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Distributed node-to-node consensus of multi-agent systems with time-varying pinning links

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ABSTRACT

In this paper, the distributed node-to-node consensus problem is addressed for a class of linear multi-agent systems with time-varying pinning links by using relative state feedback. The coordination goal in the present work is to make the states of each follower track those of its corresponding leader asymptotically where only a small fraction of followers can sense the states of their corresponding leaders through some switched communication channels. By using tools from *M*-matrix theory and stability analysis of switched systems theory, it is theoretically shown that such a node-to-node consensus in the closed-loop multi-agent systems can be guaranteed if each follower can be directly or indirectly influenced by at least one leader over some uniformly bounded time intervals, with the inner coupling matrix as well as the coupling strength being appropriately designed. Distributed node-to-node consensus for multi-agent systems with general Lipschitz-type nonlinear dynamics is also investigated. Numerical simulations are finally performed to verify the effectiveness of the theoretical results.

1. Introduction

The last few decades have witnessed a great devotion by scientists to explore and characterize the evolution mechanisms of various multi-agent systems including the multiple vehicles, WWW (World Wide Web), large-scale sensors, among many others [1–5]. One basic issue arising from multi-agent systems is to construct communication protocols based only on the local measurements that guarantees all agents to achieve a state agreement, which is known as consensus problem [6–13]. A closely related topic to consensus of multi-agent systems is synchronization of complex dynamical networks [14–17].

Distributed consensus problem has been addressed from several different perspectives in the existing literature. For instance, in [5], Vicsek et al. introduced a simple yet effective discrete-time model for the phase transition of a group of autonomous agents and numerically investigated the angle consensus behavior in such a multi-agent system. By using tools from algebraic graph theory, some theoretical interpretations for the emergence of consensus in Vicsek's mode were provided in [6]. A general framework for the study of consensus problem of networked agents was suggested in [7]. The consensus conditions derived in [7] were further relaxed in [8] by showing that consensus in first-order multi-agent systems can be ensured if and only if the network topology contains a directed spanning tree frequently enough as the network evolves with time. Robust consensus for first- and second-order multi-agent systems were studied in [9,10], respectively. Furthermore, consensus problem over random communication topology [18], consensus with output coupling [19–21], sampled-data consensus [22,23] and consensus for multi-agent systems with higher-order dynamics [24–28] were also investigated by researchers from different scientific communities. According to the presence or absence of a leader in the considered multi-agent systems, consensus problems can be divided into two categories: leaderless and leader-following consensus problems. Leaderless consensus means that the states of all agents could achieve a priori unknown agreement. In contrast, the states of the multiple agents in the context of leader-following consensus will converge to a designed state trajectory. In recent years, distributed containment control of multi-agent systems with multiple leaders has been addressed in the literature [29,30], where the states of all followers will converge into a convex hull formed by those of the leaders.

Note that it is assumed in most existing works on containment control of multi-agent systems that there is no interaction among the multiple leaders. However, in some practical applications, there may

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exist some interactions between the neighboring leaders and the states of a given follower that may only need to track the states of a particular leader but not the convex combination of all the leaders' states. Motivated by this observation and based on the existing works on leader-following consensus as well as containment control of multi-agent systems, distributed node-to-node consensus problem of multi-agent systems with multiple interacting leaders is addressed in this paper. The basic idea of node-to-node consensus was first given in [31] by assuming that all the pinning links starting from the leaders ending at followers are time-invariant. However, this assumption may not be ensured in some practical cases due to the communication disturbances and sensor range limitations. Motivated by this observation, this paper focused on solving the distributed node-to-node consensus of linear multi-agent systems with time-varying pinning links. Specifically, it is assumed that only a small fraction of the followers can sense their corresponding leaders in a switched communication manner. By using tools from nonnegative matrix theory and Lyapunov stability analysis of switched systems, some sufficient conditions for achieving node-to-node consensus are derived and analyzed. It is theoretically shown that the distributed node-to-node consensus can be ensured if some followers are appropriately pinned and the pinning links can work well over some uniformly bounded time intervals as the systems evolve with time. Furthermore, the distributed node-to-node consensus in multi-agent systems with general Lipschitz-type nonlinear dynamics is also discussed.

The rest of this paper is structured as follows. In Section 2, some preliminaries on graph theory and the problem formulation are presented. The main analytical results are given in Section 3. Numerical simulations are performed in Section 4. Concluding remarks are finally drawn in Section 5. A preliminary version of Section 3.1 of this paper can be found in [32].

Let $\mathbf{R}^{n\times n}$ and $\mathbf{N}$ be the sets of $n\times n$ real matrix space and positive natural numbers, respectively. Let $\mathrm{diag}\{d_1, d_2, \ldots, d_n\}$ be a diagonal matrix with d_i, $i=1,2,\ldots,n$, being its i-th diagonal element. Take $\mathbf{1}_n$ (or $\mathbf{0}_n$) be the n-dimensional column vector with each entry being 1 (or 0). Matrices are assumed to have compatible dimensions throughout this paper. Symbol $\|\cdot\|$ represents the Euclidean norm. For a real symmetric matrix W, $\lambda_{\max}(W)$ and $\lambda_{\min}(W)$ represent its largest and smallest eigenvalues, respectively. Furthermore, a column vector $\varepsilon=(\varepsilon_1, \varepsilon_2, \ldots, \varepsilon_n)^T\in\mathbf{R}^n$ is said to be positive if and only if $\varepsilon_i>0$, for all $i=1,2,\ldots,n$.

2. Preliminaries and model formulation

2.1. Preliminaries

Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ be a directed graph with the set of nodes $\mathcal{V}=\{1,2,\ldots,N\}$, the set of directed links $\mathcal{E}\subseteq\{(i,j), i,j\in\mathcal{V}\}$, and a weighted adjacency matrix $\mathcal{A}=[a_{ij}]_{N\times N}$ with elements $a_{ij}\geq 0$. The edge (i,j) in graph $\mathcal{G}$ is starting from node j and ending at node i. A path on $\mathcal{G}$ from node i_1 to node i_s is a sequence of ordered edges of the form $(i_{k+1},\ i_k)$, $k=1,2,\ldots,s-1$. A directed graph has or contains a directed spanning tree if there exists a node called root such that there exists a directed path from this node to every other node. The adjacency matrix $\mathcal{A}=[a_{ij}]_{N\times N}$ of a directed graph $\mathcal{G}$ is defined by $a_{ii}=0$ for $i=1,2,\ldots,N$, and $a_{ij}>0$ for $(i,j)\in\mathcal{E}$ but 0 otherwise. The Laplacian matrix $\mathcal{L}=[l_{ij}]_{N\times N}$ is defined as $l_{ij}=-a_{ij}$, $i\neq j$, and $l_{ii}=\sum_{j=1}^{N}a_{ij}$ for $i=1,2,\ldots,N$. For an arbitrarily given directed graph $\mathcal{G}$, its Laplacian matrix $\mathcal{L}$ has the following property.

Lemma 1 (*Ren and Beard [33]*). *Suppose that $\mathcal{G}$ contains a directed spanning tree. Then, 0 is a simple eigenvalue of its Laplacian matrix $\mathcal{L}$ and all the other eigenvalues have positive real parts.*

Before moving forward, the following definition and lemmas are given.

Definition 1 (*Berman and Plemmons [34]*). A nonsingular real matrix A is called an M-matrix if all of its off-diagonal elements are non-positive, and each of its eigenvalues has positive real part.

Lemma 2 (*Berman and Plemmons [34]*). *Suppose that $A\in\mathbf{R}^{N\times N}$ is an M-matrix. Then, there exists a positive vector $\phi=(\phi_1, \phi_2, \ldots, \phi_N)^T\in\mathbb{R}^N$, such that $A^T\phi=\mathbf{1}_N$ and $\Psi A+A^T\Psi>0$, where $\Psi=\mathrm{diag}\{\phi_1, \phi_2, \ldots, \phi_N\}$.*

Remark 1. It can be seen from Lemma 2 that, for an arbitrarily given positive scalar κ_0, there is a positive vector $\widehat{\phi}(\kappa_0)=(\widehat{\phi}_1(\kappa_0), \widehat{\phi}_2(\kappa_0), \ldots, \widehat{\phi}_N(\kappa_0))^T$ such that $A^T\widehat{\phi}(\kappa_0)=\kappa_0\mathbf{1}_N$. Furthermore, it is not hard to verify that $\widehat{\Psi}(\kappa_0)A+A^T\widehat{\Psi}(\kappa_0)>0$ with $\widehat{\Psi}(\kappa_0)=\mathrm{diag}\{\widehat{\phi}_1(\kappa_0), \widehat{\phi}_2(\kappa_0), \ldots, \widehat{\phi}_N(\kappa_0)\}$. The above analysis indicates that, for an arbitrarily given M-matrix $A\in\mathbf{R}^{N\times N}$, the selection of $P>0$ satisfying $PA+A^TP>0$ can be moreover with an arbitrary norm of nonzero. In other words, as long as one solution $P>0$ is obtained, the scaled matrix ℓP for any $\ell>0$ is also a solution to the above Lyapunov matrix inequality. This indicates that, for an M-matrix $A\in\mathbf{R}^{N\times N}$, the selections of Ψ in Lemma 2 such that $\Psi A+A^T\Psi>0$ are not unique.

Lemma 3. *For any given $x, y\in\mathbf{R}^n$, and matrices $P>0$, D and S of appropriate dimensions, the following inequality holds:*

$$2x^TDSy\leq x^TDPD^Tx+y^TS^TP^{-1}Sy.$$

2.2. Problem formulation

It is assumed that there are multiple interacting leaders in the present multi-agent system. To facilitate analysis, it is further assumed that the multi-agent systems under consideration consist of two layers, i.e., the leaders' layer and the followers' layer. Suppose that both the leaders' and the followers' layers contain N agents. Unlike most existing literature on distributed control of multi-agent systems that there is no interaction among the multiple leaders, in the present paper, the evolution of each leader will be affected by its neighbors located in the leaders' layer. The above statements indicate that the leaders do not receive any information from the followers. However, the dynamics of each follower will be influenced by those of its neighbors in both leaders' and followers' layers. Specifically, the evolution of the i-th $(1\leq i\leq N)$ leader is described as

$$\dot{x}_i(t)=Ax_i(t)+cBK\sum_{j=1}^{N}a_{ij}(x_j(t)-x_i(t)), \tag{1}$$

where $x_i(t)\in\mathbf{R}^m$ is the state of the i-th leader, $A\in\mathbf{R}^{m\times m}$ is the state matrix, $B\in\mathbf{R}^{m\times h}$ is the control input matrix, $K\in\mathbf{R}^{h\times m}$ is the inner linking matrix to be determined later and $c>0$ indicates the coupling strength. It is assumed that matrix pair (A, B) is stabilizable. Furthermore, the evolution of the i-th $(1\leq i\leq N)$ follower is given as

$$\dot{\hat{x}}_i(t)=A\hat{x}_i(t)+cBK\sum_{j=1}^{N}a_{ij}\left(\hat{x}_j(t)-\hat{x}_i(t)\right)+cp_i(t)BK(x_i(t)-\hat{x}_i(t)), \tag{2}$$

where $\hat{x}_i(t)\in\mathbf{R}^m$ is the state of the i-th follower, the pinning link $p_i(t)\in\{0,1\}$ and $p_i(t)=1$ if and only if the i-th follower can directly sense the i-th leader at time t, i.e., there exists a directed link from the i-th leader to the i-th follower at time t. It is assumed that, for each $i\in\{1,2,\ldots,N\}$, there exists an infinite sequence of uniformly bounded non-overlapping time intervals $[\widehat{t}_k^i, \widehat{t}_{k+1}^i)$, $k\in\mathbf{N}$, with $\widehat{t}_1^i=0$, $\widehat{t}_{k+1}^i-\widehat{t}_k^i\geq\tilde{\tau}_0$, and $\tilde{\tau}_0>0$, over which $p_i(t)$ is fixed. Here, $\tilde{\tau}_0$ is called the dwell time.

Definition 2. The distributed node-to-node consensus in multi-agent systems (1) and (2) is said to be achieved if, for any given