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Existence and stability of periodic solution of high-order discrete-time Cohen–Grossberg neural networks with varying delays

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ABSTRACT

This paper studies the existence and stability of periodic solution of the high-order discrete-time Cohen–Grossberg neural networks with varying delays. The properties of M-matrix and the contracting mapping principle are used to obtain a sufficient condition that guarantees the uniqueness and global exponential stability of the periodic solution. In addition, a numerical example is given that demonstrates the effectiveness of the proposed theoretical results.

1. Introduction

In 1983, Cohen–Grossberg initially proposed a neural network model in [1], which is usually called Cohen–Grossberg neural network model. It is well known that it has wide application in signal and image processing, pattern recognition, associative memory and combinatorial optimization. Therefore, the properties of Cohen–Grossberg neural networks (i.e., the stability and the attractiveness of the periodic solution) have been studied by many researchers, and a large number of results were obtained [2–7].

However, most researchers paid attention to low-order Cohen–Grossberg neural networks and did not consider the high-order connected terms. In fact, the low-order neural networks have many intrinsic limitations, such as slow convergence rate, small storage capacity and low fault tolerance etc. In order to solve these shortcomings, it is necessary to add high-order interactions to these neural networks [8–10]. This motivated the extensively study on the high-order Cohen–Grossberg neural networks in the past few years and many useful results on the characteristics for this type of high-order neural networks have been presented [11–16]. For example, Zhang, Jiang and Teng [16] considered the high-order Cohen–Grossberg neural networks with time-varying delays, and established some sufficient conditions on the existence and exponential stability of the anti-periodic solutions for these networks.

For the continuous-time neural networks, it is of vital importance to study the properties of its discrete-time counterparts due to the following two reasons. On the one hand, the discrete-time analogs contain much richer dynamics than its continuous-time counter parts. On the other hand, we need to discretize the continuous-time networks for computation and numerical simulation, and the discrete-time analogs can be used for the digital simulation without any loss of functionality of the continuous-time systems [17]. Recently, researchers have carried out their study on the properties of the discrete-time Cohen–Grossberg neural networks [18–22]. Liu, Xu and Wang [18] studied the periodic solution of discrete-time Cohen–Grossberg neural networks with disturbed delays by using suitable Lypunov function and the properties of M-matrix. They gave several sufficient conditions that ensure the uniqueness and global exponential stability of the periodic solutions. In addition, Li and Wang [20] studied a class of the discrete-time delayed Cohen–Grossberg neural

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network models. Some sufficient conditions that guarantee the existence and exponential stability of the periodic solution are obtained by using Mawhin's coincidence degree theory and proper Lyapunov function.

Based on the applications and features of both high-order Cohen–Grossberg neural networks and discrete-time neural networks, it is necessary to analysis the dynamic behaviors of high-order discrete-time Cohen–Grossberg neural networks. Although researchers have carried out extensively study on these networks, there are few results about the properties of high-order discrete-time Cohen–Grossberg neural networks now. This is the motivation of the study of this paper. In this paper, we investigate the existence and stability of the periodic solutions of high-order discrete-time Cohen–Grossberg neural networks with varying delays. A sufficient condition is obtained by using the properties of M-matrix and the contracting mapping principle.

The rest of this paper is organized as follows. In Section 2, the model of high-order discrete-time Cohen–Grossberg neural networks with varying delays is given. In Section 3, a sufficient condition that guarantees the uniqueness and global exponential stability of the periodic solutions is presented. In Section 4, a numerical example is given in order to demonstrate the effectiveness of our theoretical results. Finally, some concluding remarks are given in Section 5.

2. Model description and preliminaries

The model of a high-order discrete-time Cohen–Grossberg neural network with varying delays is

$$x_i(k+1)=x_i(k)-a_i(k,x_i(k))\Big[b_i(k,x_i(k))-\sum_{j=1}^{n}c_{ij}(k)f_j(x_j(k)).$$
$$-\sum_{j=1}^{n}\sum_{l=1}^{n}d_{ijl}(k)f_j(x_j(k-\tau_{ijl}(k)))\cdot f_l(x_l(k-\tau_{ijl}(k)))-I_i(k)\Big],i=1,2,\ldots,n, \tag{1}$$

where $n\geq 2$ is the number of neurons in the network, $x_i(k)$ denotes the state associated with the $i-$th neuron at the time k; $a_i(k,\cdot)$ represents the amplification function, $b_i(k,\cdot)$ is an appropriately behaved function, $c_{ij}(k)$ denotes the synaptic connection weights of unit j to unit i; $d_{ijl}(k)$ represents the synaptic connection weights of unit j to unit i and unit l; $f_j(\cdot)$ is a measure of response or activation to its incoming potential; $\tau_{ijl}(k)$ is a nonnegative constant, which means the transmission delay along the axon of unit j to unit i and unit l; $I_i(k)$ is the external bias on the $i-$th neuron at the time k; $c_{ij}(k)$, $d_{ijl}(k)$, $\tau_{ijl}(k)$ and $I_i(k)$ are $\omega-$ periodic functions; $a_i(k,\cdot)$ and $b_i(k,\cdot)$ are $\omega-$ periodic about the first argument, where ω is a positive integer.

Let Z be the set of all integers, and $Z_0^+=\{0,1,2,\ldots\}$, $Z^+=\{1,2,\ldots\}$, $[a,b]_Z=\{a,a+1,\ldots,b-1,b\},a\leq b$. For a $\omega-$ periodic function $S(k)$, we define $\overline{S}=\max_{s\in[0,\omega]_Z}|S(k)|,\underline{S}=\min_{s\in[0,\omega]_Z}|S(k)|$, where ω is a positive integer. The system (1) is supplemented with initial values given by

$$x_i(s)=\varphi_i(s),s\in[-\tau,0]_Z,\ \sup_{s\in[-\tau,0]_Z}|\varphi(s)|<+\infty, \tag{2}$$

where $\tau=\max_{1\leq i,j,l\leq n}\{\overline{\tau}_{ijl}\}$.

For convenience of description, we introduce the following assumptions that will be used in the proof of our main results.

(H_1) For the function $a_i(\cdot)$, there exist positive constants a_i and L_i^a such that

$0<a_i(\cdot)\leq\overline{a}_i,|a_i(k,x)-a_i(k,y)|\leq L_i^a|x-y|,\forall x,y\in R,i=1,2,\ldots,n;$

(H_2) There exist $\omega-$ periodic functions $\gamma_i(k)$ such that

$$\gamma_i(k)\leq\frac{a_i(k,x)b_i(k,x)-a_i(k,y)b_i(k,y)}{x-y},\forall x,y\in R,x\neq y,i=1,2,\ldots,n;$$

(H_3) For the function $f_j(\cdot)$, there exist positive constants M_j,L_j such that

$0\leq f_j(\cdot)\leq M_j,|f_j(x)-f_j(y)|\leq L_j|x-y|,\forall x,y\in R,x\neq y,j=1,2,\cdots n.$

3. Existence and stability of periodic solution

Before giving the main results of this paper, we make some preparations. Firstly, we give three useful lemmas.

Lemma 1. *Let $(X,\|\cdot\|)$ be a Banach space. If $T:(X,\|\cdot\|)\to(X,\|\cdot\|)$ is a contracting mapping, then there exists a only fixed point of T in X.*

Lemma 2. *Let $A\geq 0$ be an $n\times n$ real matrix and the spectral radius of A satisfies $\rho(A)<1$. Then, E_n-A is a non-singular M-matrix, where E_n is an $n\times n$ identity matrix.*

Lemma 3. *Let $A=(a_{ij})_{n\times n}$ with $a_{ij}\leq 0,i,j=1,2,...,n$, and $i\neq j$. The following statements are equivalent:*

(1) A is a non-singular M-matrix;
(2) There exists a vector $\xi=[\xi_1,\xi_2,...,\xi_n]^T>[0,0,...,0]^T$ such that $A\xi>0$;
(3) $A^{-1}\geq 0$.

Secondly, we define a space $\overline{C}([-\tau,0]_Z,R^n)$. For $\forall\Psi=[\varphi_1(s),\varphi_2(s),\ldots,\varphi_n(s)]^T\in\overline{C}$, let $\|\Psi\|=\sup_{s\in[-\tau,0]_Z}\sum_{i=1}^n|\varphi_i(s)|$. It is easy to verify that $\overline{C}$ is a Banach space. In addition, we denote the solution of system (1) with initial condition Ψ by $x(k,\Psi)=[x_1(k,\Psi),x_2(k,\Psi),\ldots,x_n(k,\Psi)]^T$. Now, we explain how to use the properties of M-matrix and the contracting mapping principle to prove the existence and stability of the periodic solution of (1).

Theorem 1. *Assume that $(H_1)-(H_3)$ hold and we let $W=A-L^aB-L\Phi$. If W is a M-matrix, then system (1) has exactly one $\omega-$ periodic solution, and it is globally exponentially stable, where*

$$A=diag(\underline{\gamma}_1,\underline{\gamma}_2,\cdots,\underline{\gamma}_n),L^a=diag(L_1^a,L_2^a,\cdots,L_n^a),$$

$$B=diag\Bigg(\sum_{j=1}^n\overline{c}_{1j}M_j+\sum_{j=1}^n\sum_{l=1}^n\overline{d}_{1jl}M_jM_l+\overline{I}_1,\ \sum_{j=1}^n\overline{c}_{2j}M_j+\sum_{j=1}^n\sum_{l=1}^n\overline{d}_{2jl}M_jM_l+\overline{I}_2,$$
$$\cdots,\sum_{j=1}^n\overline{c}_{nj}M_j+\sum_{j=1}^n\sum_{l=1}^n\overline{d}_{njl}M_jM_l+\overline{I}_n\Bigg),$$

$$L=diag(L_1,L_2,\cdots,L_n),\Phi=[\phi_{ij}]_{n\times n},\phi_{ij}=\overline{a}_j\overline{c}_{ji}+2\overline{a}_j\sum_{l=1}^n\overline{d}_{jil}M_l.$$

Proof. Suppose that $x(k,\Psi_1),x(k,\Psi_2)$ are the solutions of system (1) with initial values $\Psi_1=[\varphi_1,\varphi_2,\cdots,\varphi_n]^T,\Psi_2=[\xi_1,\xi_2,\cdots,\xi_n]^T\in\overline{C}$, respectively. From (1), we have

$$x_i(k+1,\Psi_1)-x_i(k+1,\Psi_2)=x_i(k,\Psi_1)-x_i(k,\Psi_2)$$
$$-[a_i(k,x_i(k,\Psi_1))b_i(k,x_i(k,\Psi_1))-a_i(k,x_i(k,\Psi_2))b_i(k,x_i(k,\Psi_2))]$$
$$+a_i(k,x_i(k,\Psi_1))\sum_{j=1}^n c_{ij}(k)\big[f_j(x_j(k,\Psi_1))-f_j(x_j(k,\Psi_2))\big]$$
$$+[a_i(k,x_i(k,\Psi_1))-a_i(k,x_i(k,\Psi_2))]\sum_{j=1}^n c_{ij}(k)f_j(x_j(k,\Psi_2))$$
$$+a_i(k,x_i(k,\Psi_1))\sum_{j=1}^n\sum_{l=1}^n d_{ijl}(k)[f_j(x_j(k-\tau_{ijl}(k),\Psi_1))f_l(x_l(k-\tau_{ijl}(k),\Psi_1))$$