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Letters

New approach to stability criteria for generalized neural networks with interval time-varying delays

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ABSTRACT

This paper is concerned with the problem of delay-dependent stability of delayed generalized continuous neural networks, which include two classes of fundamental neural networks, i.e., static neural networks and local field neural networks, as their special cases. It is assumed that the state delay belongs to a given interval, which means that the lower bound of delay is not restricted to be zero. An improved integral inequality lemma is proposed to handle the cross-product terms occurred in derivative of constructed Lyapunov–Krasovskii functional. By using the new lemma and delay partitioning method, some less conservative stability criteria are obtained in terms of LMIs. Numerical examples are finally given to illustrate the effectiveness of the proposed method over the existing ones.

1. Introduction

In recent years, neural networks have been widely applied in various fields such as image processing, pattern recognition, associative memory and combinatorial optimization [1–4]. During the implementation of artificial neural networks, time delays naturally occur due to the finite switching speed of amplifiers and might cause oscillation, divergence, and even instability [5–7]. Therefore, the stability of neural networks with time delay has become an important topic. According to whether the neuron states of the neurons are chosen as basic variables to depict the dynamical evolution rule or not, neural networks can be classified as static neural networks and local fields neural networks. These two models can be transferred equivalently from one to the other under some assumptions, but these assumptions cannot always be satisfied in many applications [8]. That is, local field neural network models and static neural network models are not always equivalent. So, it is necessary and important to study them separately. There are many results about the stability of local neural networks [9–35] and static neural networks [36–39]. Recently, some researcher constructed a unified model to combined these two system together [28,29]. For example, in [29], the delay-dependent stability criteria for generalized neural networks with two delay components were studied by using two delay-partitioning, free-weighting matrix and reciprocally convex combination method.

On the other hand, the stability criteria of neural networks are classified into two categories, i.e., delay-dependent stability and delay-independent ones. The delay-dependent stability conditions received much attention, since they are usually less conservative than delay-independent ones, especially when the time delays are relatively small or it varies within an interval. The main aim of delay-dependent stability criteria is to get maximum delay bounds such that the designed networks are asymptotically stable for any delay which is less than the maximum delay bounds. The reduction of the conservativeness mainly affected by two aspects: choosing the Lyapunov–Krasovskii functional and estimating its derivative. Various types of Lyapunov–Krasovskii functional have been constructed to the stability of delayed neural networks, such as discretized Lyapunov–Krasovskii functional [12], delay-partitioning Lyapunov–Krasovskii functional [29], augmented Lyapunov–Krasovskii functional [32,33], and so on. On the other hand, how to obtain the upper bound of its derivative of the Lyapunov–Krasovskii functional is also paly a key role in for deriving the less conservatism stability criteria. For this reason, numerous techniques have been developed for the delayed neural networks, such as free-weighting matrix [31], Jensen inequality [8–39], convex combination technique [26], and so on. However, these methods suffer some common shortcomings: (1) the delay-interval divided into the same size [26,31], which may lead to conservative in some degree. Furthermore, it can be predicted that it will lead to less conservatism if we divide more subintervals in the delay-partitioning Lyapunov–Krasovskii functional. But the reduction of the conservatism tends to inapparent

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as increasing of the number of delay subintervals in some degree, which leads to a large computational burden. (2) Jensen inequality [8–39], which neglect some terms, was utilized to estimate the upper bound of some derivative of Lyapunov–Krasovskii functional. Thus, it is important and necessary to further study the stability of neural networks with time-varying delays, which is one of the motivation of this paper. It is worth to pointed out that, in practice, a time-varying interval delay is often encountered, that is, the lower bound is not restricted zero. Recently, some researchers studied about the stability of neural networks with the interval time varying delay [32–39].

Motivated by the above statement, this paper is concerned with the improved stability criteria for general neural networks with interval time-varying delay. Based on a modified Lyapunov–Krasovskii functional and new integral inequality, some less conservative delay-dependent stability criteria are developed in terms of linear matrix inequalities. It is worth pointing out that all of these criteria are applicable not only to the static neural networks but also to the local neural networks. Two numerical examples are provided to demonstrate the effectiveness and the reduced conservatism of the proposed method.

The main contributions of this paper are summarized as follows:

(1) Inspired by the work [28,29], a unified model, i.e., general neural networks, which include static neural networks and local neural networks, is considered in our work.

(2) The aim is to exploit new methods to achieve less conservative stability criteria. Different with the method considered in [8–39], an improved inequality [41], which can obtain more accurate upper bound than Jensen inequality for dealing with the cross-term, is employed first in delayed neural networks. Furthermore, a new inequality, which is introduced based on the improved inequality [41] and the reciprocally convex approach [40], is used to derive the stability criteria.

(3) For delay-partitioning method, different with the papers considered in [29,31], two delay-partitioning method is employed and the delay-interval does not divide into the same size, which can get less conservative results.

Notations: Throughout this paper, * denotes the elements below the main diagonal of a symmetric block matrix. I denotes the identity matrix with appropriate dimensions, $\mathcal{R}^n$ denotes the n dimensional Euclidean space, and $\mathcal{R}^{m\times n}$ is the set of all $m\times n$ real matrices, $\|\cdot\|$ refers to the Euclidean vector norm and the induced matrix norm. For symmetric matrices A and B, the notation $A>B$ (respectively, $A\ge B$) means that the matrix A–B is positive definite (respectively, nonnegative). $diag\{\dots\}$ denotes the block diagonal matrix.

2. Problem statement

The delayed neural network is described by

$$\dot{y}(t)=-Ay(t)+W_0g(W_2y(t))+W_1g(W_2y(t-h(t)))+J,$$
$$y(t)=\psi(t)\in[-h_2,0] \tag{1}$$

where $y(t)=[y_1(t),y_2(t),\dots,y_n(t)]^T\in\mathcal{R}^n$ is the neuron state vector associated with n neurons, $A=diag\{a_1,a_2,\dots,a_n\}>0$, W_0,W_1 and W_2 are the connection weight matrix, $g(\cdot)=[g_1(\cdot),g_2(\cdot),\dots,g_n(\cdot)]^T\in\mathcal{R}^n$ denotes the continuous activation function, $J=[J_1,J_2,\dots,J_n]^T$ is an exogenous input vector. $\psi(t)$ is the initial condition. $h(t)$ denotes the time-varying delay and satisfies

$$h_1\le h(t)\le h_2,\quad \dot{h}(t)\le\mu, \tag{2}$$

where h,μ are known constants.

The activation function $g_i(\cdot)$ satisfies

$$k_i^-\le\frac{g_i(W_{2i}\xi_1)-g_i(W_{2i}\xi_2)}{W_{2i}\xi_1-W_{2i}\xi_2}\le k_i^+\ (k_i^+>k_i^-),$$
$$g_i(0)=0,\ \ \xi_1,\xi_2\in\mathcal{R},\ \ \xi_1\ne\xi_2,\ \ i=1,2,\dots,n, \tag{3}$$

where k_i^-,k_i^+ are some known constants.

From Brouwer's fixed-point theorem [9], there exists an equilibrium point for the neural networks. Assume that $y^*=[y_1^*,y_2^*,\dots,y_n^*]$ is an equilibrium point of system (1), and using the transformation $x(\cdot)=y(\cdot)-y^*$, (1) can be converted to the following system:

$$\dot{x}(t)=-Ax(t)+W_0f(W_2x(t))+W_1f(W_2x(t-h(t))),$$
$$x(t)=\psi(t)\in[-h_2,0] \tag{4}$$

where $f(s)=[f_1(s)f_2(s)\dots f_n(s)]^T$ and $f(W_2x(t))=g(W_2x(t)+y^*)-g(W_2x(t))$. By (3), we obtain

$$k_i^-\le\frac{f_i(W_{2i}\xi_1)-f_i(W_{2i}\xi_2)}{W_{2i}\xi_1-W_{2i}\xi_2}\le k_i^+,$$
$$f_i(0)=0,\ \ \xi_1,\xi_2\in\mathcal{R},\ \ \xi_1\ne\xi_2,\ \ i=1,2,\dots,n, \tag{5}$$

where k_i^-,k_i^+ are some known constants.

Lemma 2.1 (*Seuret and Gouaisbaut [41]*). *For a given matrix $M>0$, the following inequality holds for all continuously differentiable function $x(t)$ in $[a,b]\in\mathcal{R}^n$:*

$$-(b-a)\int_a^b\dot{x}^T(s)M\dot{x}(s)\,ds\le-[x(b)-x(a)]^TM[x(b)-x(a)]-3\Omega^TM\Omega \tag{6}$$

where $\Omega=x(b)+x(a)-(2/(b-a))\int_a^b x(s)\,ds$.

Lemma 2.2. *For a given matrix $R>0$, $h_m\le h(t)\le h_M$, and any appropriate dimension matrix $\mathcal{X}$, which satisfies $\begin{bmatrix}\overline{R} & \mathcal{X}\\ * & \overline{R}\end{bmatrix}\ge0$. Then, the following inequality holds for all continuously differentiable function $x(t)$:*

$$-(h_M-h_m)\int_{t-h_M}^{t-h_m}\dot{x}^T(s)R\dot{x}(s)\,ds\le-\alpha^T(t)\begin{bmatrix}\overline{R} & \mathcal{X}\\ * & \overline{R}\end{bmatrix}\alpha(t)$$

where $\alpha(t)=[\alpha_1^T(t)\ \alpha_2^T(t)\ \alpha_3^T(t)\ \alpha_4^T(t)]^T$, $\alpha_1(t)=x(t-h_m)-x(t-h(t))$, $\alpha_2(t)=x(t-h_m)+x(t-h(t))-(2/(h(t)-h_m))\int_{t-h(t)}^{t-h_m}x(s)\,ds$, $\alpha_3(t)=x(t-h(t))-x(t-h_M)$, $\alpha_4(t)=x(t-h(t))+x(t-h_M)-(2/(h_M-h(t)))\int_{t-h_M}^{t-h(t)}x(s)\,ds$, $\overline{R}=\begin{bmatrix}R & 0\\ * & 3R\end{bmatrix}$.

Proof. Based on Lemma 2.1, we have

$$\begin{aligned}
&-(h_M-h_m)\int_{t-h_M}^{t-h_m}\dot{x}^T(s)R\dot{x}(s)\,ds\\
&=-(h_M-h_m)\int_{t-h(t)}^{t-h_m}\dot{x}^T(s)R\dot{x}(s)\,ds-(h_M-h_m)\int_{t-h_M}^{t-h(t)}\dot{x}^T(s)R\dot{x}(s)\,ds\\
&\le-\frac{h_M-h_m}{h(t)-h_m}(\alpha_1^T(t)R\alpha_1(t)+3\alpha_2^T(t)R\alpha_2(t))-\frac{h_M-h_m}{h_M-h(t)}(\alpha_3^T(t)R\alpha_3(t)\\
&\quad+3\alpha_4^T(t)R\alpha_4(t))\\
&=-\frac{h_M-h_m}{h(t)-h_m}\begin{bmatrix}\alpha_1(t)\\ \alpha_2(t)\end{bmatrix}^T\overline{R}\begin{bmatrix}\alpha_1(t)\\ \alpha_2(t)\end{bmatrix}-\frac{h_M-h_m}{h_M-h(t)}\begin{bmatrix}\alpha_3(t)\\ \alpha_4(t)\end{bmatrix}^T\overline{R}\begin{bmatrix}\alpha_3(t)\\ \alpha_4(t)\end{bmatrix}\\
&=-\begin{bmatrix}\alpha_1(t)\\ \alpha_2(t)\end{bmatrix}^T\overline{R}\begin{bmatrix}\alpha_1(t)\\ \alpha_2(t)\end{bmatrix}-\frac{h_M-h(t)}{h(t)-h_m}\begin{bmatrix}\alpha_1(t)\\ \alpha_2(t)\end{bmatrix}^T\overline{R}\begin{bmatrix}\alpha_1(t)\\ \alpha_2(t)\end{bmatrix}\\
&\quad-\begin{bmatrix}\alpha_3(t)\\ \alpha_4(t)\end{bmatrix}^T\overline{R}\begin{bmatrix}\alpha_3(t)\\ \alpha_4(t)\end{bmatrix}-\frac{h(t)-h_m}{h_M-h(t)}\begin{bmatrix}\alpha_3(t)\\ \alpha_4(t)\end{bmatrix}^T\overline{R}\begin{bmatrix}\alpha_3(t)\\ \alpha_4(t)\end{bmatrix}.
\end{aligned} \tag{7}$$

If

$$\begin{bmatrix}\overline{R} & \mathcal{X}\\ * & \overline{R}\end{bmatrix}>0,$$