A}(t) \in [\underline{A}, \overline{A}]$, $\hat{B}(t) \in [\underline{B}, \overline{B}]$ such that

$$\frac{\mathrm{d}x(t)}{\mathrm{d}t} \in -\hat{D}(t)x(t) + \hat{A}(t)f(x(t)) + \hat{B}(t)f(x(t - \tau(t))) + I(t), \quad t \ge 0, \tag{6}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$, $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t)))^T$, $\tau(t) = (\tau_1(t), \tau_2(t), \ldots, \tau_n(t))$, $I(t) = (I_1(t), I_2(t), \ldots, I_n(t))^T$.

Definition 1. A vector-value function (in Filippov's sense) $x^*(t) = (x_1^*(t), x_2^*(t), \ldots, x_n^*(t))^T$ is a solution of system (1), with the initial conditions $\phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T \in \mathcal{C}([-\tau,0],\mathbb{R}^n)$, if $x^*(t)$ is an absolutely continuous function on $[0, +\infty)$ and satisfies the