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High-order tracking problem with a time-varying topology and communication delays ☆

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ABSTRACT

In this paper, tracking problem for high-order agents under uncertain communication topology and time varying delays is investigated. Each tracking agent is assumed only its own and neighbors' information is used in their tracking strategy design. The uncertain communication topology is modelled by a finite number of Laplacian matrices with their corresponding scheduling functions. Sufficient conditions for the existence of a tracking strategy have been expressed in terms of the solvability of linear matrix inequalities. Finally, numerical examples are given to verify the effectiveness and advantages of proposed tracking strategy. Through a set of simulations with various control parameters γ, the relationship between γ and the maximum allowable communications delays is tabulated.

1. Introduction

The consensus problem for multi-agent systems has attracted considerable attention from various disciplines of engineering, biology and science over the past decade. This may due to their collective behavior have a variety of applications such as flocking [1], formation control [2] of mobile robots, and distributed sensor fusion [3,4]. The first-order and second-order consensus problem for multi-agent systems has been intensively investigated [5–14]. In [5,6] the consensus problem of first-order multi-agent systems with multiple time delays has been studied and in [7–14], the second-order consensus or formation algorithms for multi-agent systems with or without time delays has been studied. Specifically, [7–9] explore the case of fixed topologies without time delays, [10–12] investigate the case of fixed topologies with time-varying delays and [12–14] deal with the case of switching topologies.

There are considerable results on tracking problem of multi-agent systems with first-order or second-order kinematics. However, few works have been published on high-order multi-agent systems. Considering the behavior of a group of mobile vehicles, in practical applications, they are in the form of high-order [15]. Consequently, some researchers become interested in the high-order multi-agent systems [16–21]. Specifically, in [17], the authors investigate the high-order leader-following consensus problem of nonlinear multi-agent systems in undirected network topologies, and subsequently authors in [20] consider consensus problems of high-order multi-agent systems with switching topologies.

In most of existed literatures, the topologies of the multi-agent systems are supposed to be fixed topologies or switching topologies. [11–16,20] investigate consensus problems for multi-agent systems under switching topologies, which are modelled by switching between several fixed topologies. While, in practices, since all the agents are moving and the communication radius of each agent is finite, the communication topology between the target and the tracking agents may change from time to time rather than switches among several fixed topologies. Then, tracking problem of a maneuvering target under time-varying topologies is of vital necessity and it is also a new challenging topic.

It is worth to note that so far only few results dealt with the consensus problem of multi-agent systems under time-varying topologies. In [22], the authors propose a consensus controller using the local information and its neighbor's corresponding state information with time delay for second-order multi-agent systems with dynamically changing topologies. In [23], the authors address the consensus problem of multi-agent systems, consisting of a set of identical multiple–input multiple–output (MIMO), linear time-invariant (LTI) systems, under a time-varying network which has a well-defined average. Ref. [23] investigates linear systems without communication time delays. However, in our paper, we focus on the performance of nonlinear systems with communication delays. Also [23] is only

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focused on stability analysis of the linear multi-agent systems under fast switching network whereas the controller synthesis is not focused. In our previous work, we have investigated tracking problem of first-order, second-order and high-order nonlinear multi-agent systems under a time-varying topology in [24–26] respectively. Sufficient and necessary conditions of consensus by employing some knowledge of complex dynamical network are given in [24]. Based on the eigenvalues of the Laplacian matrix of the time-varying topology, some sufficient conditions and controller design principles which ensure the leader-following consensus of multi-agent systems are explored in [25,26]. Sufficient conditions in [26] are based on the assumptions that Laplacian matrices are non-negative and their time derivatives are non-positive. However, these assumptions are not easy to satisfy, which limits the range of application of the proposed theorem in [26]. In these papers, only [22] considers communication delays, which, in real-world systems, when information is exchanged between the agents and the target through the wireless communication network are inevitable.

Motivated by the related work, we aim to develop another algorithm to solve the high-order nonlinear tracking problem under time-varying topology and communication delays. In this work, the time-varying topology is modelled by a finite Laplacian matrices with their corresponding scheduling functions. It should be noted that, to the best of our knowledge, we have not found any published papers on the high-order multi-agent consensus problem with a time-varying topology and communication delays. The rest of this paper is organized as follows. In Section 2, some related preliminaries and the problem formulations for high-order systems are presented. In Section 3, the main results are obtained. Section 4 shows numerical examples and conclusions are drawn in Section 5.

Throughout this paper, the following notations will be used: $\mathbb{R}^{p\times q}$ is the set of $p\times q$ real matrix. $\mathbf{I}_n \in \mathbb{R}^{n\times n}$ denotes an $n\times n$ identity matrix. The Kronecker product is denoted by $\otimes$. Euclidean norm of X is denoted by $\|X\|$.

2. Problem description

Considering a team of n identical tracking agents which are distributed at n distinct position in the m-dimensional space and denoted by $\mathbf{P}:=\{x_{\mathbf{p}i}\in\mathbb{R}^m, i\in\mathcal{N}\}$. The kinematic equation of the i-th tracking agent is described by

$$\begin{aligned} \dot{x}_{\mathbf{p}i}^{(0)}(t) &= x_{\mathbf{p}i}^{(1)}(t)\\ \dot{x}_{\mathbf{p}i}^{(1)}(t) &= x_{\mathbf{p}i}^{(2)}(t)\\ &\vdots\\ \dot{x}_{\mathbf{p}i}^{(k-1)}(t) &= f(t, x_{\mathbf{p}i}^{(0)}(t), x_{\mathbf{p}i}^{(1)}(t), \ldots, x_{\mathbf{p}i}^{(k-1)}(t)) + u_{\mathbf{p}i}(t), \end{aligned} \tag{1}$$

where $x_{\mathbf{p}i}^{(d)}\in\mathbb{R}^m$, $d=\{0,1,\ldots,k-1\}$ are the states of the i-th tracking agent.

$f(t, x_{\mathbf{p}i}^{(0)}(t), x_{\mathbf{p}i}^{(1)}(t), \ldots, x_{\mathbf{p}i}^{(k-1)}(t))$ is a nonlinear function which denotes the dynamics of the tracking agents. $u_{\mathbf{p}i}\in\mathbb{R}^m$ is the control input of the i-th tracking agent.

In the view of graph theory, every agent can be treated as a node, as shown in Fig. 1. Then the communication topology of tracking agents and the target can be treated as a dynamic graph. A weighted graph $G:=\{\mathcal{N},\mathcal{E},\mathcal{A}\}$ is denoted by with a node set $\mathcal{N}=\{1,2,\ldots,n\}$, an edge set $\mathcal{E}\subseteq\mathcal{N}\times\mathcal{N}$ and a weighted adjacency matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{n\times n}$ with nonnegative elements. The nodes within the communication range of node i are called the set of neighbors of node i, which is denoted by $\mathcal{N}_i=\{j|j\in\mathcal{N},(j,i)\in\mathcal{E}\}$. When $j\notin\mathcal{N}_i$, which means node j is beyond the communication range of node i, $a_{ij}=0$, otherwise $a_{ij}>0$.

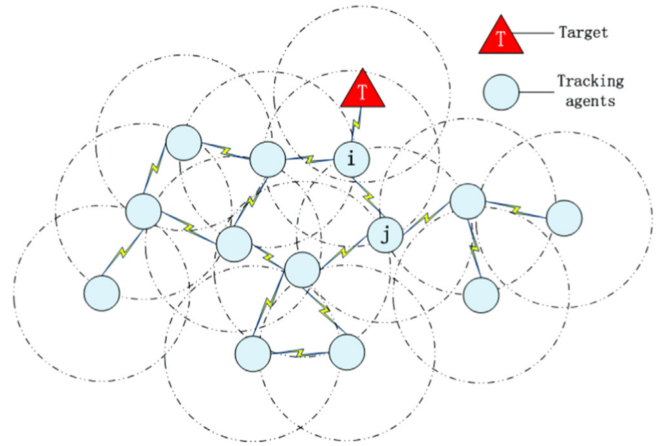

Fig. 1. A wireless communication network of high-order agents in which agent i receives the information of its neighbor, agent j, if there is a link (i,j) connecting the two nodes. This means the two nodes are within each other's communication range, which is denoted by dash circle. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this article.)

The Laplacian matrix $\mathcal{L}=[l_{ij}]\in\mathbb{R}^{n\times n}$ of graph G is defined as $\mathcal{L}:=\mathcal{D}-\mathcal{A}$, where

$\mathcal{D}:=diag\{\sum_{j\in\mathcal{N}_1} a_{1j}, \sum_{j\in\mathcal{N}_2} a_{2j}, \ldots, \sum_{j\in\mathcal{N}_n} a_{nj}\}$.

Since the wireless communication possesses uncertainty and the agents are moving in real-time, the topology among the agents is not invariant. In this paper, we consider a_{ij}, the weighted value of agent j to agent i, as time-varying, and it is denoted as $a_{ij}(t)$. The communication topology for the tracking agents is assumed to be modeled by a time-varying graph $G(t):=\{\mathcal{N},\mathcal{E}(t),\mathcal{A}(t)\}$ with node set $\mathcal{N}$, time-varying edge $\mathcal{E}(t)\subseteq\mathcal{N}\times\mathcal{N}$ and continuous adjacency matrix $\mathcal{A}(t)=[a_{ij}(t)]\in\mathbb{R}^{n\times n}$. The topology is symmetric, i.e., the corresponding Laplacian matrix $\mathcal{L}(t)$ satisfying $\mathcal{L}(t)=\mathcal{L}^T(t)$.

The goal of each tracking agent, which is regarded as a point of the dynamic graph, is to track a maneuvering target, which is denoted by a red triangle in Fig. 1. The maneuvering target is assumed to possess the following high-order kinematics.

$$\begin{aligned} \dot{x}_{\mathbf{t}}^{(0)}(t) &= x_{\mathbf{t}}^{(1)}(t)\\ \dot{x}_{\mathbf{t}}^{(1)}(t) &= x_{\mathbf{t}}^{(2)}(t)\\ &\vdots\\ \dot{x}_{\mathbf{t}}^{(k-1)}(t) &= f(t, x_{\mathbf{t}}^{(0)}(t), x_{\mathbf{t}}^{(1)}(t), \ldots, x_{\mathbf{t}}^{(k-1)}(t)), \end{aligned} \tag{2}$$

where $x_{\mathbf{t}}^{(d)}\in\mathbb{R}^m$, $d=\{0,1,\ldots,k-1\}$ are the states of the target agent.

$f(t, x_{\mathbf{t}}^{(0)}(t), x_{\mathbf{t}}^{(1)}(t), \ldots, x_{\mathbf{t}}^{(k-1)}(t))$ denotes the change of the force which is imposed on the target.

$b_i(t)>0$ indicates the case that the i-th tracking agent can receive the states of the target at time t, and $b_i(t)=0$ indicates the case that information of the maneuvering target is not accessible by the i-th tracking agent. Denote $\mathcal{B}(t):=diag\{b_1(t), b_2(t), \ldots, b_n(t)\}$.

When information is exchanged between the agents and the target through the wireless communication network, time delays are inevitable. The objective of this paper is to design a tracking strategy which ensures that the tracking agents effectively track the target agent in m-dimensional space with a time-varying topology and communication delays. In this work, we assume $f(\cdot)$ satisfies the following assumption:

Assumption 1. There exists a finite constant $l\geq 0$ such that $|f(t,\hat{x}_{\mathbf{p}i}(t)) - f(t,\hat{x}_{\mathbf{t}}(t))| \leq l|\hat{x}_{\mathbf{p}i}(t) - \hat{x}_{\mathbf{t}}(t)|$.

To analyze the tracking problem, the following definitions and lemmas are essential.