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# Diverting homoclinic chaos in a class of piecewise smooth oscillators to stable periodic orbits using small parametrical perturbations

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ABSTRACT

This paper investigates the mechanisms of small parametrical perturbations in controlling chaos in a class of non-autonomous piecewise smooth oscillators, which describe a large class of nonlinear dynamical systems in the real world. The analytical expressions of two homoclinic orbits of unperturbed piecewise smooth oscillators, which connect the same hyperbolic saddle point are solved analytically. Firstly, when there are no small parametrical perturbations, by using Melnikov's approach, it is rigorously proven that the homoclinic chaos in the Smale horseshoes sense exists when the system's parameters are selected above the threshold for chaos occurrence. Secondly, under the small parametrical perturbations, by using Melnikov's approach, a sufficient criterion is derived, serving as designing the parameters of the control signal, i.e., amplitude and phase position. In the process of computing Melnikov's functions, it is found that the expressions of Melnikov's functions could not be solved analytically because the homoclinic orbits are highly complicated. To this end, a numerical algorithm is proposed. Numerical simulations are presented to verify the theoretical results. The results of this paper can be used to explore the underlying chaotic behaviors of the inertial neural network model.



## 1. Introduction

Since the past decade, chaos control has aroused much interest and its applications have widely been found in various fields such as engineering, biology chemistry, physics and economics [1–4], and so on [29]. One of the major chaos control schemes is the small parametrical perturbations [5] and it belongs to non-feedback control, in which the parameters are selected and modified according to the properly designed control laws. Such a control scheme is attractive since it is based on general properties of chaotic dynamics and it is applicable to a variety of physical systems [6]. However, due to the complexity and diversity of the chaotic systems, it is not easy to find a general approach to control chaos.

It is well known that Melnikov's approach, one of the analytical approaches, has been used to predict the onset of chaotic motions in nonlinear dynamical systems. By measuring the distance between the stable and unstable manifolds of dynamical systems, Melnikov's approach is a powerful tool to find the occurrence of chaos in Hamiltonian systems or near Hamiltonian systems. Moreover, it has been successfully applied to the analysis of chaos in smooth systems [7]. The existence of a simple zero root of the corresponding Melnikov's function implies the occurrence of chaos in the sense of Smale horseshoes. Conversely, if some control terms are added in the original system such that the resultant Melnikov's functions do not have a simple zero root, the chaos in the sense of Smale horseshoes will disappear in this controlled system. In recent years, Melnikov's approach has been used to analyze the control mechanisms of suppressing chaos in the two-dimensional systems by many researchers [15–20,27]. Yagasaki et al. [15,16] investigated the chaos control for a pendulum equation with two external excitations by using the feedback control. It is shown that the chaotic dynamics resulting from transversal intersection between the stable and unstable manifolds can be stabilized to the target saddle-type periodic orbit by using OGY and SOGY methods. Brainman et al. [17] further studied the chaos control for a pendulum equation with two external

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forces by using oscillatory perturbations and provided an effective way to reduce chaos (i.e., by reducing the Lyapunov exponent) or to eliminate chaos even in a deep chaotic state. Baker [15] studied the control of chaotic damped driven pendulum (with one external excitation) by using the OGY method, stabilizing an unstable periodic orbit through a feedback mechanism which periodically adjusts the damping parameter of the pendulum. Yang et al. [19] considered the inhibition conditions of homoclinic and heteroclinic chaos in the pendulum equation for primary and subharmonic resonance. Recently, Yang et al. further investigated the control of chaos in a pendulum equation with parametrical excitations by using Melnikov's approach [20]. However, all the objects in the above-mentioned literatures are based on a class of smooth nonlinear systems.

There are many non-smooth systems in the real world, for example, vibro-impact systems [8], collision dynamics [9], stick–slip motions [10], mechanical systems with dry friction [11], generalized Duffing oscillator with fractional-order deflection [12], and so on. These systems often exhibit very complicated dynamics, such as periodic-adding cascades [13], and non-smooth bifurcations [7,14]. In particular, for a general class of nonlinear impact oscillators, Du et al. [21] found the subharmonic bifurcation by using Melnikov's approach. Awrejcewicz [22] discussed the chaos prediction in non-smooth systems with sliding. Li et al. [23] investigated the Type-II periodic motions for a general class of nonlinear impact oscillators. Cveticanin et al. [12] analyzed the occurrence of chaos by using Melnikov's approach, then applied the delayed feedback control approach to the dynamical systems with fractional-order deflection. According to the recent research [28], in a neural network, there may exist some nerve cells which should be modeled as a second-order nonlinear oscillator. To understand the dynamical properties (including stability, bifurcation and chaos) of such kinds of neural networks, it is the first step to reveal the dynamics of an isolated nerve cell and seek some useful control approaches.

Meanwhile, sinusoidal control signal is a kind of smooth signal source, which has a high degree of accuracy and small waveform distortion, which is easy to be implemented in practice. In addition, based on the Fourier series theorem, almost all the continuous and non-continuous periodic control signals can be expressed in the form of Fourier series.

Combining all the above, for a generalized class of non-autonomous second-order systems with fractional-order deflection, this paper serves the purpose of studying the role of chaos control of small parametrical perturbations using sinusoidal control signal, thus helping to analyze the chaos control by other kinds of signals. Two theorems for chaos occurrence and chaos suppression are proposed using Melnikov's approach. A numerical algorithm is presented for calculating the highly complicated Melnikov's function. A threshold for chaos in such kinds of systems is found and a region for choosing amplitude and phase position of sinusoidal signal is also given. Finally, some numerical simulations are presented to show the correctness of the theoretical results.

The outline of this paper is as follows. In Section 2, the generalized Duffing type oscillator with fractional-order deflection is described and a small sinusoidal parametrical perturbation is introduced to the damping coefficient. In Section 3, the analytical expressions of two homoclinic orbits of the unperturbed system with fractional-order deflection which connect the same hyperbolic saddle point are solved in detail. The critical parameter curve for the existence of chaos in the Smale horse sense is shown and the regions in which the parameters in the sinusoidal signal for controlling chaos to the stable periodic orbits is also found in Section 4. In Section 5, the effectiveness of the proposed method has also been confirmed with numerical simulations. We end our investigation in Section 6 with a brief conclusion.

## 2. Description and analysis of the model

Consider the following generalized Duffing type oscillator with fractional-order deflection:

$$\ddot{x}-ax+bx|x|^{\alpha-1}=\varepsilon(-\delta\dot{x}+\gamma\cos\omega t), \tag{1}$$

where $a>0$ is the linear feedback gain, $b>0$ is the nonlinear feedback gain, $\delta$ is the damping coefficient, $\omega$ and $\gamma$ are the angular frequency and amplitude of the periodic perturbation function, respectively, which model the synthetic effect either from big charges which produce strong anti-electronic force periodically or from errors caused by machinery rotation in the generators. The constant $\alpha>1$ is an integer or a fraction, and $\varepsilon$ is assumed to be a sufficiently small positive constant, i.e., $0<\varepsilon\ll<1$, such that the right term in Eq. (1) can be considered as a perturbation term.

**Remark 1.** Some inertial neural network models [29–31] can be seen as special cases of system (1).

Introducing a new variable $y=\dot{x}$, Eq. (1) can be transformed into the following equivalent form:

$$\begin{cases}\dot{x}=y,\\ \dot{y}=ax-bx|x|^{\alpha-1}+\varepsilon(-\delta y+\gamma\cos\omega t).\end{cases} \tag{2}$$

In some specific range of parameters, system (2) can display chaotic behaviors. In this paper, our motivation is to find the threshold for chaos occurrence and suppress the chaotic behavior to the stable periodic motions by adding a small parametrical perturbation to the damping coefficient $\delta$. As stated in the above, we choose the sinusoidal signal as the parametric perturbation source. After adding the perturbation term, the perturbed system can be described by the following equation:

$$\begin{cases}\dot{x}=y,\\ \dot{y}=ax-bx|x|^{\alpha-1}+\varepsilon(-(\delta+F\sin(\omega t+\Phi))y+\gamma\cos\omega t),\end{cases} \tag{3}$$

where $F$ is the amplitude, $\omega$ is the angular frequency same as that in Eq. (2), and $\Phi$ is the phase position of the sinusoidal control signal.

If $\varepsilon=0$, Eq. (3) is considered as an unperturbed system and can be written as

$$\begin{cases}\dot{x}=y,\\ \dot{y}=ax-bx|x|^{\alpha-1}.\end{cases} \tag{4}$$

Through the analysis of the equilibrium points and their stability for system (4), it is shown that there exist three equilibrium points for $\alpha>1$: $C_{1,2}=(\pm(a/b)^{1/\alpha-1},0)$ being centers and $O=(0,0)$ being a hyperbolic saddle as shown in Fig. 1(a). System (4) is a Hamiltonian system with a Hamiltonian function:

$$H(x,y)=\frac{1}{2}y^2-\frac{a}{2}x^2+\frac{b}{\alpha+1}x^2|x|^{\alpha-1}, \tag{5}$$

and the potential function can be described as

$$V(x)=-\frac{a}{2}x^2+\frac{b}{\alpha+1}x^2|x|^{\alpha-1}. \tag{6}$$

The corresponding potential function $V(x)$ is depicted in Fig. 1(b). In spite of $|x|$ appearing in Eq. (6), the function $V(x)$ is of $C^2$ class at $x=0$ because it is approaching 0 like $(\pm x)^{\alpha+1}$ tends to 0 as $x\to 0$. We call the potential function $V(x)$ as a double-well potential, and its two wells being separated by a potential barrier. The orbits that emerge from and converge to the hyperbolic saddle point $O=(0,0)$ are the homoclinic orbits. Periodic orbits confined to a well evolve inside a homoclinic orbit around one of the centers $C_1$ and $C_2$; those that cross the potential barrier evolve outside the homoclinic orbits. The homoclinic orbits are separatrices between these two types of motions. Hyperbolic saddle point $O=(0,0)$ is connected to itself by two