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Robust reliable H_∞ control for stochastic neural networks with randomly occurring delays

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ABSTRACT

This paper investigates the problem of robust stabilization for a class of discrete-time stochastic neural networks with randomly occurring discrete and distributed time-varying delays. More precisely, the neuron activation functions are assumed to be more general and satisfy sector-like nonlinearities. Moreover, the effects of both variation range and probability distribution of mixed time-delays are taken into consideration in the proposed problem. The main objective of this paper is to design a state feedback reliable H_∞ controller such that for all admissible uncertainties as well as actuator failure cases, the resulting closed-loop form of considered neural network is robustly asymptotically stable while satisfying a prescribed H_∞ performance constraint. Linear matrix inequality approach together with proper construction of Lyapunov–Krasovskii functional is employed for obtaining delay dependent sufficient conditions for the existence of robust reliable H_∞ controller. The obtained results are formulated in terms of linear matrix inequalities (LMIs) which can be easily solved by using the MATLAB LMI toolbox. Finally, a numerical example with simulation results is provided to illustrate the effectiveness of the obtained control law and less conservativeness of the proposed results.

1. Introduction

Neural networks have been extensively studied because of their potential applications in various fields such as signal processing, pattern recognition, static image processing, associative memory and combinatorial optimization [4,5,8]. Also, it is well known that the achieved applications heavily depend on the dynamic behavior such as stability and in particular, if a neural network is employed to solve some optimization problems, it is highly desirable for the neural networks to have a unique globally stable equilibrium point [7]. Further, it is noted that neural network problems are extensively studied with continuous-time cases. The discrete-time neural networks become more important than the continuous-time counterparts when implementing the neural networks in a digital life [2]. On the other hand, the study on time delay systems has become a topic of theoretical and practical importance since time delays are inherent features of many physical process and may lead to instability or significantly affect performances of the corresponding system. It should be pointed that the time delays in some neural networks often exist in a stochastic manner and its probabilistic characteristic, such as Binomial distribution in case of discrete-time system or normal distribution in case of continuous time system, can regularly be obtained by using the statistical methods. Also, the deviations and perturbations in parameters are the main sources of uncertainty which are unavoidable due to mainly modeling inaccuracies, variations of the operating point and aging of the devices. More specifically, the connection weights of the neurons are naturally dependent on certain resistance and capacitance values that unavoidably bring uncertainties at some stage in the parameter identification process. Therefore, it is important to study robustness issue of the neural networks against the uncertainties and random time delays.

It is well known that stability is one of the most important qualitative properties of control systems, because unstable systems have no practical importance. It should be noted that every control system must be primarily stable and then the other qualitative properties can be studied [20–22]. Therefore, design of control for neural networks is a subject of both practical and theoretical importance. On the other side, feedback can modify the natural dynamics of a system and reduce sensitivity to external disturbances or that to changing parameters in the system itself and stabilizes the system easily. Therefore, it is very useful to design a state feedback controller such that the closed-loop form of neural network can converge as fast as possible. Thus, the robust stability and stabilization analysis for neural networks with time delay and uncertainty has attracted much attention and lots

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of related results have been developed on this topic based on the linear matrix inequality approach [6,9,10,15,23–25,31,32]. Wu and Zeng [28] derived some sufficient conditions in terms of linear matrix inequalities in order to achieve exponential stabilization of a general class of memristive neural networks with time delays. The global exponential stabilization results for a class of neural networks with various activation functions and time-varying distributed delays have been studied in [30]. It should be noted that the aforementioned stabilization results are under a complete reliability assumption that all control components of the systems are in perfect working conditions. Because of the growing complexity of automated control systems, various faults are likely to be encountered, in particular faults from actuators and sensors [33]. A reliable control system possesses the ability to accommodate system failures automatically and maintain overall system stability and acceptable performance in the presence of component failures [27]. Further, analysis and synthesis in H_∞ setting have excellent advantages such as efficient disturbance rejection and reduced sensitivity to uncertainties [16]. Recently, Sakthivel et al. [17] derived a set of sufficient conditions for uncertain discrete-time stochastic neural networks with time-varying delays via a reliable H_∞ control law. Ahn [1] proposed a H_∞ weight learning law for the asymptotic stability of switched Hopfield neural networks.

Moreover, from the practical point of view, the stability problem for stochastic discrete time neural networks has become important and several related results have been reported on this topic (see [12,13] and references therein). A robust delay-distribution-dependent stochastic stability analysis has been studied in [26] for a class of discrete-time stochastic delayed neural networks with parameter uncertainties. Yue et al. [19] investigated the global exponential stability in the mean square sense for a class of linear discrete-time recurrent neural networks with stochastic delay by using the Lyapunov–Krasovskii functional and exploiting some new analysis techniques. The state estimation problem for a class of discrete-time stochastic neural networks with random delays has been reported in [3]. Meng et al. [29] obtained a set of conditions for exponential stability of stochastic neural networks with time-varying delays by using the Young inequality, M-matrix technique and the semimartingale convergence theorem. The exponential stability problem for a class of uncertain stochastic neural networks with discrete time-varying delays and unbounded distributed delays has been investigated in [14]. More recently, Tang et al. [18] studied stability analysis problem for a new class of discrete-time neural networks with randomly occurring time delays due to the fact that the time delay may be subject to random changes in environmental circumstances. To the best of our knowledge, no work has been reported on robust stabilization for discrete-time stochastic neural networks with randomly occurring time delays. However, the problems of robust reliable H_∞ stabilization for discrete time neural network with the discrete and distributed delay have not been fully investigated, and there is still room open for further improvements of the stability criteria.

Motivated by this consideration, in this paper we deal with the problem of reliable H_∞ control for stochastic neural network with randomly occurring discrete and distributed time-varying delays in the discrete case. The main objective of this paper is to design a reliable H_∞ controller such that the resulting closed loop form of neural network is robustly asymptotically stable with a desired H_∞ performance level. Also, it is assumed that the neuron activation functions satisfy sector-like nonlinearities. Moreover, the effects of both variation range and probability distribution of time-delays are taken into account in the proposed work. Based on a proper Lyapunov–Krasovskii functional and LMI technique, a robust reliable H_∞ controller is designed for obtaining the required result. The derived results are established in terms of LMI which can be easily calculated by MATLAB-LMI toolbox.

This paper is organized as follows. The discrete-time stochastic neural networks formulation is presented in Section 2. Section 3 proposes a reliable H_∞ control for stochastic uncertain neural networks and robust asymptotic stabilization with known as well as unknown actuator fault. In Section 4, the effectiveness of the proposed control methodology is verified by numerical simulations. The paper is concluded in Section 5.

2. Problem formulation and preliminaries

In this section, we start by introducing some notations and basic results that will be used in this paper. The superscripts T and (-1) stand for matrix transposition and matrix inverse, respectively; $\mathbb{R}^{n\times n}$ denotes the $n\times n$-dimensional Euclidean space; $P>0$ means that P is real, symmetric and positive definite; I and 0 denote the identity and zero matrix with compatible dimensions, respectively; diag$\{\cdot\}$ denotes the block-diagonal matrix; we use an asterisk $(*)$ to represent a term that is induced by symmetry. Moreover, $(\Omega,\mathcal{F},\mathbb{P})$ is a probability space, where Ω is the sample space, $\mathcal{F}$ is the σ-algebra of subsets of the sample space and $\mathbb{P}$ is the probability measure on $\mathcal{F}$ [18]. $\mathbb{E}[\cdot]$ stands for the mathematical expectation operator with respect to the given probability measure $\mathbb{P}$. The notation $\|\cdot\|$ stands for the usual $l_2(0,\infty)$ norm. Matrices which are not explicitly stated are assumed to be compatible for matrix multiplications.

Consider the following n-neuron discrete-time stochastic neural network with discrete and distributed time varying delays with output signal:

$$\begin{aligned} x(k+1) &= \overline{C}x(k)+\overline{A}f(x(k))+\overline{B}g(x(k-\tau(k)))\\ &\quad +\overline{D}\sum_{i=-d(k)}^{-1}h(x(k+i))+\overline{E}u^f(k)\\ &\quad +\overline{F}v(k)+\sigma(k,x(k))w(k),\\ z(k) &= C_1x(k), \end{aligned}\tag{1}$$

where k is the time; $x(k)\in\mathbb{R}^n$ is the state vector; $f(x(k))=[f_1(x_1(k)),f_2(x_2(k)),\ldots,f_n(x_n(k))]^T$, $(x(k))=[g1(x_1(k)),g_2(x_2(k)),\ldots,g_n(x_n(k))]^T$ and $h(x(k))=[h_1(x_1(k)),h_2(x_2(k)),\ldots,h_n(x_n(k))]^T$ are the neuron activation functions; $u^f(k)$ is the control input of actuator fault; $z(k)$ is the output vector; $v(k)\in l_2(0,\infty)$ is the disturbance input vector; $\sigma(.,.):\mathbb{R}\times\mathbb{R}^n\to\mathbb{R}^n$ is the noise intensity function; $w(k)$ is a scalar Wiener process defined on a probability space $(\Omega,\mathbb{F},\mathbb{P})$ with

$$\mathbb{E}[w(k)]=0,\quad \mathbb{E}[w^2(k)]=1,\quad \mathbb{E}[w(i)w(j)]=0,\quad i\neq j.$$

In model (1), $\overline{A}(k)=A+\Delta A(k)$, $\overline{B}(k)=B+\Delta B(k)$, $\overline{C}(k)=C+\Delta C(k)$, $\overline{D}(k)=D+\Delta D(k)$, $\overline{E}(k)=E+\Delta E(k)$ and $\overline{F}(k)=F+\Delta F(k)$, where $C=\text{diag}\{c_i\}$, $i=1,2,\ldots,n$, is the diagonal matrix representing the self-feedback term with $|c_i|<1$; the matrices $A=[a_{ij}]_{n\times n}$, $B=[b_{ij}]_{n\times n}, D=[d_{ij}]_{n\times n}, E=[e_{ij}]_{n\times n}$ and $F=[f_{ij}]_{n\times n}$ are known real constant weight matrices and C_1 is the output matrix. Further, the matrices $\Delta C(k)$, $\Delta A(k)$, $\Delta B(k)$, $\Delta D(k)$, $\Delta E(k)$ and $\Delta F(k)$ represent time varying parameter uncertainties which are defined as follows:

$$[\Delta C(k)\ \Delta A(k)\ \Delta B(k)\ \Delta D(k)\ \Delta E(k)\ \Delta F(k)]=MH(k)[N_1\ N_2\ N_3\ N_4\ N_5\ N_6],$$

where N_1, N_2, N_3, N_4, N_5, N_6 and M are known constant matrices of appropriate dimensions and $H(k)$ are unknown time-varying matrix with Lebesgue measurable elements bounded by $H^T(k)H(k)\le I$.

Remark 2.1. Comparing to the discrete-time neural networks discussed in the literature [3,18], in this paper, without loss of