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Geometric active curve for selective entropy optimization

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ABSTRACT

Recently, with the development of high dimensional large-scale medical imaging devices, the need of fast, robust and accurate segmentation methods is increasing. In this paper, we propose a new level set method (LSM) for image segmentation. The basic idea is to design a selective entropy-based energy functional which is effective and robust against noise, from which we will derive the level set equations and a new selective entropy external forces for the lattice Boltzmann D2Q5 partial differential equation (PDE) solver. The method is accurate and highly parallelizable. The local nature of the lattice Boltzmann method (LBM) allows it to be suitable for fast segmentation methods implemented using some parallel devices such as the graphics processing unit. The proposed algorithm is effective, robust against noise and highly parallelizable. Furthermore, the method can easily be extended to perform an effective image filtering based on Gaussian fuzzy selection. Experimental results on medical images demonstrate subjectively and objectively the performance of the proposed method.

1. Introduction

Segmentation [1] is one of the most important steps in image processing systems. It aims to extract object boundaries in a given scene or to partition a given image into several distinct regions. Due to their advantages, such as the ability to obtain closed contours, the active contours models (ACM) [2–5] have been widely used for contour based image segmentation. They are divided in two representative families. The snakes' models [6] which use the parametric representation of ACM, and the geometric active curve models or level set method (LSM) [7,8] which use the geometric or implicit representation of ACM.

In this paper we use the LSM because they present more advantages than the snakes' model, such as the ability to automatically handle topological changes. The original idea of the LSM was the Hamilton Jacobi approach [9], i.e., a time-dependent equation for a moving surface in the seminal work of Osher and Sethian [10]. In two-dimensional (2D) space, the LSM represents a closed curve in the plane as a zero level set of a three-dimensional (3D) function. For instance, starting with a curve around the object to be detected, the curve moves toward its interior normal and has to stop on the boundary of the object [11–13]. Usually two approaches are employed to stop the evolving curve on the boundary of the desired object. The first one uses an edge indicator depending on the gradient of the image like in classical snakes and active contours models [10]; and the second one uses some regional statistics to stop the evolving curve on the actual boundary [14]. The latter is more robust against noise and can detect objects with weak boundaries and without edges. One of the most used region-based technique was proposed in [15] where Chan and Vese introduced a level set formulation to minimize the Mumford and Shah functional [16] that converted the problem into a mean curvature flow problem like in the active contours but the results are more effective than the classical active contours because the stopping term did not depend on the gradient of the image which reduces the dependency on clear edges.

The LSM has several advantages. For example, it can easily handle complex shapes and topological changes. Nevertheless, the method is computational expensive. The movement of the zero level set is driven by the level set equation (LSE), which is a partial differential equation (PDE). For solving the LSE, most classical methods such as the upwind scheme use some finite difference, finite element or finite volume based approximations and an explicit computation of the curvature of the evolving contour [17]. Unfortunately, these methods are computational intensive.

Lately, the LBM is used as an alternative approach for solving LSE ([14], [18]). It can better handle the problem of time consuming because the curvature is implicitly computed, the algorithm is simple and highly parallelizable. It has only few limits, such as the memory consumption.

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In this paper, entropy minimization framework [19] and the LBM is used to solve a multiphase LSE. We firstly design a selective entropy-based energy functional from which we derive the level set equations, and then the new selective entropy external forces for the LBM-PDE solver [20]. The selective entropy-based energy functional is designed so that it is robust against noise. The basic idea is, instead to use the intensity of all the pixels to compute the entropy inside or outside the evolving curve, we first give a local Gaussian fuzzy membership to be either a background or an object pixel, either an edge or a noise to each pixels. Only the pixels with high membership values, i.e, belonging to the background or an object, are used to compute the entropy. The evolution of the active contour is effectively stopped when the entropy is at minimum. The proposed method combines both the advantages of the LSM and the LBM. It is effective, efficient and robust against noise. The local nature of LBM makes it suitable for massively parallel architectures. An NVIDIA graphics processing unit is used to accelerate the algorithm. Furthermore, we will see that the proposed framework can be easily extended to an effective image filtering method.

This paper is organized as follows. In Section 2 an overview of the LSM and the LBM is presented. Section 3 explains the formulation of the proposed method. Section 4, shows how an extension to image filtering can be done. Section 5 demonstrates the performance of the proposed method through experimental results. The last is the conclusion.

2. Background

The proposed method uses mainly two techniques belonging to different frameworks: the level set method and the lattice Boltzmann method.

2.1. Level set method

The level set method (LSM) is a numerical technique for tracking interfaces and shapes. Using an implicit representation of active contours, it has the advantage of handling automatically topological changes of the tracked shape. In 2D image segmentation, the LSM represents a closed curve as the zero level set of a given level set function (LSF) ϕ. The active curve evolution starts from an arbitrary contour, and is driven by the level set equation (LSE), which is a convection–diffusion equation expressed as

$$\frac{\partial \phi}{\partial t}+\vec{V}\nabla\phi=\eta\Delta\phi, \tag{1}$$

where $\nabla\phi$ and $\Delta\phi$ are respectively the gradient and the Laplacian of ϕ. The term $\eta\Delta\phi$ is called artificial viscosity. It is suggested to replace it with $\eta k|\nabla\phi|$ which can better handle the evolution of lower dimensional interfaces [21], with k is the curvature of the LSF. The LSE can therefore be rewritten as

$$\frac{\partial \phi}{\partial t}+\vec{V}\nabla\phi=\eta k|\nabla\phi| \tag{2}$$

As an alternative method for solving PDEs, the LBM has several advantages such as parallelizability and simplicity.

2.2. Lattice Boltzmann method

The LBM is firstly designed to simulate Navier–Stokes equations for an incompressible fluid ([20,22,23]). The evolution equation of LBM is

$$f_i(\vec{r}+\vec{e}_i,t+1)-f_i(\vec{r},t)=\left(\frac{\partial f}{\partial t}\right)_{coll}, \tag{3}$$

where t is the time, f_i is the particle distribution function and $(\partial f/\partial t)_{coll}$ is the Bhatnager–Gross–Krook (BGK) collision model [24–27] with a body force $\vec{F}$.

$$\left(\frac{\partial f}{\partial t}\right)_{coll}=\frac{1}{\tau}[f_i^{eq}(\vec{r},t)-f_i(\vec{r},t)]+\frac{D}{bc^2}\vec{F}\,\vec{e}_i, \tag{4}$$

where D is the grid dimension, b is the link at each grid point, c is the length of each link which is set to 1 in this paper, τ represents the relaxation time and f_i^{eq} the local Maxwell–Boltzmann equilibrium particle distribution function expressed in its continuous form as

$$f^{eq}=\rho(2\pi RT)^{-3/2}\exp[-(\vec{\upsilon}-\vec{u})^2/2RT], \tag{5}$$

where $\vec{\upsilon}$ is the particle velocity and $\vec{u}$ the macroscopic velocity. The equilibrium distribution can be expressed in discrete form as follows when modeling typical diffusion phenomenon,

$$f_i^{eq}(\rho)=\rho A_i \quad \text{with } \rho=\sum_i f_i,, \tag{6}$$

where ρ is the macroscopic fluid density. By performing the Chapman–Enskog expansion [28], the following diffusion equation can be recovered from LBM [20],

$$\frac{\partial \rho}{\partial t}=\beta\,\mathrm{div}(\nabla\rho)+F. \tag{7}$$

Substituting ρ by the signed distance function ϕ in Eq. (7), the LSE can be recovered. Thus LBM can be used to solve the level set equation. Eq. (4) is used to recover f_i and then Eq. (6) is used to update the signed distance function ϕ. In our model we use the D2Q5 ($D=2$, $b=5$) LBM lattice structure. The body force F acts as the image data link for the LBM solver. Fig. 1 displays a typical D2Q5 model where each link has its velocity vector $e_i(\vec{r},t)$.and $\vec{r}$ is the position of the cell.

Ref. [21] used another approach to perform the level set image segmentation. Eq. (3) is the general evolution equation of LBM. However, in level set based image segmentation a stop function $g(\vec{r})$ is necessary for stopping the evolving curve or surface at the boundaries of the object. In order to introduce the stop function into LBM, they considered a medium between the nodes of the lattice. The particles can pass through the medium with a possibility of $g_i(\vec{r})$, and will be punched back where they were with a possibility of $1-g_i(\vec{r})$. The LBM evolution equation is modified as

$$f_i(\vec{r}+\vec{e}_i,t+1)=g_i(\vec{r})[f_i(\vec{r},t)+\frac{1}{\tau}[f_i^{eq}(\vec{r},t)-f_i(\vec{r},t)]+\sigma] \\ +(1-g_i(\vec{r}))f_i(\vec{r}+\vec{e}_i,t), \tag{8}$$

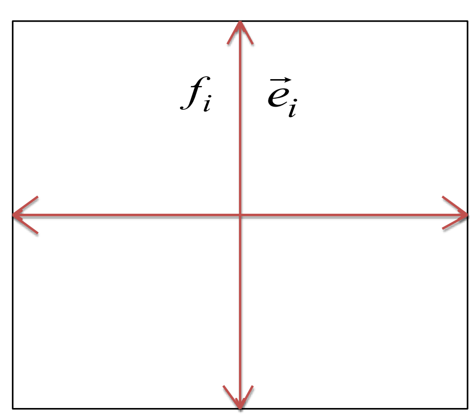

Fig. 1. Spatial structure of the D2Q5 LBM lattice.