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Letters

Finite-time lag synchronization of delayed neural networks

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ABSTRACT

In this paper, a finite-time lag synchronization of coupled neural networks with time delay is investigated. By means of the Lyapunov stability theory, a feedback controller is designed for achieving lag synchronization between two delayed neural networks systems in finite time. Paper extends some basic results from the area of finite time to time-delay systems. Numerical simulations on coupled Lu neural systems illustrate the effectiveness of the results.

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1. Introduction

Since the seminal works of Pecora and Carroll [1,2], synchronization of chaotic systems has been extensively investigated due to their successful application in many areas, such as communication, modeling brain activity, signal processing, combinatorial optimization and so on. Different kinds of synchronization including complete, lag, projective, generalized, phase, and anticipated synchronization have been proposed [1–9]. In lag synchronization, there is an exact time shift between master and slave systems [10–12]. In Ref. [10], the authors proposed the lag synchronization of Cohen–Grossberg neural networks with discrete delays. The lag synchronization problem of fuzzy cellular neural networks with time-varying delays was obtained using a nonlinear measure method [11]. In Ref. [12], the authors studied the lag synchronization of 3D chaotic delayed neural networks via impulsive control. Several new impulsive control laws were obtained by using the stability theory of impulsive functional differential. Recently, we investigated the lag synchronization of coupled delayed systems with parameter mismatches [13]. We also estimated the error bound of the lag synchronization by rigorous theoretical analysis.

On the other hand, in some practical situations, stabilization and synchronization should be achieved in finite time. So, it is necessary to make a study for finite-time synchronization. Some authors have investigated synchronization based on finite-time stability theory [14,15]. The finite-time synchronization of dynamical networks with complex-variable chaotic systems is investigated [14]. In Ref. [15], the authors showed how to obtain finite-time stabilization of linear systems with delays in the input by using an extension of Artstein's model reduction to nonlinear feedback. In Ref. [16], the authors studied the finite-time synchronization problem for linearly coupled complex networks with discontinuous non-identical nodes. In Ref. [17], the authors proposed the finite-time synchronization between two complex networks with non-delayed and delayed coupling by using the impulsive control and the periodically intermittent control. The authors of [18] investigated bounded synchronization of coupled discrete time varying stochastic complex networks over a finite horizon. They designed a state estimator to estimate the network states such that the dynamics of the estimation error was bounded in an H_∞ sense. To the best of our knowledge, there are few reports on the program of finite-time synchronization of delayed neural networks. In this paper, we shall deal with the analysis issue for finite-time lag synchronization of neural networks with time delay. A general theoretical result involving the Lyapunov functional gives a general sufficient condition for the finite-time synchronization of delayed neural networks and two examples are addressed.

The rest of the paper is organized as follows. In the next section, we formulate the problem of lag synchronization of coupled neural networks. In Section 3, a general scheme for the finite-time lag synchronization is presented. Numerical simulations are given in Section 4. Finally, conclusions are given in Section 5.

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2. Problem formulation and preliminaries

In this paper, we consider the chaotic cellular neural networks described by

$$\begin{cases} \dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(x_j(t-\tau)), & i = 1, 2, 3, \ldots, n; \quad t > 0, \\ x_i(t) = \varphi_i(t), & -\tau \le t \le 0, \end{cases} \tag{1}$$

Or, in a compact form

$$\begin{cases} \dot{x}(t) = Cx(t) + Af(x(t)) + Bg(x(t-\tau)), & t > 0, \\ x(t) = \varphi(t), & -\tau \le t \le 0, \end{cases} \tag{2}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in R^n$ denotes the state vector, C, A, and $B \in R^{m \times m}$ are constant matrices, $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots f_n(x_n(t))]^T \in R^n$, $g(x(t)) = [g_1(x_1(t)), g_2(x_2(t)), \ldots g_n(x_n(t))]^T \in R^n$, τ is the time delay, and $f, g : R^m \to R^m$ are nonlinear functions satisfying the Lipstchiz condition, namely, there exist positive constants L_f, L_g such that, for all x_i, $y_i \in R^n$,

$$|f_i(x_i) - f_i(y_i)| \le L_f |x_i - y_i|, \quad |g_i(x_i) - g_i(y_i)| \le L_g |x_i - y_i|.$$

Consider the corresponding slave system given in the following form:

$$\begin{cases} \dot{y}(t) = Cy(t) + Af(y(t)) + Bg(y(t-\tau)) + u(t), & t > 0, \\ y(t) = \psi(t), & -\tau \le t \le 0, \end{cases} \tag{3}$$

where $y(t) \in R^n$ denotes the state vector, C, A, and $B \in R^{n \times n}$ are constant matrices, and $u(t)$ denotes the feedback control defined as follows:

$$u(t) = -k_1(y(t) - x(t-\theta)) - \lambda sign(y(t) - x(t-\theta)) |(y(t) - x(t-\theta))|^{\eta}$$
$$- \lambda \left(\int_{t-\tau}^{t} (y(s) - x(s-\theta))^T (y(s) - x(s-\theta)) d_s \right)^{\frac{1+\eta}{2}} \left(\frac{y(t) - x(t-\theta)}{\|y(t) - x(t-\theta)\|^2} \right) \tag{4}$$

where $|(y(t) - x(t-\theta))|^{\eta} = (|(y_1(t) - x_1(t-\theta))|^{\eta}, \ldots, |(y_n(t) - x_n(t-\theta))|^{\eta})^T$, $\lambda > 0$, $0 < \eta < 1$ are all constants, k_1 denotes control strength, θ be the transmittal delay, and

$$sign(y(t) - x(t-\theta)) = diag(sign(y_1(t) - x_1(t-\theta)), \ldots, sign(y_n(t) - x_n(t-\theta))).$$

Defining the lag synchronization error between systems (2) and (3) as $e(t) = y(t) - x(t-\theta)$, we have the following error dynamical system:

$$\begin{aligned} \dot{e}(t) &= \dot{y}(t) - \dot{x}(t-\theta) \\ &= Cy(t) + Af(y(t)) + Bg(y(t-\tau)) + u(t) \\ &\quad - (Cx(t-\theta) + Af(x(t-\theta)) + Bg(x(t-\tau-\theta))) \\ &= Ce(t) + Af(y(t)) - Af(x(t-\theta)) + Bg(y(t-\tau)) - Bg(x(t-\tau-\theta)) \\ &\quad - k_1(y(t) - x(t-\theta)) - \lambda sign(y(t) - x(t-\theta)) |(y(t) - x(t-\theta))|^{\eta} \\ &\quad - \lambda \left(\int_{t-\tau}^{t} e(s)^T e(s) d_s \right)^{\frac{1+\eta}{2}} \left(\frac{e(t)}{\|e(t)\|^2} \right). \end{aligned} \tag{5}$$

Definition 1. The master system (2) and the slave system (3) are said to be lag synchronization in finite time if there exists a constant $T > 0$such that

$$\lim_{t \to T} \|e(t)\| = \lim_{t \to T} \|y(t) - x(t-\theta)\| = 0 \text{ and } \|e(t)\| = 0 \text{ if } t > T. \tag{6}$$

Lemma 1. *Assume that a continuous, positive definite function $V(t)$ satisfies the following differential inequality:*

$$\dot{V}(t) \le -\lambda V^{\eta}(t), \quad \forall t \ge t_0, \ V(t_0) \ge 0, \tag{7}$$

where $\lambda > 0$, $0 < \eta < 1$are all constants. Then, for any given t_0, $V(t)$ satisfies the following inequality:

$$V^{1-\eta}(t) \le V^{1-\eta}(t_0) - \lambda(1-\eta)(t-t_0), \ t_0 \le t \le t_1, \tag{8}$$

and $V(t) = 0$, $\forall t \ge T$, with t_1 given by

$$T = t_0 + \frac{V^{1-\eta}(t_0)}{\lambda(1-\eta)}. \tag{9}$$

Lemma 2. *([19]): Given any real matrices Σ_1,Σ_2,$\Sigma_3$of appropriate dimensions and a scalar $s > 0$, such that $0 < \Sigma_3 = {\Sigma_3}^T$. Then the following inequality holds:*

$${\Sigma_1}^T \Sigma_2 + {\Sigma_2}^T \Sigma_1 \le s {\Sigma_1}^T \Sigma_3 \Sigma_1 + s^{-1} {\Sigma_2}^T {\Sigma_3}^{-1} \Sigma_2.$$

We now state our main results

3. Finite-time lag synchronization

This section addresses the finite-time lag synchronization problem of coupled neural networks.

Theorem 1. *Suppose that there exist constants s_1,s_2, the coupling strength k_1,k_2, time delay θ such that*

(i) $.C + C^T - 2k_1 I + s_1 AA^T + {s_1}^{-1} L_f + s_2 BB^T + k_2 I \le 0$;
(ii) $.{s_2}^{-1} L_g - k_2 \le 0$.

Then, the lag synchronization error system $e(t)$ is globally finite-time stability, and the lag synchronization between system (2) and system (3) can be obtained in a finite time, and the finite time is estimated by

$$T = t_0 + V^{1-(1+\eta)/2}(t_0)/2\lambda(1-(1+\eta)/2)$$

Proof. Consider the following Lyapunov function:

$$V(t) = e(t)^T e(t) + \int_{t-\tau}^{t} k_2 e(s)^T e(s) d_s. \tag{10}$$

The derivative of Eq. (10) with respect to time t along the trajectories of system (5) is calculated and estimated as follows:

$$\begin{aligned} \dot{V}(t) &= e(t)^T \dot{e}(t) + \dot{e}(t)^T e(t) + k_2 e(t)^T e(t) - k_2 e(t-\tau)^T e(t-\tau) \\ &= e(t)^T \Big[Cy(t) + Af(y(t)) + Bg(y(t-\tau)) - Cx(t-\theta) - Af(x(t-\theta)) \\ &\quad - Bg(x(t-\tau-\theta)) - k_1 e(t) - \lambda \left(\int_{t-\tau}^{t} e(s)^T e(s) d_s \right)^{1+\eta/2} \left(\frac{e(t)}{\|e(t)\|^2} \right) \\ &\quad - \lambda sign(e(t)) |e(t)|^{\eta} \Big] + \Big[Cy(t) + Af(y(t)) + Bg(y(t-\tau)) \\ &\quad - Cx(t-\theta) - Af(x(t-\theta)) - Bg(x(t-\tau-\theta)) \\ &\quad - k_1 e(t) - \lambda \left(\int_{t-\tau}^{t} e(s)^T e(s) d_s \right)^{1+\eta/2} \left(\frac{e(t)}{\|e(t)\|^2} \right) \\ &\quad - \lambda sign(e(t)) |e(t)|^{\eta} \Big]^T e(t) \\ &\quad + k_2 e(t)^T e(t) - k_2 e(t-\tau)^T e(t-\tau) \\ &= e(t)^T [C + C^T - 2k_1 I] e(t) + e(t)^T [Af(y(t)) - Af(x(t-\theta))] \\ &\quad + [Af(y(t)) - Af(x(t-\theta))]^T e(t) + e(t)^T [Bg(y(t-\tau)) \\ &\quad - Bg(x(t-\tau-\theta))] + [Bg(y(t-\tau)) - Bg(x(t-\tau-\theta))]^T e(t) \\ &\quad - \lambda e(t)^T sign(e(t)) |e(t)|^{\eta} - \lambda [sign(e(t)) |e(t)|^{\eta}]^T e(t) \\ &\quad - 2\lambda \left(\int_{t-\tau}^{t} e(s)^T e(s) d_s \right)^{1+\eta/2} \left(\frac{e(t)^T e(t)}{\|e(t)\|^2} \right) \\ &\quad + k_2 e(t)^T e(t) - k_2 e(t-\tau)^T e(t-\tau) \end{aligned}$$

Using Lemma 2, we have the following estimation:

$$e(t)^T [Af(y(t)) - Af(x(t-\theta))] + [Af(y(t)) - Af(x(t-\theta))]^T e(t)$$

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